

A Time Dependent Local Isospin Density Approximation (TDLIDA) for the response function in asymmetric nuclear matter

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Collaborators

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Overview

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Response
function in
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and neutrino
physics

Evaluation of
the response
function:
TDLI(S)DA

Derivation of
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- 2 Evaluation of the response function: TDLI(S)DA
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Some Motivations...

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- Computation of from first principles of the response function in longitudinal and transverse isospin (spin) channels is still very hard in many-body systems. Mean field approximation is still useful
- The response in asymmetric medium-heavy nuclei shows interesting and non-obvious features (e.g. pigmy resonances), and sum rules are related to quantities relevant for the EoS (e.g. relationship between m_{-1} and compressibility)

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- In homogeneous matter the response can be related to the neutrino-nucleon scattering rate, and to the neutrino mean free path in compact stars.



Some references...

- **Tamm-Dancoff response function and ν mean free path**
Cowell and Pandharipande, *Phys. Rev. C* **70**, 035801;
A. Lovato, O. Benhar, S. Gandolfi, C. Losa, *Phys. Rev. C* **70**, 025804 (2014)
- **TDLSDA for many electron systems:**
E. Lipparini and L. Serra, *Phys. rev. B* **57** R6830(R) (1998)
- **This work:**
E. Lipparini and F. Pederiva, *Phys. Rev. C* **88**, 024318 (2013);
E. Lipparini and F. Pederiva, submitted to PRC
L. Riz, M.Sc. thesis (to be published)

General charge-density excitations in nuclear matter

We are interested in studying the **density response** of the system. For nucleons the response can be splitted in different channels, described by the following operators (Cowell, Pandhariphande, PRC 67, 035504 (2004)):

$$O_F = \sum_i O_F(i) = \sum_i \tau_i^\pm e^{i\mathbf{q}\cdot\mathbf{r}_i} \text{ "Fermi"}$$

$$O_{GT} = g_A \sum_i \mathbf{O}(i) = \sum_i \sigma_i \tau_i^\pm e^{i\mathbf{q}\cdot\mathbf{r}_i} \text{ "Gamow-Teller"}$$

$$O_{NV} = \sum_i O_{NV}(i) \text{ "Neutral-vector"}$$

$$= \sum_i \left[-\sin^2 \theta_W + \frac{1}{2}(1 - 2\sin^2 \theta_W)\tau_i^z \right] e^{i\mathbf{q}\cdot\mathbf{r}_i}$$

$$O_{NA} = g_A \sum_i \mathbf{O}_{NA}(i) = g_A \sum_i \frac{1}{2}\tau_i^z \sigma_i e^{i\mathbf{q}\cdot\mathbf{r}_i} \text{ "Neutral-axial-vector"}$$

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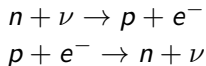
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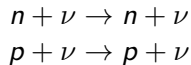
Conclusions

These operators can be easily related to weak processes, possibly involving spin- or charge-exchange or both.

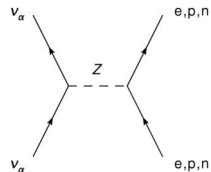
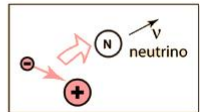
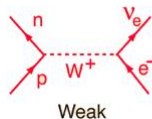
- *Fermi operator:*



- *Neutral-vector current*



and so on...



Weinberg-Salam model

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The relation between **weak scattering processes** and **nuclear density response** descends from the *Weinberg-Salam Lagrangian* coupling a nucleon of mass m with neutrinos through weak currents (charged or neutral)(see e.g. Iwamoto & Pethick, PRD 25, 313 (1982), Burrows & Sawyer, PRC 58, 554 (1998)).

Weinberg-Salam model

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$$\mathcal{L}_W = \frac{G_W}{\sqrt{2}} \bar{\psi}_\nu(x) \gamma_\mu (1 - \gamma_5) \psi_\nu(x) \frac{1}{2} \bar{\psi}_n(x) \gamma^\mu (1 - C_A \gamma_5) \psi_n$$

ν scattering rate

The WS Lagrangian couples neutrinos to *density* and *spin density* fluctuations of neutrons.

In the **non-relativistic limit** the baryonic current can be approximated by:

$$\bar{\psi}_n(x)\gamma^\mu(1 - C_A\gamma_5)\psi_n \sim \psi_n^\dagger(x)\psi_n(x)\delta_0^\mu - C_A\psi_n^\dagger(x)\sigma_i\psi_n(x)\delta_i^\mu$$

One can easily recognize in these two contribution a **density fluctuation** and a **spin-density** fluctuation operators.

