Van der Waals interactions for $1S$-bottomonium

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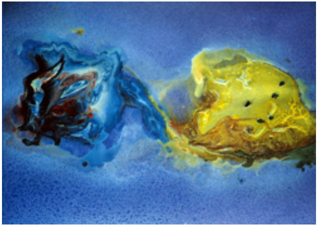

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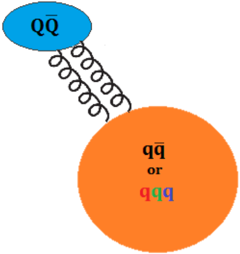
Motivation: color van der Waals potential



- A proposed multiquark exotic quarkonium picture is that of a compact quarkonium state bound to a light–quark hadron.
- A similarly picture may explain the recently discovered pentaquark resonances $P_c^+(4380)$ and $P_c^+(4450)$.
- The binding of J/ψ to nuclear matter.
- Quarkonium-quarkonium bound states.

1S–Bottomonium

- It can be described perturbatively in pNRQCD.
- It is color neutral and has no multipolar moments.
- The two–1S–Bottomonium system presents a set of well separated scales.
- We can sequentially integrate out the different scales and build the EFT most adequate at each scale.

Potential Non–Relativistic QCD

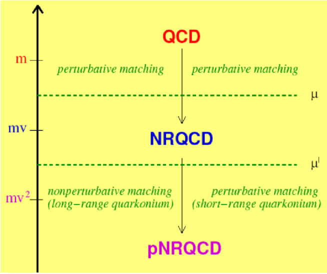
Motivation

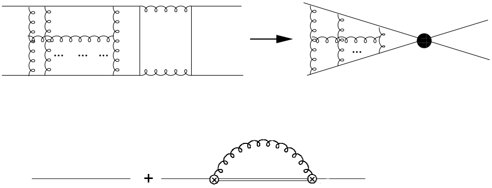
- Quarkonium systems are non–relativistic bound states.
- **Multiscale system**: $m \gg mv \gg mv^2$, and Λ_{QCD}. m is the heavy–quark mass, $v \ll 1$ the heavy quark velocity.
- We can exploit the **scale hierarchies** by building an **Effective Field Theory** (EFT).

Matching procedure

- Integrating out the m scale leads to the well known NRQCD. Caswell, Lepage 1986; Bodwin, Braaten and Lepage 1995. Since $m \gg \Lambda_{QCD}$ the matching is perturbative.
- The degrees of freedom in pNRQCD are a **color singlet** (S) and **octet fields** (O^a) and the ultrasoft gluons.
- $\boldsymbol{R}$ is the CoM coordinate and $\boldsymbol{r}$ the relative coordinate of the quark pair.
- **Multipole expansion**: Since $r \sim 1/mv$, integrating out mv implies a multipole expansion of the gluon fields.

Potential Non–Relativistic QCD





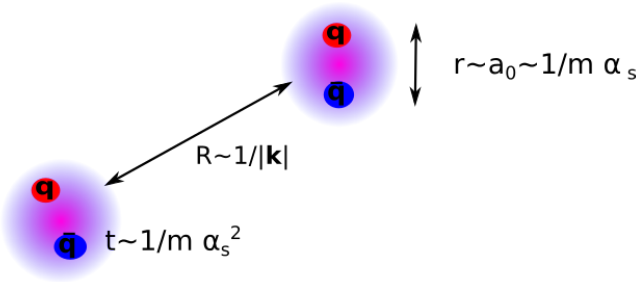
Weakly-coupled pNRQCD Lagrangian

- Weakly-coupled pNRQCD is valid for short distances: $mv \sim 1/r \gg \Lambda_{QCD}$.

$$\mathcal{L}_{\rm pNRQCD} = \int d^3r\, {\rm Tr}\left[S^\dagger \left(i\partial_0 + \frac{\boldsymbol{\nabla}_r^2}{M} - V_s(r) \right) S + O^\dagger \left(iD_0 + \frac{\boldsymbol{\nabla}_r^2}{M} - V_o(r) \right) O \right]$$
$$+ gV_A(r){\rm Tr}\left[O^\dagger \boldsymbol{r}\cdot\boldsymbol{E} S + S^\dagger \boldsymbol{r}\cdot\boldsymbol{E} O \right] + \frac{g}{2} V_B(r){\rm Tr}\left[O^\dagger \boldsymbol{r}\cdot\boldsymbol{E} O + O^\dagger O \boldsymbol{r}\cdot\boldsymbol{E} \right]$$
$$-\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu\, a} + \sum_{i=1}^{N_f} \bar{q}_i i \not{D} q_i$$

Pineda, Soto 1998; Brambilla, Pineda, Soto, Vairo 2000

Scales of the system



One $1S$-bottomonium system

- $mv \gg \Lambda_{QCD} \Rightarrow$ Coulombic state.
- $v \sim \alpha_s \ll 1$.
- Descrived by weakly coupled pNRQCD.

