Scalar glueball decay rates from string theory

(through the top-down holographic Witten-Sakai-Sugimoto model)

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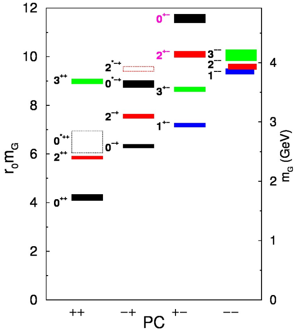
Still elusive: Glueballs

Spectrum of *bare* glueballs
(prior to mixing with $q\bar{q}$ states)
more or less known from lattice:

$m_{0^{++}} \sim 1.7$ GeV
$m_{2^{++}} \sim 2.4$ GeV

Morningstar & Peardon hep-lat/9901004



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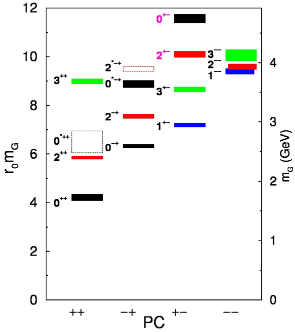

Interactions of glueballs still unclear:

- Are glueballs broad or narrow?
- Do they mix with $q\bar{q}$ strongly or weakly?

$\rightarrow$ no conclusive identification of any glueball in meson spectrum

most discussed lowest 0^{++} candidates:

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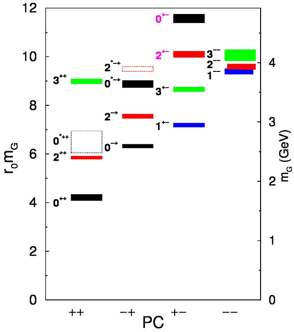

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various phenomenological models describe $f_0(1500)$ or $f_0(1710)$
alternatingly as ∼50-70% or ∼75-90% glue

Large-N_c QCD

Lattice: glueball spectrum at $N_c = 3$ rather similar to large N_c

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Consider *'t Hooft limit (1974):* $N_c \to \infty$ with $\lambda = g^2 N_c$ (and N_f) fixed
— Only planar diagrams left (quarks, if any, at the edges)
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Finite, large N:
— mixing of mesons and glueballs at most $\sim N^{-1/2}$
— meson decay rates $\sim N^{-1}$
— glueball decay rates $\sim N^{-2}$
If large-N limit appropriate starting point for approximations:
glueballs should be weakly mixed and relatively stable
(though in Veneziano limit $N_c \sim N_f \gg 1$ strong mixing!)

Large-N_c QCD

$1/N_c$ expansion may be good expansion even at $N_c = 3$,

if coefficients conspire like in QED
(QED: $e \approx 0.303$, but expansion parameter $e^2/4\pi = 1/137$)

Alas, no direct way to sum planar diagrams

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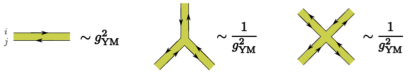
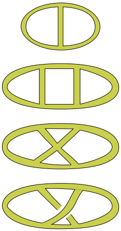
early suspicion of relation to string theory by 't Hooft
(expansion parameter is genus of diagrams like in string perturbation theory)



or: ~ 1 $\sim \frac{\sqrt{\lambda}}{\sqrt{N_c}}$ $\sim \frac{\lambda}{N_c}$



$\sim (g^2_{\rm YM})^{3-2} N_c^3 = \lambda N_c^2$

$\sim (g^2_{\rm YM})^{6-4} N_c^4 = \lambda^2 N_c^2$

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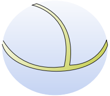

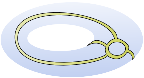

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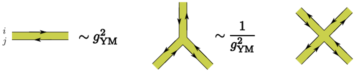

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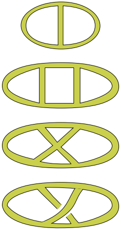

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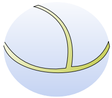
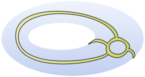




1997: Maldacena's **AdS/CFT (gauge/gravity) duality** eventually established such a link
(Witten: "Thus the old prophecy comes to pass.")