The scattering rate from a system of neutrons of a neutrino with 4-momentum $q^\mu \equiv (q^0, \vec{q})$ can be computed from the Fermi golden rule, averaging on the initial (neutron and/or proton) states and summing over all the final states.

ν scattering rate

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The result gives the *neutrino scattering rate*. For a neutrino of incident energy E , the contribution to the scattering rate σ in a given channel can be written as:

$$\sigma = \frac{G^2}{2} \frac{1}{E} \int dq \int d\omega (E - \omega) q \left(1 + \frac{E^2 + (E - \omega)^2 - q^2}{2E(E - \omega)} \right) S(q, \omega)$$

where $S(q, \omega)$ is the *dynamical structure factor (DSF)* for the excitation operators describing the process. These in turn can be written as a combination of the DSF relative to a *density*, *spin-density*, *isospin-density*, and *spin-isospin-density* excitations.

Evaluation of the structure factor

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There are several previous papers.

- **Landau Theory:**
 - N. Iwamoto and C. Pethick, PRD 25, 313 (1982) (PNM)
 - A. Burrows and R.F. Sawyer, PRC 58, 554 (1998) (SNM)
 - O. Benhar, A. Cipollone, A. Loretì, PRC 87, 014601 (2013) (PNM, +CBF)
- **Correlated basis Functions + Tamm-Dancoff**
 - S. Cowell and V.R. Pandharipande, PRC 70, 035801 (2004) (SNM)
 - A. Lovato, O. Benhar, S. Gandolfi, C. Losa, PRC 89, 025804 (2014)

Time Dependent Local Density Approximation

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We use here the **Time Dependent Local Density Approximation (TDLDA)** approach to compute the response function and the DSF.

We have so far worked out the response function in the **transverse** and **longitudinal** isospin channels in asymmetric nuclear matter, and the corresponding response functions for spin excitations in spin polarized neutron matter. Since the operators we consider contain a dependence on spin/isospin, we need a theory in which both density and spin/isospin density will appear. We will then be talking about a **Time-Dependent Local (Iso)Spin Density Approximation (TDLI(S)DA)**.

TDLIDA

Following the *Kohn-Sham* method, we introduce a **Local Isospin Density Approximation (LIDA)** for the homogeneous nuclear matter defining the **energy functional** as:

$$E(\rho, \xi) = T_0(\rho, \xi) + \int \epsilon_V(\rho, \xi) \rho \, d\mathbf{r}$$

where $T_0(\rho, \xi)$ is the kinetic energy of the *non interacting* system with density $\rho = \rho_n + \rho_p$, and isospin polarization $\xi = \rho_1/\rho$ with $\rho_1 = \rho_n - \rho_p$.

Energy minimization with respect to the neutron and proton densities gives the set of equations:

$$\left[-\frac{1}{2m} \nabla_{\mathbf{r}}^2 + v(\mathbf{r}) + w(\mathbf{r})\eta_{\tau} \right] \varphi_i^{\tau}(\mathbf{r}) = \epsilon_{i,\tau} \varphi_i^{\tau}(\mathbf{r}),$$

where $\eta^{\tau} = \pm 1$ depending on the isospin of the nucleon.

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The effective interactions $v(\mathbf{r})$ and $w(\mathbf{r})$ are related to the energy-density functional by:

$$v(\mathbf{r}) = \frac{\partial \{\rho \epsilon_V[\rho(\mathbf{r}), \xi(\mathbf{r})]\}}{\partial \rho(\mathbf{r})} \quad w(\mathbf{r}) = \frac{\partial \{\rho \epsilon_V[\rho(\mathbf{r}), \xi(\mathbf{r})]\}}{\partial \xi(\mathbf{r})}$$

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The Hohenberg-Kohn theorem provides a *variational principle on the energy-density functional*.

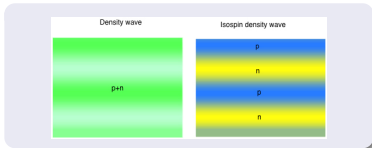
Longitudinal channel

We seek for solutions for a homogeneous matter in presence of a (isospin-)density excitation. For example, in the longitudinal channels:

$$\sum_{k=1}^A \lambda_{\tau}^k \left(e^{i(\mathbf{q}\cdot\mathbf{r}_k - \omega t)} + e^{-i(\mathbf{q}\cdot\mathbf{r}_k - \omega t)} \right)$$

where λ is the excitation strength and $\lambda_{\tau}^k = \lambda$ in the scalar channel and $\lambda_{\tau}^k = \lambda \eta_{\tau}$ in the isovector channel. The time dependent Kohn-Sham equations become:

$$i \frac{\partial}{\partial t} \varphi_i^{\tau}(\mathbf{r}, t) = \left\{ -\frac{1}{2m} \nabla_{\mathbf{r}}^2 + v[\rho_n(\mathbf{r}, t), \rho_p(\mathbf{r}, t)] \right. \\ \left. + w[\rho_n(\mathbf{r}, t), \rho_p(\mathbf{r}, t)] \eta_{\tau} + \lambda_{\tau} [e^{i(\mathbf{q}\cdot\mathbf{r} - \omega t)} \right. \\ \left. + e^{-i(\mathbf{q}\cdot\mathbf{r} - \omega t)}] \right\} \varphi_i^{\tau}(\mathbf{r}, t)$$



Longitudinal channel

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Since we are interested in homogeneous systems, we seek for solutions of the form:

$$\begin{aligned}\rho_n(\mathbf{r}, t) &= \rho_n + \delta\rho_n(\mathbf{r}, t) \\ \rho_p(\mathbf{r}, t) &= \rho_p + \delta\rho_p(\mathbf{r}, t)\end{aligned}$$

where

$$\begin{aligned}\delta\rho_n(\mathbf{r}, t) &= \delta\rho_n \left[e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)} + e^{-i(\mathbf{q}\cdot\mathbf{r}-\omega t)} \right] \\ \delta\rho_p(\mathbf{r}, t) &= \delta\rho_p \left[e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)} + e^{-i(\mathbf{q}\cdot\mathbf{r}-\omega t)} \right]\end{aligned}$$

The quantities $\delta\rho_n$ and $\delta\rho_p$ have to be determined from the KS equations.

Longitudinal channel

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By definition, the **density-density** and **isovector density-isovector density** response function in the linear regime are given by:

$$\frac{\chi^s(\mathbf{q}, \omega)}{V} = \frac{\delta\rho_n + \delta\rho_p}{\lambda} \equiv \frac{1}{V} [\chi^n(\mathbf{q}, \omega) + \chi^p(\mathbf{q}, \omega)] ,$$

and

$$\frac{\chi^v(\mathbf{q}, \omega)}{V} = \frac{\delta\rho_n - \delta\rho_p}{\lambda} \equiv \frac{1}{V} [\chi^n(\mathbf{q}, \omega) - \chi^p(\mathbf{q}, \omega)] .$$

respectively.

Longitudinal channel

The solution of the linearized KS equations lead to the following result:

TDLIDA longitudinal response functions (isoscalar and isovector)

$$\frac{\chi^s(q, \omega)}{V} = \frac{\frac{\chi_0^n}{V} [1 - (V_{pp} - V_{np}) \frac{\chi_0^p}{V}] + \frac{\chi_0^p}{V} [1 - (V_{nn} - V_{pn}) \frac{\chi_0^n}{V}]}{(1 - V_{pp} \frac{\chi_0^p}{V})(1 - V_{nn} \frac{\chi_0^n}{V}) - V_{np} \frac{\chi_0^n}{V} V_{pn} \frac{\chi_0^p}{V}}$$

$$\frac{\chi^v(q, \omega)}{V} = \frac{\frac{\chi_0^n}{V} [1 - (V_{pp} + V_{np}) \frac{\chi_0^p}{V}] + \frac{\chi_0^p}{V} [1 - (V_{nn} + V_{pn}) \frac{\chi_0^n}{V}]}{(1 - V_{pp} \frac{\chi_0^p}{V})(1 - V_{nn} \frac{\chi_0^n}{V}) - V_{np} \frac{\chi_0^n}{V} V_{pn} \frac{\chi_0^p}{V}}$$

The effective potentials $V_{\alpha\beta}$ with $\alpha, \beta = p, n$ are derivatives of the KS effective interaction with respect to density and isospin (spin) asymmetry, while χ_0^α is the free Fermi gas response function for neutrons and protons.

Transverse channel

A similar derivation can be done for the transverse channel (corresponding to excitations by the Fermi operator).