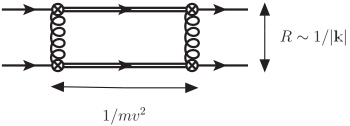
Two $1S$-bottomonium system

- New scale: $|\boldsymbol{k}| \sim 1/R \ll mv$
- Different regimes depending on the relative sizes of $|\boldsymbol{k}|$, mv^2 and Λ_{QCD}.

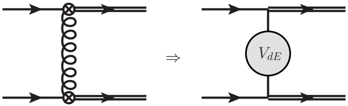
Perturbative case:

$$|\boldsymbol{R}| \ll \frac{1}{\Lambda_{QCD}}$$

London regime



- The London regime is characterized by $mv \gg |\boldsymbol{k}| \gg mv^2$.
- The gluons are exchanged nearly instantaneously.



- We integrate out $|\boldsymbol{k}|$
- The one-gluon exchange can be matched into a chromo-electric dipole-dipole potential in the pNRQCD Lagrangian.

Chromo-electric dipole-dipole potential

$$L_{pNRQCD} = -\frac{1}{2}\int d^3\boldsymbol{R_1} d^3\boldsymbol{R_2} d^3\boldsymbol{r_1} d^3\boldsymbol{r_2} S^\dagger O(t, \boldsymbol{r_1}, \boldsymbol{R_1})\, V_{dE}(\boldsymbol{R_1} - \boldsymbol{R_2})\, S^\dagger O(t, \boldsymbol{r_2}, \boldsymbol{R_2}) + \text{h.c.}$$

$$\widetilde{V}_{dE}(\boldsymbol{q}) = -g^2 \frac{\boldsymbol{r}_1 \cdot \boldsymbol{q} \boldsymbol{r}_2 \cdot \boldsymbol{q}}{\boldsymbol{q}^2}$$

London regime

Van der Waals EFT for $1S$-bottomonia

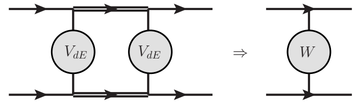
- We can define a van der Waals EFT at the energy scale of the dynamics of two $1S$-bottomonia $E \sim \boldsymbol{k}^2/m_\phi \ll mv^2$
- Since we ignore the spin we represent both η_b and $\Upsilon(1S)$ by 0^{-+} field ϕ

$$L^{\phi}_{\text{WEFT}} = \int d^3\boldsymbol{R}\, \phi^\dagger(t,\boldsymbol{R}) \left(i\partial_0 + \frac{\boldsymbol{\nabla}^2}{2m_\phi}\right) \phi(t,\boldsymbol{R})$$

$$L^{\phi\phi}_{\text{WEFT}} = -\frac{1}{2}\int d^3\boldsymbol{R}_1 d^3\boldsymbol{R}_2\, \phi^\dagger\phi(t,\boldsymbol{R}_1)\, W(\boldsymbol{R}_1,\boldsymbol{R}_2)\, \phi^\dagger\phi(t,\boldsymbol{R}_2)$$

London potential

• Matching pNRQCD to WEFT at LO we obtain



$$W_{London}(\boldsymbol{R}) = -\frac{3\alpha_s^2 w}{2\boldsymbol{R}^6}$$

London regime

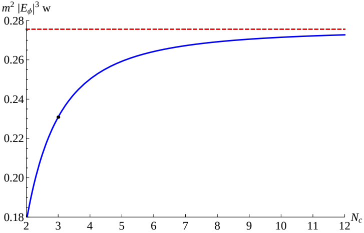
$$w = i\frac{(N_c^2-1)}{9N_c^2}\langle\phi_1\phi_2|\int\frac{dq^0}{2\pi}\boldsymbol{r}_1^i\frac{1}{q^0-E_\phi+h_{o,1}-i\epsilon}\boldsymbol{r}_1^i\boldsymbol{r}_2^j\frac{1}{q^0+E_\phi-h_{o,2}+i\epsilon}\boldsymbol{r}_2^j|\phi_1\phi_2\rangle$$