Holographic QCD

Celebrated AdS/CFT duality relates strongly coupled large-N_c supersymmetric Yang-Mills theories to supergravity on anti-de Sitter space in 5 dimensions (AdS$_5 \times S^5$)

Holographic QCD: generalization to nonconformal nonsupersymmetric case
Options:

- **Bottom-up**: breaking of conformal invariance (necessary for confinement) by hand and matching to QCD with holographic dictionary, e.g.
  hard-wall model (Erlich-Katz-Son-Stephanov 2005)
  soft-wall model (Karch-Katz-Son-Stephanov 2006)
- **Top-down**: first-principles constructions from superstring theory with nonconformal D-branes
  - here: **Witten**[1998]-**Sakai-Sugimoto**[2004] **model**

Both approaches surprisingly successful quantitative description of low-energy QCD with minimal set of parameters

WSS model: almost parameter-free (1 coupling at a certain mass scale)!

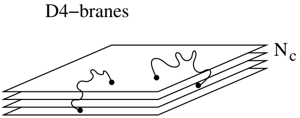
New results on: TU WIEN

- Glueball decay pattern [with F. Brünner & D. Parganlija]
- Effects from finite quark masses [with F. Brünner]:
  lifting of flavor blindness; $\eta\eta'$ decay rates

Witten model: Holographic nonsupersymmetric QCD

E. Witten, Adv. Theor. Math. Phys. 2, 505 (1998):

Type-IIA string theory with $N_c \to \infty$ $D4$ branes dual to $4+1$-dimensional super-Yang-Mills theory



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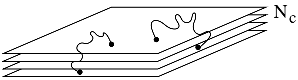

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supersymmetry completely broken by compactification
on "thermal-like" circle $x_4 \equiv x_4 + 2\pi/M_{\text{KK}}$ $_{(\text{Kaluza–Klein})}$

- antisymmetric b.c. for adjoint fermions: masses $\sim M_{\text{KK}}$
- adjoint scalars not protected by gauge symmetry: also masses $\sim M_{\text{KK}}$

$\to$ dual to pure-glue YM theory
3+1-dimensional at scales $\ll M_{\text{KK}}$

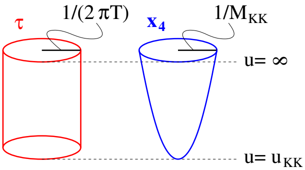
but supergravity approximation needs weak curvature,
cannot take limit $M_{\text{KK}} \to \infty$

Deconfinement phase transition

Thermal circle in Euclidean time τ in addition to compactified x_4
Hawking-Page transition when $2\pi T = M_{\rm KK}$ (thus ~ 1 GeV ?)

Confined phase

$$ds^2 = \left(\frac{u}{R}\right)^{3/2} [d\tau^2 + d\mathbf{x}^2 + f(u) dx_4^2] + \left(\frac{R}{u}\right)^{3/2} \left[\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right]$$

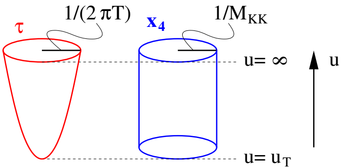


$$M_{\rm KK} = \frac{3}{2}\frac{u_{\rm KK}^{1/2}}{R^{3/2}} \qquad f(u) \equiv 1 - \frac{u_{\rm KK}^3}{u^3}$$

Cigar topology in x_4-u subspace $\longrightarrow$

Deconfined phase

$$ds^2 = \left(\frac{u}{R}\right)^{3/2} [\tilde f(u) d\tau^2 + \delta_{ij} d\mathbf{x}^2 + dx_4^2] + \left(\frac{R}{u}\right)^{3/2} \left[\frac{du^2}{\tilde f(u)} + u^2 d\Omega_4^2\right]$$



$$T = \frac{3}{4\pi}\frac{u_T^{1/2}}{R^{3/2}} \qquad \tilde f(u) \equiv 1 - \frac{u_T^3}{u^3}$$

Cigar in τ-u = **Euclidean black hole**

Glueballs in confined phase

$\exists$ scalar and tensor glueballs corresponding to 5D dilaton Φ and graviton G_{ij}
Csaki, Ooguri, Oz & Terning 1999