In this case the LIDA-KS equations are [see e.g. A.K. Rajagopal, PRB 17, 2980 (1978)]:

$$\left[-\frac{1}{2}\nabla_{\mathbf{r}}^2 + \frac{1}{2}\omega_C\tau_z + v(\mathbf{r}) + w(\mathbf{r})\tau_z \right] \varphi_i^T(\mathbf{r}) = \varepsilon_{i,\tau} \varphi_i^T(\mathbf{r})$$

The second term in the l.h.s is an **effective isovector potential** accounting for the equilibrium isospin polarization. In a star matter it is fixed by **charge neutrality** and **beta-equilibrium**, in nuclei by the proton Coulomb potential. The parameter ω_C can be related to the equilibrium asymmetry by imposing that the variation of the LIDA energy with respect to ξ be zero. One gets

$$\int d\mathbf{r}(\rho_n - \rho_p) = N - Z = \omega_C \frac{\frac{3A}{4\epsilon_F}}{1 + \frac{3\rho}{2\epsilon_F} \frac{\partial w}{\partial m}},$$

Transverse channel

The final result for the response function is the following:

$$\frac{\chi_t(q, \omega)}{V} = \frac{\chi_t^0(q, \omega)}{1 - \frac{2}{V} \mathcal{W}(\rho, m) \chi_t^0(q, \omega)}$$

where the $\chi_t^0(q, \omega)$ is the transverse response of the free Fermi gas. In the $qv_F \ll \epsilon_F$ limit, where $v_F = k_F/m$ is the Fermi velocity, it is given by:

$$\frac{\chi_t^0(q, \omega)}{V} = -\frac{3}{4} \frac{\rho}{\epsilon_F} \left(1 + \frac{\omega}{2qv_F} \ln \frac{\omega - \omega_a - qv_F}{\omega - \omega_a + qv_F} \right),$$

where

$$\omega_a = \frac{\omega_C}{\left(1 + \frac{3\rho\mathcal{W}(\rho, m)}{2\epsilon_F}\right)} = \frac{2}{3} \frac{k_F^2}{m} \xi,$$

Nuclear matter with AV6'+ DDI

The energy-density functional is derived in a very simple way, relying on the calculations of Gandolfi et al., Mon. Not. R. Astron. Soc. **404**, L35 (2010). Nuclear matter is here modeled with an AV6' interaction with the addition of a density dependent term.

$$\hat{H} = \hat{T} + AV6' + TNA$$

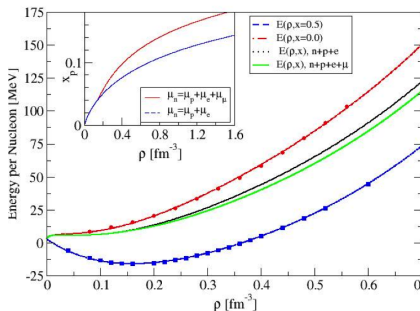
$$TNA = \gamma_2 \rho^2 e^{-\gamma_3 \rho} \left[3 - 2 \left(\frac{\rho_n - \rho_p}{\rho} \right)^2 \right]$$

$$\frac{E_{SNM}[\rho]}{N} = E_0 + b(\rho - \rho_0)^2 + c(\rho - \rho_0)^3 e^{\gamma(\rho - \rho_0)}$$

$$\frac{E[\rho]}{N} = \frac{E_{SNM}[\rho]}{N} + C_s \left(\frac{\rho}{\rho_0} \right)^{\gamma_s} (1 - 2x_p)^2$$

Parameter values:

$$\begin{aligned} E_0 &= -16.0 \text{ MeV} & \rho_0 &= 0.160 \text{ fm}^{-3} \\ b &= 520.0 \text{ MeV} \cdot \text{fm}^6 & c &= -1297.4 \text{ MeV} \cdot \text{fm}^9 \\ \gamma &= -2.213 \text{ fm}^3 & C_s &= 31.3 \text{ MeV} \\ \gamma_s &= 0.64 \end{aligned}$$



Energy density functional

The interaction part of the EDF is assumed to be of the form:

$$\epsilon_V(\rho, \xi) = \epsilon_0(\rho) + \xi^2 [\epsilon_1(\rho) - \epsilon_0(\rho)] ,$$

where

$$\epsilon_q(\rho) = \epsilon_q^0 + a_q(\rho - \rho_0) + b_q(\rho - \rho_0)^2 + c_q(\rho - \rho_0)^3 e^{\gamma_q(\rho - \rho_0)}$$

The saturation density is assumed to be $\rho_0 = 0.16\text{fm}^{-3}$. The values of the parameters we have extracted from the fit are the following:

q	ϵ_q^0 (MeV)	a_q (MeV·fm ³)	b_q (MeV·fm ⁶)	c_q (MeV·fm ⁹)	γ_q (fm ³)
0	-38.1	-92.1	630.1	-1717.2	-2.360
1	-19.8	-21.0	533.0	-1327.7	-2.201

This parametrization reproduces very well the AFDMC calculations in a wide range of density ρ (from $\rho_0/2$ to $3\rho_0$) and for both $\xi = 0, 1$.