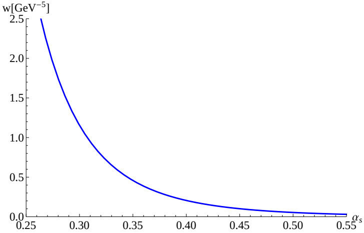
- Intermediate octet states are given by Coulombic continuum wavefunctions.

$$w = \frac{2^{16}\rho(\rho+2)^4}{3^2N_c^2}\frac{1}{m^2\left|E_\phi\right|^3}I_L$$

From α_s(1 GeV) to α_s(2 GeV) we obtain

$$w = 0.37^{+0.49}_{-0.29}\ \mathrm{GeV}^{-5}$$

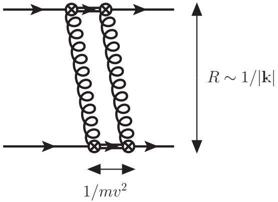




$$I_L = \int_0^\infty dp dl\, p^3 l^3 \frac{\left(1+\rho^2/p^2\right)\left(1+\rho^2/l^2\right)e^{4\rho/p\arctan p+4\rho/l\arctan l}}{\left(e^{2\pi\rho/p}-1\right)\left(1+p^2\right)^6\left(e^{2\pi\rho/l}-1\right)\left(1+l^2\right)^6\left(2+p^2+l^2\right)}\,,\quad \rho = 1/(N_c^2-1)$$

Preliminary!

Casimir-Polder regime



- Casimir-Polder regime is characterized by the hierarchy of scales $mv^2 \gg |\boldsymbol{k}|$
- The gluons are emitted nearly at the same time.
- Retardation effects have to be incorporated.

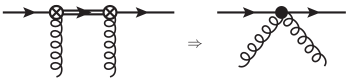
• We integrate out mv^2 and match pNRQCD to gWEFT

Gluonic van der Waals EFT

$$L_{\rm gWEFT} = \int d^3\boldsymbol{R} \left\{ \phi^\dagger(t,\boldsymbol{R}) \left[i\partial_0 - E_\phi + \frac{\boldsymbol{\nabla}^2_{\boldsymbol{R}}}{4m} + \frac{1}{2}\beta g^2 \boldsymbol{E}^2_a + \cdots \right] \phi(t,\boldsymbol{R}) \right\}$$

- Defined at the energy scale $E \sim |\boldsymbol{k}|$
- ϕ is a 0^{-+} field representing $1S$-bottomonium states with binding energy E_ϕ.
- β is the $1S$-bottomonium chromo-polarizability.

Tree–level matching: Polarizability

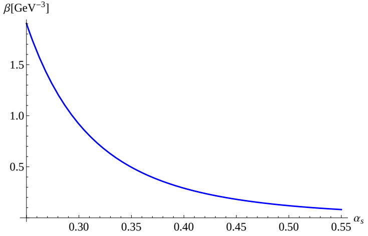


- Matching the two gluon emission in pNRQCD and gWEFT we obtain the polarizability.
- Intermediate octet states are given by Coulombic continuum wavefunctions.

$$\beta = -\frac{1}{3N_c}\langle\phi|\boldsymbol{r}\frac{1}{E_\phi - h_o}\boldsymbol{r}|\phi\rangle = 256\frac{\rho(\rho+2)^2}{3N_c}\frac{1}{mE_\phi^2}I_{CP}$$

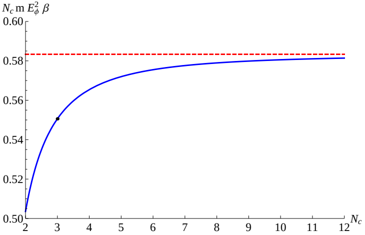
$$I_{CP} = \int_0^\infty dp\, p^3 \frac{\left(1+\rho^2/p^2\right) e^{4\rho/p \arctan p}}{\left(e^{2\pi\rho/p}-1\right)(1+p^2)^7}$$

$$\rho = 1/(N_c^2-1)$$



From $\alpha_s(1\text{ GeV})$ to $\alpha_s(2\text{ GeV})$ we obtain

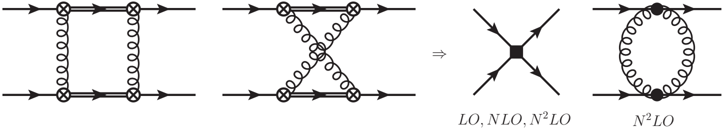
$$\beta = 0.50^{+0.42}_{-0.38}\ \text{GeV}^{-3}$$