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Type-IIA supergravity compactified on x_4-circle many more modes:
Constable & Myers 1999; Brower, Mathur & Tan 2000

Mode	S$_4$	T$_4$	V$_4$	N$_4$	M$_4$	L$_4$
Sugra fields	G_{44}	Φ, G_{ij}	C_1	B_{ij}	C_{ij4}	G^α_α
J^{PC}	0^{++}	$0^{++}/2^{++}$	0^{-+}	1^{+-}	1^{--}	0^{++}
n=0	7.30835	22.0966	31.9853	53.3758	83.0449	115.002
n=1	46.9855	55.5833	72.4793	109.446	143.581	189.632
n=2	94.4816	102.452	126.144	177.231	217.397	277.283
n=3	154.963	162.699	193.133	257.959	304.531	378.099
n=4	228.709	236.328	273.482	351.895	405.011	492.171

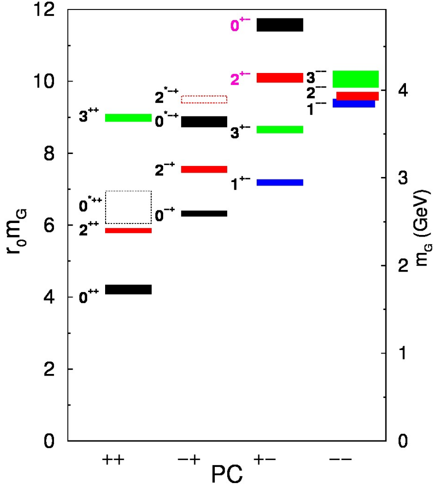
Lowest mode not from dilaton, but from "exotic polarization" – in 11D notation:

$$\underline{\delta g_{44}} = -\frac{r^2}{L^2} f\, H(r) G(x), \quad \delta g_{\mu\nu} = \frac{r^2}{L^2}\left[\frac{1}{4}H(r)\eta_{\mu\nu} - \left(\frac{1}{4} + \frac{3R^6}{5r^6-2R^6}\right) H(r)\frac{\partial_\mu\partial_\nu}{M^2}\right] G(x)$$

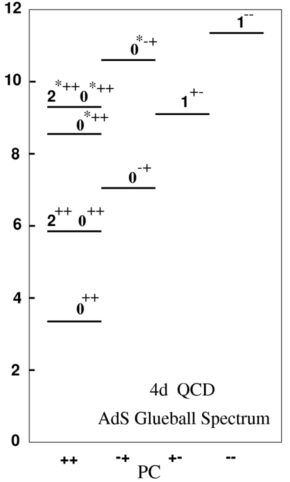
$$\delta g_{11,11} = \frac{r^2}{L^2}\frac{1}{4}H(r)G(x), \quad \delta g_{rr} = -\frac{L^2}{r^2} f^{-1}\frac{3R^6 H(r) G(r)}{5r^6-2R^6}, \quad \delta g_{r\mu} = \frac{90 r^7 R^6 H(r)\partial_\mu G(x)}{M^2 L^2 (5r^6-2R^6)^2}$$

Lattice glueballs vs. supergravity glueballs

Morningstar & Peardon hep-lat/9901004:



Brower, Mathur & Tan 2000:

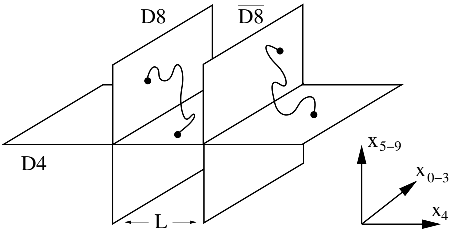


(mass scales matched on 2^{++}) → seemingly good qualitative agreement!