Excitation strengths and sum rules

- From the response function it is possible to determine the **dynamic structure factor** via the relation:

$$S^{s,v}(q, \omega) = -\frac{1}{\pi} \Im m[\chi^{s,v}]$$

- From the DSF it is also possible to compute the **energy weighted sum rules**:

$$m_k^{s,v} = \int_0^\infty d\omega \omega^k S^{s,v}(q, \omega) = \sum_n \omega_{no}^k |\langle 0 | F^{s,v} | n \rangle|^2$$

In particular the ratio m_{-1}/m_0 gives the *compressibility* of the system.

- The poles of $\chi(q, \omega)$ give the spectrum and the dispersion $\omega(q)$ of the **collective excitations**, for which we can also evaluate the **strength**.

Excitation strengths: longitudinal channel

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It is convenient to express the response function in term of the adimensional variable $s = \omega/qv_F$, where v_F is the Fermi velocity at a given density ρ . The longitudinal Fermi gas response function as a function of s in the **low q limit**, and for an **arbitrary isospin polarization** reads:

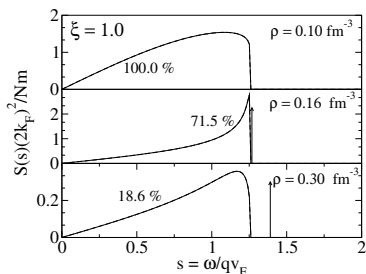
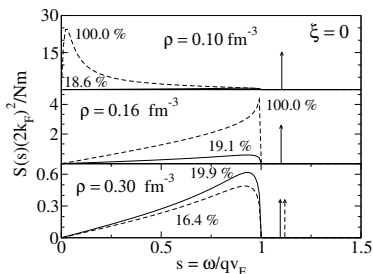
$$\frac{\chi_0^{n,p}(\mathbf{q}, \omega)}{V} = -\nu^{n,p} \left[1 + \frac{s}{2(1 \pm \xi)^{1/3}} \ln \frac{s - (1 \pm \xi)^{1/3}}{s + (1 \pm \xi)^{1/3}} \right]$$

where $\nu^{n,p} = mk_F^{n,p}/\pi^2 = mk_F(1 \pm \xi)^{1/3}/\pi^2$, $k_F = (\frac{3\pi^2}{2}\rho)^{1/3}$.

The plus sign holds for χ_0^n and the minus sign for χ_0^p .

Since $\chi^{s,v}$ depends on q and ω only via the dependence of χ_0^n and χ_0^p on these variables, **also the response functions of the interacting system turn out to be functions of s only.**

Longitudinal response



TDLISDA Dynamical structure factor for density (solid lines) and spin density (dashed lines) in the longitudinal channel. Left panel: results for SNM. Right panel: results for PNM. Arrows indicate the location of the collective excitations. The percentages represent the fraction of the total strength pertinent to the particle-hole excitations.

Longitudinal response

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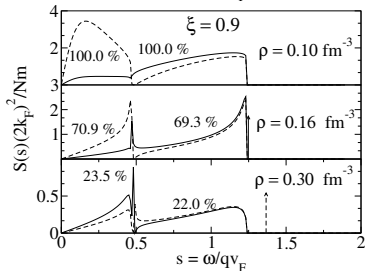
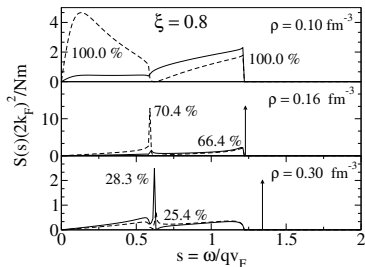
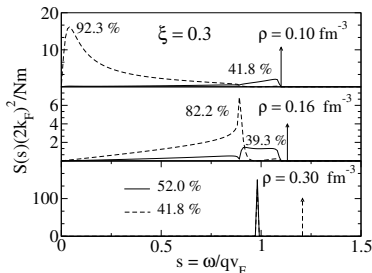
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TDLIDA DSFs in the longitudinal channel for nuclear matter with three different isospin polarizations $\xi = 0.3, 0.8, \text{ and } 0.9$.

Compressibility

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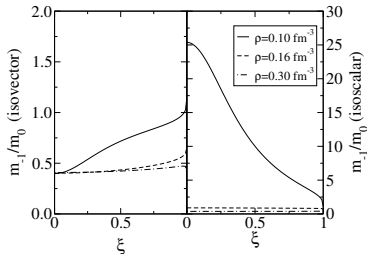
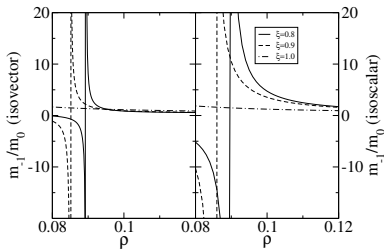
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Compressibility ratio m_{-1}/m_0 as a function of the density (left panel), and of the isospin polarization (right panel) in the isovector and isoscalar channels. Notice that the β -equilibrium induces a mechanical instability at densities of order 0.1 fm^{-3} (clustering?).