∗ Black dot $N_c = 3$ Leutwyler 1981; Voloshin 1982

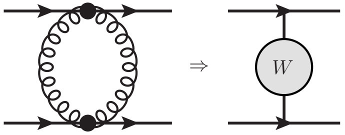
∗ Dashed line: large N_c limit Peskin 1979; Bhanot, Peskin 1979

One-loop matching: Contact terms



- Matching the 4-point functions in pNRQCD and gWEFT we find contributions to contact terms.
- Origin in pNRQCD loop momentum region $l^0 \sim |\boldsymbol{l}| \sim mv^2$.
- Contact terms are parametrically larger than the two-gluon exchange in gWEFT.
- Contact terms correspond to Dirac-delta potential and its derivatives.
- The N^{2}LO contact term cancels out the of the loop diagram in gWEFT.

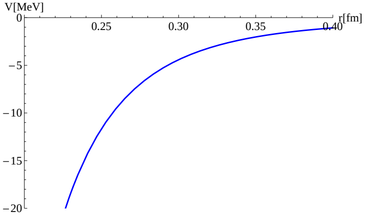
Casimir-Polder regime



$$W_{\text{C-P}} = -\frac{23(N_c^2-1)\alpha_s^2(\nu)\beta^2}{4\pi R^7} + \text{(local pieces)}$$

- R^{-7} dependence as in two photon exchange at long range Casimir, Polder 1948; Feinberg, Sucher 1970.
- Different coefficient from Fuji, Mima 1978 obtained.

- We integrate out $|\boldsymbol{k}|$ scale to go from gWEFT to WEFT.
- The van der Waals potential can be computed as a matching coefficient.



Using $\beta = 0.12\ \text{GeV}^{-3}$

Nonperturbative case:

$$|\boldsymbol{R}| \gg \frac{1}{\Lambda_{QCD}}$$

Nonperturbative regime

• At energies much below Λ_{QCD} the degrees of freedom are the Goldstone bosons and the $1S$-bottomonium states.

χEFT

$$\mathcal{L}^{\phi}_{\chi\text{EFT}} = \phi^\dagger\left(i\partial_0 + \frac{\boldsymbol{\nabla}^2}{2m_\phi}\right)\phi\,,$$

$$\mathcal{L}^{\pi}_{\chi\text{EFT}} = \frac{F^2}{4}\left\{\text{Tr}\left[\partial_\mu U \partial^\mu U^\dagger\right] + \text{Tr}\left[\chi^\dagger U + \chi U^\dagger\right]\right\}\,,$$

$$\mathcal{L}^{\phi-\pi}_{\chi\text{EFT}} = \phi^\dagger\phi\frac{F^2}{4}\left\{c_{d0}\text{Tr}\left[\partial_0 U\partial_0 U^\dagger\right] + c_{di}\text{Tr}\left[\partial_i U\partial^i U^\dagger\right] + c_m\text{Tr}\left[\chi^\dagger U + \chi U^\dagger\right]\right\}\,,$$

$$U = e^{i\Phi/F}\,,\quad \Phi = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}\,,\ \chi = m_\pi^2\mathbf{1}$$

- Defined at the energy scale $E \sim m_\pi$.
- ϕ is a 0^{-+} field representing $1S$-bottomonium states with mas m_ϕ.
- ϕ is a scalar under chiral symmetry.
- c_{d0}, c_{di} and c_m are matching coefficients.