Sakai-Sugimoto model: Adding chiral quarks

T. Sakai, S. Sugimoto, Prog. Theor. Phys. 113, 843 (2005)
add N_f D8- and $\overline{\text{D8}}$-branes, separated in x_4, $N_f \ll N_c$ (probe branes)

	0	1	2	3	4	5	6	7	8	9
D4	x	x	x	x	x					
D8/$\overline{\text{D8}}$	x	x	x	x		x	x	x	x	x



4-8, 4-$\bar{8}$ strings
$\to$ fundamental, massless chiral fermions

flavor symmetry
$U(N_f)_L \times U(N_f)_R$

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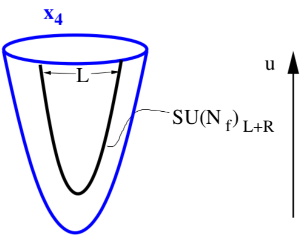

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spontaneously broken because D8-$\overline{\text{D8}}$ have to join in cigar-shaped topology

for now: maximal separation in x_4 (antipodal on x_4 circle): $L = \pi/M_{\text{KK}}$

Quantitative predictions

Parameter-free prediction of (axial-)vector meson mass pattern:

Isotriplet Meson	$\lambda_n = m^2/M_{\rm KK}^2$	m/m_ρ	$(m/m_\rho)^{\rm exp.}$	$(m/m_\rho)^{N\to\infty}$
1^{--} (ρ)	0.669314	1	1	1
1^{++} (a_1)	1.568766	1.531	1.59(5)	1.86(2)
1^{--} (ρ^*)	2.874323	2.072	1.89(3)	2.40(4)
1^{++} (a_1^*)	4.546104	2.606	2.12(3)	2.98(5)

(last column from lattice study by Bali et al. JHEP 06, 071 (2013))

agreement within $\lesssim 20\%$

not bad, given that WSS is not yet large-N QCD (in particular at scales $\gtrsim M_{\rm KK}$)

(near-perfect agreement for m_{a_1}/m_ρ with real QCD certainly fortuitous)

Quantitative predictions

Other predictions depend on value of 't Hooft coupling λ at scale $M_{\rm KK}$

Matching

1. $m_\rho \approx 776$ MeV fixes $\boxed{M_{\rm KK} = 949 \text{ MeV}}$ ($\Rightarrow T_{deconf} = 151$ MeV)
2. $f_\pi^2 = \frac{\lambda N_c}{54\pi^4} M_{\rm KK}^2$ gives $\boxed{\lambda = g_{\rm YM}^2 N_c \approx 16.63}$ [Sakai&Sugimoto 2005-7]
(matching instead large-N_c lattice result [Bali et al. 2013] for $m_\rho/\sqrt{\sigma}$ gives $\lambda \approx 12.55$)

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(matching instead large-N_c lattice result [Bali et al. 2013] for $m_\rho/\sqrt{\sigma}$ gives $\lambda \approx 12.55$)

yields (for $N_c = 3$ and $\lambda = 16.63 \ldots 12.55$):

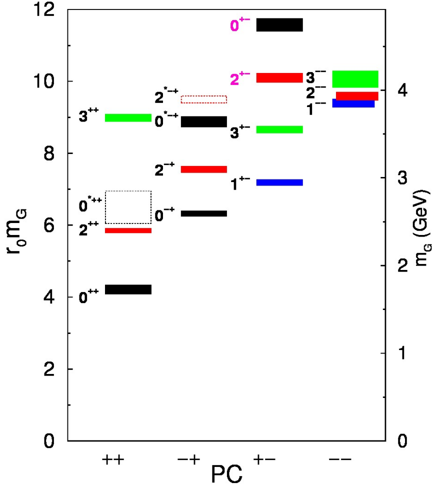
- LO decay rate of ρ meson $\sim \lambda^{-1} N_c^{-1}$
$\Gamma_{\rho\to 2\pi}/m_\rho = 0.1535 \ldots 0.2034$ (exp.: 0.191(1))
- decay rate for $\omega \to 3\pi$ (from Chern-Simons part of D8 action) $\sim \lambda^{-4} N_c^{-2}$
$\Gamma_{\omega\to 3\pi}/m_\omega = 0.0033 \ldots 0.0102$ (exp.: 0.0097(1))
- gluon condensate [Kanitscheider, Skenderis & Taylor JHEP 0809]
$C^4 \equiv \langle \frac{\alpha_s}{\pi} F_{\mu\nu}^2 \rangle = \frac{4}{3^7 \pi^4} N_c \lambda^2 M_{\mathrm{KK}}^4 \simeq 0.0126 \ldots 0.0072 \text{ GeV}^4$
classical SVZ value: 0.012 GeV^4 (lattice higher but with large subtraction ambiguities)