Compressibility

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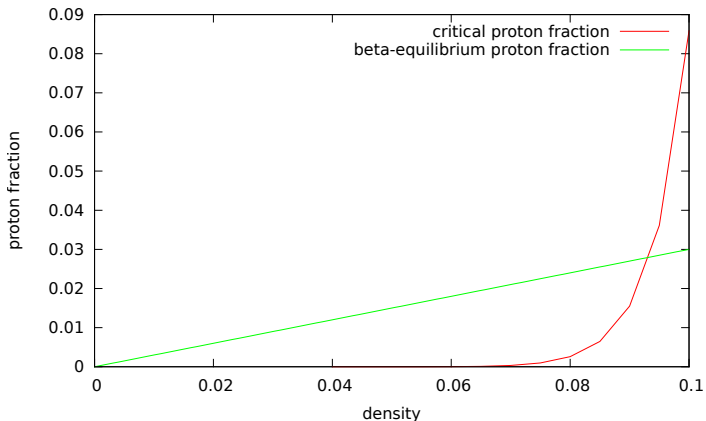
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If we compare the critical proton fraction predicted by our calculations with the proton fraction imposed by β -equilibrium, we see that the predicted limit of stability of nuclear matter is at a density of about 0.09fm^{-3} .

Transverse response

Introducing the adimensional quantities $s = \frac{\omega}{qv_F}$, $z = \frac{3q}{2k_F\xi}$, the free Fermi gas transverse response function reads:

$$\frac{\chi_t^0(q, \omega)}{V\nu} \equiv \frac{\chi_t^0(s, z)}{V\nu} = \Omega_{\pm}(s, z), \quad (1)$$

with

$$\nu = mk_F/\pi^2,$$

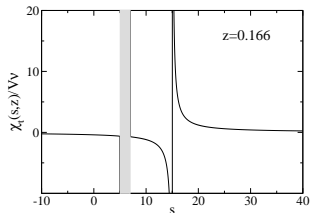
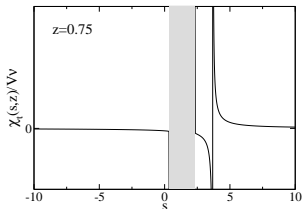
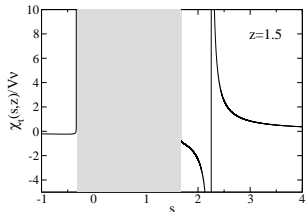
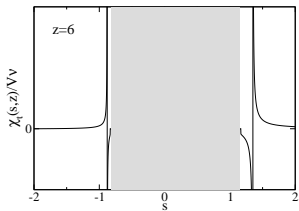
$$\Omega_{\pm}(s, z) = - \left(1 + s/2 \ln \frac{s-1-1/z}{s+1-1/z} \right),$$

The interacting transverse response function becomes then:

$$\frac{\chi_t(q, \omega)}{V\nu} \equiv \frac{\chi_t(s, z)}{V\nu} = \frac{\Omega_{\pm}(s, z)}{1 - 2\nu\mathcal{W}(\rho, m)\Omega_{\pm}(s, z)}.$$

Transverse response

Real part of the response function



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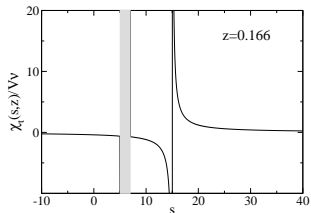
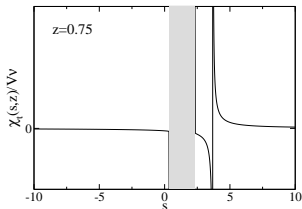
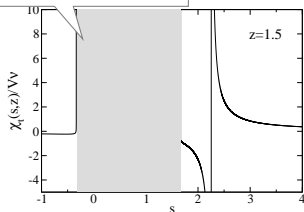
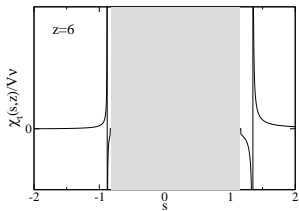
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Real part