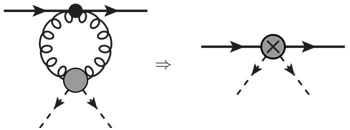
Matching of gWEFT to χEFT using the QCD trace anomaly

- The matrix element of the QCD energy–momentum tensor $\theta_{\mu\nu}$ over pions $\langle \pi^+\pi^- | \theta_{\mu\nu} | 0 \rangle$ is fully determined by symmetry considerations, on-mass shell conditions, normalization and the Adler zero.
- κ parametrizes is the part of $\langle \pi^+\pi^- | G_{\mu\nu} G_{\lambda\sigma} | 0 \rangle$ that is not determined by the anomaly of the trace.

$$g^2 \langle \pi^+(p_1)\pi^-(p_2) | \boldsymbol{E}_a^2 | 0 \rangle = \frac{8\pi^2}{b} \left(\kappa_1 p_1^0 p_2^0 - \kappa_2 p_1^i p_2^i + 3m_\pi^2 \right)$$

with $\kappa_1 = 2 - 9\kappa/2$, $\kappa_2 = 2 + 3\kappa/2$ Chanowitz, Ellis 1972, 1973; Crewther 1972; Freedman, Muzinich, Weinberg 1974; Collins, Duncan, Joglekar 1977; Voloshin, Zakharov 1980; Novikov, Shifman 1981; Voloshin 2008

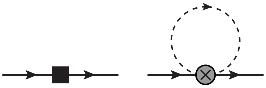
- κ can be obtained from pionic transitions of quarkonium states ($\kappa \sim 0.2$).
- We integrate out Λ_{QCD} and match gWEFT to a χEFT.



$$c_{d0} = -\frac{4\pi^2\beta}{b}\kappa_1 \,, \; c_{di} = -\frac{4\pi^2\beta}{b}\kappa_2 \,, \; c_m = -\frac{12\pi^2\beta}{b}$$

Leading chiral log of the $1S$-bottomonium mass

- Since the LEC π-ϕ sector are determined, a prediction of the leading chiral log for the $1S$-bottomonium mass is possible.



$$\delta m_\phi = -F^2 c_m m_\pi^2 + \text{counterterms of } \mathcal{O}(m_\pi^4) + \frac{3m_\pi^2}{8}(c_{d0} + 3c_{di} - 4c_m)\, A[m_\pi^2] + \frac{3m_\pi^4\,(c_{d0} - c_{di})}{256\pi^2}$$

$$\delta m_\phi|_{\text{chiral log}} = -\frac{3}{8}\frac{\beta}{b} m_\pi^4 \log\frac{m_\pi^2}{\nu^2}$$

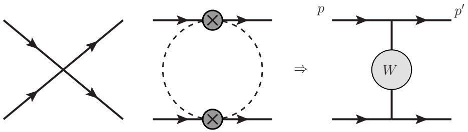
- Extra counterterms needed to renormalize the self-energy amplitude.
- Independent of κ.
- We disagree from the previous result of Grinstein, Rothstein 1996.

Van der Waals potential in the nonperturbative regime

- Since $R \sim 1/m_\pi$, $m_\pi^2/(m_\phi) \ll m_\pi$, i.e, the pion dynamics takes place at a higher energies scale than $1S$–bottomonium dynamics.
- Van der Waals potential is defined at the level of WEFT.

$$\mathcal{L}_{\rm WEFT} = \phi^\dagger \left(i\partial_0 - \frac{\boldsymbol{\nabla}^2}{2m_\phi} \right) \phi - \frac{1}{2} \int d^3\boldsymbol{R}_1 d^3\boldsymbol{R}_2\, \phi^\dagger \phi(t, \boldsymbol{R}_1)\, W(\boldsymbol{R}_1 - \boldsymbol{R}_2)\, \phi^\dagger \phi(t, \boldsymbol{R}_2)$$

- The last step is to integrate out the m_π scale and match χEFT to WEFT.



- The two pion exchange is a N^{2}LO contribution to the $\phi - \phi$ scattering amplitude.
- However the two pion exchange is the leading non-local term in the amplitude.

Van der Waals potential N. Brambilla, G. Krein, J.T., A. Vairo. arXiv:1510.05895

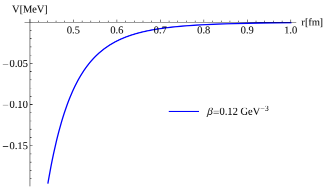
- To obtain a coordinate space representation of the potential we use a dispersive representation using the Cauchy theorem.