Lattice vs. supergravity glueballs

seemingly good qualitative agreement by matching up 2^{++}

(but AdS spectrum somewhat stretched and slightly too many 0^{++})

Morningstar & Peardon hep-lat/9901004:



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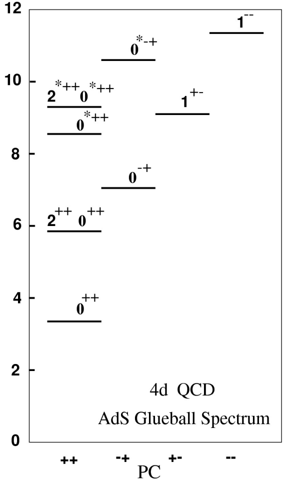

Lattice vs. supergravity glueballs in Sakai-Sugimoto model

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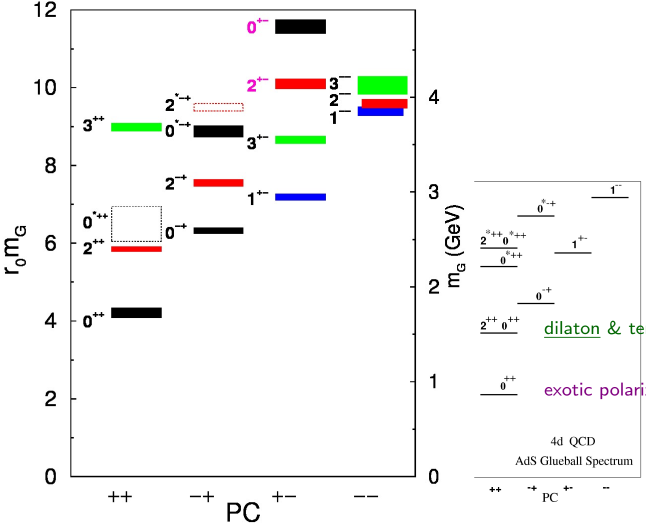

dilaton & tensor 1487 MeV

exotic polarization 855 MeV

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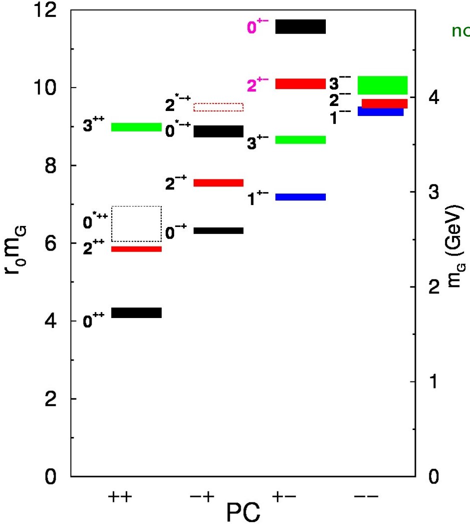
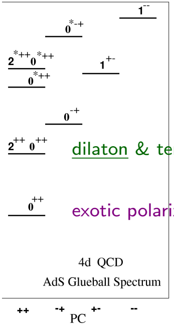


now good match lowest $0^{++} \leftrightarrow$ dilaton



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- possibly not part of spectrum of holographic QCD in limit $M_{\text{KK}} \to \infty, \lambda \to 0$ (already asked by Constable & Myers)
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Nonrealistic degeneracy of dilatonic 0^{++} and tensor 2^{++} suggests that supergravity approximation insufficient for masses

Take good results for (dimensionless) mesonic Γ/m as encouragement for calculation of relative width

Glueball-$\bar{q}q$ couplings in Sakai-Sugimoto model

Gravitational modes stable in confined background, but
can calculate *effective action for glueball-$\bar{q}q$ interactions*

done for lowest (exotic) mode by
Hashimoto, Tan & Terashima, Phys.Rev. D77 (2008) 086001 [arXiv:0709.2208]

revisited, corrected, and extended to other modes by
Brünner, Parganlija & AR, Phys.Rev. D91 (2015) 106002 [arXiv:1501.07906]