The shaded area represents the region of the continuum p-h excitation spectrum



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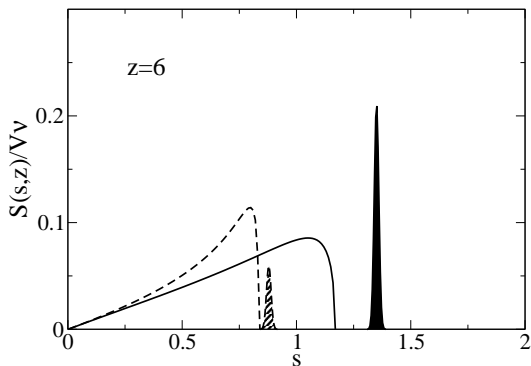
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Excitation strengths for $z = 3q/(2k_F\xi) = 6$ The full and dashed lines indicate the quasi-particle/quasi-hole and collective strengths in the $\Delta T_z = -1$ and $\Delta T_z = +1$ channels (i.e. $s < 1$).

Transverse response

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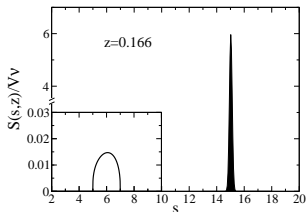
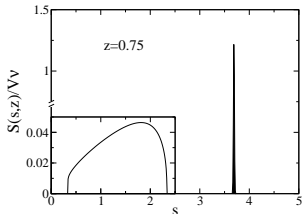
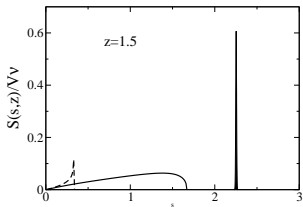
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For a given momentum q , a larger isospin polarizability ξ implies a decreasing value of z . We can notice how the strength of the collective mode is strongly enhanced in conditions typical of matter at β -equilibrium.

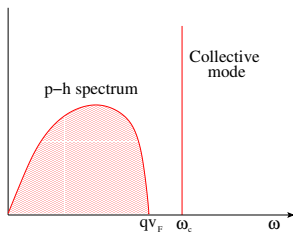
Neutrino mean free path

As previously discussed, the **scattering rate** of neutrinos can be obtained by computing the integral:

$$\sigma = \frac{G^2}{2} \frac{1}{E} \int dq \int d\omega (E - \omega) q \left(1 + \frac{E^2 + (E - \omega)^2 - q^2}{2E(E - \omega)} \right) S(q, \omega)$$

The **neutrino mean free path** λ is related to σ by the following relation:

$$\lambda = \frac{1}{\sigma \rho}$$



The scattering rate is made up of two contributions:

- Contribution from the particle-hole excitations
- Contribution from the collective mode

Neutrino mean free path

The integration must be performed on the values of momentum kinematically accessible to neutrinos.

Notation:

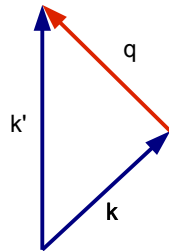
$$k^\mu = (k^0, \vec{k}) \quad k'^\mu = (k'^0, \vec{k}')$$

are the incoming and outgoing 4-momenta of the neutrino.

$$q^\mu = (\omega, \vec{q})$$

is the transferred 4-momentum.

Neutrinos are assumed to be *ultra-relativistic*. For the moment we only considered contributions from the **transverse channel**.



Neutrino mean free path

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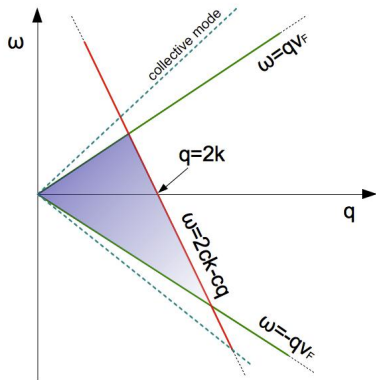
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Kinematic limits



The transferred momentum must satisfy the following inequality:

$$|\omega| < cq < |\omega - 2ck|$$

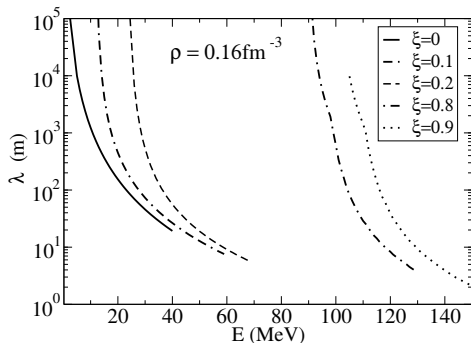
This implies that:

$$\omega < c(2k - q)$$

This represents the integration bound, that has to be intersected with the limits coming from $S(q, \omega)$.

This holds for **non-degenerate** neutrinos.