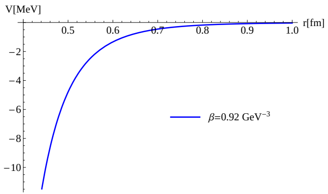
$$W(\boldsymbol{R}) = \frac{1}{2\pi^2 R} \int_{2m_\pi}^{\infty} d\mu \, e^{-\mu R} \mu \, \mathrm{Im} \left[\widetilde{W}(\epsilon - i\mu) \right]$$

- Two subtractions needed.
- The contribution from the two–pion cut gives us the non–local part of the potential.

$$W(R) = -\frac{3\pi\beta^2 m_\pi^2}{8b^2 R^5} \Big[\left(4\left(\kappa_2+3\right)^2 (m_\pi R)^3 + \left(3\kappa_1^2 + 43\kappa_2^2 + 14\kappa_1\kappa_2\right) m_\pi R\right) K_1(2m_\pi R) + 2\left(2\left(\kappa_2+3\right)\left(\kappa_1+5\kappa_2\right)(m_\pi R)^2 + \left(3\kappa_1^2 + 43\kappa_2^2 + 14\kappa_1\kappa_2\right)\right) K_2(2m_\pi R) \Big]$$

$K_n(x)$ are the modified Bessel functions of the second kind.





Limits of the van der Waals potential

- For $R \gg 1/m_\pi$ the potential reduces to

$$W(R) = -\frac{3(3+\kappa_2)^2 \pi^{3/2} \beta^2}{4b^2} \frac{m_\pi^{9/2}}{R^{5/2}} e^{-2m_\pi R}$$

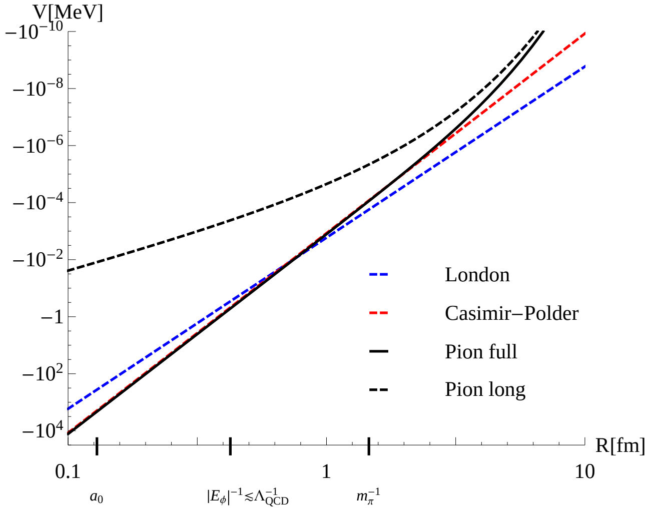
  - Same dependence in m_π and R as Fujii, Kharzeev 1999 who used dispersive techniques.
  - Different coefficient due to:
    - The spectral function used by Fujii, Kharzeev 1999 did not include the whole m_π dependence.
    - $\kappa \neq 0$.
    - The polarizability was estimated using the results in the large N_c from Bhanot, Peskin 1979.

- For $R \ll 1/m_\pi$ the potential reduces to a Casimir-Polder-like potential:

$$W(R) = -\frac{3\pi\beta^2}{8b^2 R^7} \left(3\kappa_1^2 + 43\kappa_2^2 + 14\kappa_1\kappa_2\right)$$

equivalent to taking $\alpha_s \approx 0.5$.

Comparison of the potentials in different regimes



$m = 5$ GeV, $N_c = 3$, $N_f = 3$ and $\mu = 1$ GeV

Conclusions

- We have integrated out the different scales and built the EFTs for two $1S$–bottomonium dynamics.
- In the perturbative case $R \ll 1/\Lambda_{QCD}$ we obtain:
  - The London potential for $1/(mv^2) \gg R \gg 1/(mv)$
  - Casimir-Polder potential for $R \gg 1/(mv^2)$

  with extra color factors.
- We have computed the $1S$–bottomonium polarizabilities with Coulombic continuum octet intermediate state.
- In the perturbative case $R \gg 1/\Lambda_{QCD}$ we studied the van der Waals potential using a χ EFT.
  - We obtain the π-ϕ LEC using the calculation of the polarizability and hadronizing the gluons with help of the QCD trace anomaly.
  - The van der Waals potential has been computed from a two–pion exchange diagram.

Thank you for your attention