For example: Vertices of one glueball and two (massless) pions
for "exotic" mode:

$$S_{G_E\pi\pi} = \mathrm{Tr}\int d^4x \frac{1}{2}\partial_\mu\pi\,\partial_\nu\pi\left(\breve{c}_1\eta^{\mu\nu} - c_1\frac{\partial^\mu\partial^\nu}{M_E^2}\right)G_E$$

for "predominantly dilatonic" mode:

$$S_{G_D\pi\pi} = \mathrm{Tr}\int d^4x \frac{1}{2}\partial_\mu\pi\,\partial_\nu\pi\,\tilde{c}_1\left(\eta^{\mu\nu} - \frac{\partial^\mu\partial^\nu}{M_D^2}\right)G_D$$

with $\{c_1, \breve{c}_1, \tilde{c}_1\} = \{62.66, 16.39, 17.23\} \times \lambda^{-1/2}N_c^{-1}M_{\mathrm{KK}}^{-1}$

and many more: $S_{G\rho\rho} \propto \lambda^{-1/2}N_c^{-1}$, $S_{G\rho\pi\pi} \propto \lambda^{-1}N_c^{-3/2}$,...

Glueball decay rates in Sakai-Sugimoto model

Results for decay into two pions:

Exotic mode: $\Gamma_{G_E \to \pi\pi}/M_E \approx \dfrac{13.79}{\lambda N_c^2} \approx 0.092 \dots 0.122 \quad (M_E \approx 855\text{MeV})$

Dilaton mode: $\Gamma_{D \to \pi\pi}/M_D \approx \dfrac{1.359}{\lambda N_c^2} \approx 0.009 \dots 0.012 \quad (M_D \approx 1487\text{MeV})$

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Most likely *experimental candidates* for meson with dominant scalar glueball content: $f_0(1500)$ or $f_0(1710)$

$$\Gamma^{(\text{ex})}(f_0(1500) \to \pi\pi)/(1505\text{MeV}) = 0.025(3)$$
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- another hint that G_E should be discarded?

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- another hint that G_E should be discarded?
- or could it perhaps correspond to broad glueball component of σ-meson à la Narison 1998: QCD sum rules need very broad glueball around 1 GeV plus narrow glueball around 1.5 GeV

  (cp.: Janowski et al. 1408.4921: eLSM fit of $f_0(1710)$ as predominantly glue, but only with extremely large gluon condensate)

Glueball decay rates in Sakai-Sugimoto model (cont'd)

Full decay pattern:

decay $G_D \to 4\pi$ suppressed (below 2ρ threshold): $\Gamma_{G\to 4\pi}/\Gamma_{G\to 2\pi} \sim \lambda^{-1} N_c^{-1}$,
while $f_0(1500) \to 4\pi$ dominant:

decay	Γ/M (PDG)	$\Gamma/M[G_D]$
$f_0(1500)$ (total)	0.072(5)	0.027...0.037
$f_0(1500) \to 4\pi$	0.036(3)	0.003...0.005
$f_0(1500) \to 2\pi$	0.025(2)	0.009...0.012
$f_0(1500) \to 2K$	0.006(1)	0.012...0.016
$f_0(1500) \to 2\eta$	0.004(1)	0.003...0.004

$\Rightarrow$ $f_0(1500)$ seemingly disfavored, at least when nearly pure glue

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$f_0(1710) \to \pi\pi$ OK,
but $f_0(1710)$ decays predominantly into $K\bar{K}$!

— not reproduced by (chiral) WSS model,
but may be due to mechanism of "chiral suppression of scalar glueball decay" (Chanowitz 2005)

Nonchiral enhancement in mass-deformed WSS?

F. Brünner & AR, PRL 115 (2015) 131601 [1504.05815]

Current quark masses can be introduced in principle through deformations of the WSS model by either world-sheet instantons or with bifundamental background scalar $\mathcal{T}$

both lead to

$$\int d^4x \int_{u_{\rm KK}}^{\infty} du\, h(u)\, {\rm Tr}\, \left(\mathcal{T}(u)\, {\rm P}\, e^{-i\int dz A_z(z,x)} + h.c. \right),$$

where $h(u)$ includes metric (glueball) fields

Choosing appropriate boundary conditions for $\mathcal{T}$, the quark mass matrix arises through

$$\int_{u_{\rm KK}}^{\infty} du\, h(u)\, \mathcal{T}(u) \propto \mathcal{M} = {\rm diag}(m_u, m_d, m_s),$$

thereby realizing a Gell-Mann-Oakes-Renner relation.