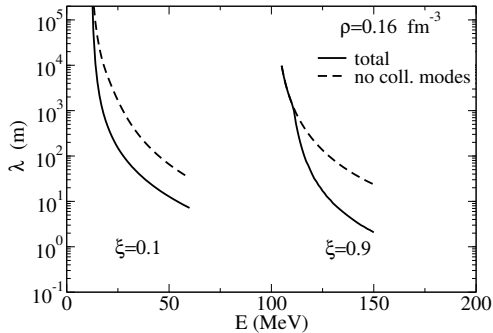
Neutrino mean free path



Neutrino mean free paths at different isospin polarizations.

- The contribution of the transverse channel to the neutrino scattering rate is strongly suppressed in the β -equilibrium regime.
- Results for $\xi = 0$ are comparable and compatible with the results of Cowell and Pandharipande.

Neutrino mean free path



It is interesting to look at the contribution of the collective modes. While this is quite relevant, in particular at higher temperatures.

Neutrino mean free path

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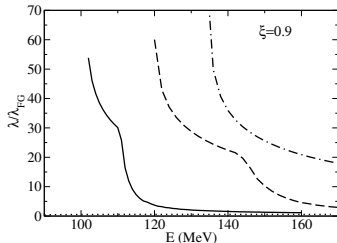
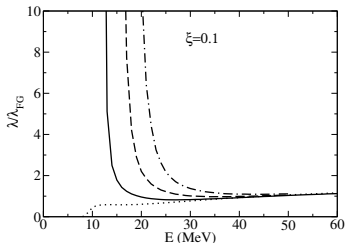
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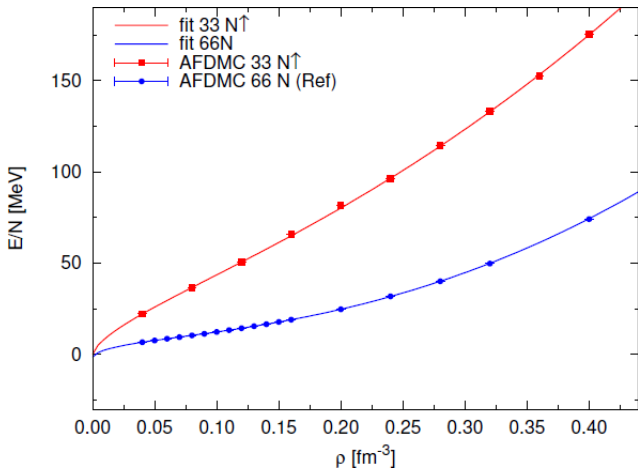
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Ratio of the NMFP in an interacting nucleonic matter and in a free Fermi gas at different densities ($\rho/\rho_0 = 0.5$ (dotted), 1 (solid), 1.5 (dashed), 2 (dotted-dashed)).

Neutron matter



Equation of state of spin polarized (red) and spin unpolarized PNM computed with AV18+UIX.

Neutron matter

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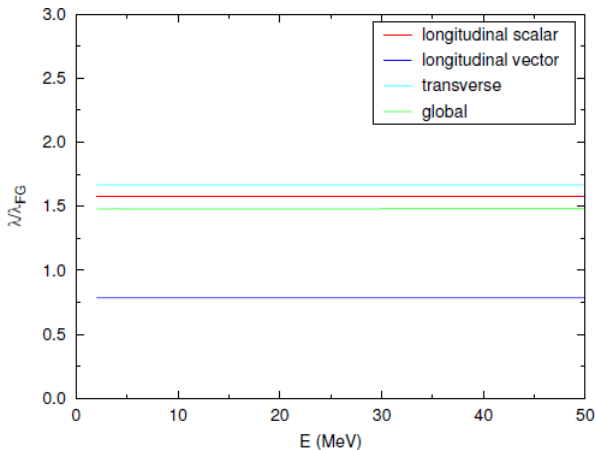
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Ratio of the NMFP in PNM at $\rho = 0.16\text{fm}^{-3}$ as a function of the incident energy

Conclusions

- The time dependent local density approximation was successfully applied to estimate the response function of asymmetric nuclear matter at an arbitrary value of isospin asymmetry.
- The longitudinal response shows a non trivial structure of the ρ h spectrum and of the collective modes for intermediate isospin polarizations.
- We computed the contribution of the Fermi channel to the suppression of the neutrino mean free path in nuclear matter. At the NS core conditions matter is essentially transparent, while relevant effects could be seen in the NS crust.
- We are computing the response function in the longitudinal and transverse channel in pure neutron matter, starting from accurate QMC calculations of (spin polarized) neutron matter.