Witten-Veneziano mass term

Already in chiral model:

WSS contains (fully determined) Witten-Veneziano mass term for singlet η_0 pseudoscalar from U(1)$_A$ anomaly contributions $\sim 1/N_c$

$$m_0^2 = \frac{N_f}{27\pi^2 N_c}\lambda^2 M_{\rm KK}^2$$

from $S_{C_1} = -\frac{1}{4\pi(2\pi l_s)^6}\int d^{10}x\sqrt{-g}|\tilde{F}_2|^2$ with $\tilde{F}_2 = \frac{6\pi u_{\rm KK}^3 M_{\rm KK}^{-1}}{u^4}\left(\theta + \frac{\sqrt{2N_f}}{f_\pi}\eta_0\right) du \wedge dx^4$,
where θ is the QCD theta angle and $\eta_0(x) = \frac{f_\pi}{\sqrt{2N_f}}\int dz\, {\rm Tr} A_z(z,x)$.

With $N_f = N_c = 3$, $M_{\rm KK} = 949$ MeV, $\lambda = 16.63 \ldots 12.55$: $\boxed{m_0 = 967 \ldots 730 \text{ MeV}}$

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With $N_f = N_c = 3$, $M_{\rm KK} = 949$ MeV, $\lambda = 16.63\ldots 12.55$: $\boxed{m_0 = 967\ldots 730 \text{ MeV}}$

With finite quark masses η_0 and η_8 no longer mass eigenstates.

Diagonalizing (with $\mathcal{M} = {\rm diag}(\hat{m},\hat{m},m_s)$, fixing $m_\pi = 140$ MeV and $m_K = 497$ MeV) $\rightarrow$

$$m_\eta = 518\ldots 476\ \text{MeV}, \quad m_{\eta'} = 1077\ldots 894\ \text{MeV},$$
$$\theta_P = -14.4^\circ \ldots -24.2^\circ,$$

nice ballpark!

Nonchiral enhancement in mass-deformed WSS!

Holographic realization of mass terms give additional vertices between glueballs and pseudoscalars
Rigorously calculable for $G_D \eta_0^2$,

$$\mathcal{L}^{\text{chiral}}_{G_D \eta_0 \eta_0} = \frac{3}{2} d_0 m_0^2 \, \eta_0^2 G_D, \qquad d_0 \approx \frac{17.915}{\lambda^{1/2} N_c M_{\text{KK}}}$$

but not (yet) calculable for octet.
Parametrize uncertainty by free parameter x:

$$\mathcal{L}^{\text{massive}}_{G_D \pi\pi} = \frac{3}{2} d_m G_D \mathcal{L}^{\mathcal{M}}_m, \qquad d_m \equiv x d_0$$

Most symmetric choice $x = 1$ ($\Leftrightarrow$ no $G_D \to \eta\eta'$)
$\to$ relatively strong enhancement factor for kaons and η mesons:

$$\Gamma^{\text{chiral}}_{G\to PP} \to \Gamma^{\text{chiral}}_{G\to PP} \times \left(1 - 4\frac{m_P^2}{M_G^2}\right)^{1/2} \left(1 + 8.480\frac{m_P^2}{M_G^2}\right)^2$$

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cp. to J.Ellis & J.Lanik [PLB 150 (1985) 289]:
glueballs from effective dilaton theory also with some "nonchiral enhancement"

but too weak to overcome kinematic suppression: $\left(1 - 4\frac{m_P^2}{M_G^2}\right)^{1/2} \left(1 + \frac{m_P^2}{M_G^2}\right)^2 = 1 - O(m_P^4/M_G^4)$

Comparison with $f_0(1710)$

decay	Γ/M **(PDG)**	$\Gamma/M[G_D]$ (chiral)	$\Gamma/M[G_D]$ (massive)
$f_0(1710)$ (total)	**0.081(5)**	0.059...0.076	0.083...0.106
$f_0(1710) \to 2K$	(*) 0.029(10)	0.012...0.016	0.029...0.038
$f_0(1710) \to 2\eta$	0.014(6)	0.003...0.004	0.009...0.011
$f_0(1710) \to 2\pi$	$0.012(^{+5}_{-6})$	0.009...0.012	0.010...0.013
$f_0(1710) \to 2\rho, \rho\pi\pi \to 4\pi$	?	0.024...0.030	0.024...0.030
$f_0(1710) \to 2\omega$	$0.010(^{+6}_{-7})$	0.011...0.014	0.011...0.014
$f_0(1710) \to \eta\eta'$	?	0	if 0 : $\Uparrow$
$\Gamma(\pi\pi)/\Gamma(K\bar{K})$	**0.41** $^{+0.11}_{-0.17}$	**3/4**	**0.35**
$\Gamma(\eta\eta)/\Gamma(K\bar{K})$	**0.48**± 0.15	1/4	0.28

* PDG ratios for decay rates + $\mathrm{Br}(f_0(1710) \to KK) = 0.36(12)$ [Albaladejo&Oller 2008]

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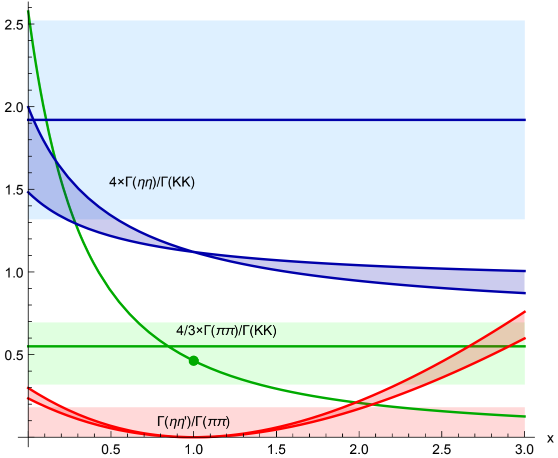
- decays into 2 pseudoscalars: massive WSS perfectly compatible with PDG data!
- significant decay into 4 pions (after extrapolation to beyond 2ρ threshold): falsifiable prediction of this model!
  ($f_0(1710) \to 2\rho^0$ forthcoming from CMS-TOTEM!)

Constraints on $\eta\eta'$ rates for $f_0(1710)$ as ≈pure glueball

Relaxing $x = 1$: [F. Brünner & AR, PRD92, 1510.07605]
WSS model gives flavor asymmetries consistent with experimental results for $f_0(1710)$ in as long as $\Gamma(G \to \eta\eta')/\Gamma(G \to \pi\pi) \lesssim 0.04$ (upper limit from WA102: < 0.18)



Tensor glueball decay rates in Sakai-Sugimoto model

Tensor glueball in WSS, and extrapolated to higher mass:

decay	M	$\Gamma/M[T(M)]$
$T \to 2\pi$	1487	0.013...0.018
$T \to 2K$	1487	0.004...0.006
$T \to 2\eta$	1487	0.0005...0.0007
T (total)	1487	$\approx 0.02 \dots 0.03$
$T \to 2\rho \to 4\pi$	2000	0.135...0.178
$T \to 2\omega \to 6\pi$	2000	0.045...0.059
$T \to 2\pi$	2000	0.014...0.018
$T \to 2K$	2000	0.010...0.013
$T \to 2\eta$	2000	0.0018...0.0024
T (total)	2000	$\approx 0.16 \dots 0.21$

With a mass of 2 GeV, the relative width turns out to be comparable with that of the rather broad tensor meson $f_2(1950)$, which has $\Gamma/M = 0.24(1)$.

Very narrow (unconfirmed) candidate $f_J(2220)$ not compatible with WSS

Summary – Glueballs in Witten-Sakai-Sugimoto model

After fitting just m_ρ to fix $M_{\rm KK} = 949$ MeV

- good prediction of higher vector and axial vector mesons masses,
- reasonable prediction of glueball masses if “exotic mode” discarded

after fitting f_π or $m_\rho/\sqrt{\sigma}$ to also fix 't Hooft coupling at $\lambda = 16.63 \ldots 12.55$

- good prediction of ρ and ω decay rates
- good prediction of anomalous $m'_\eta \propto N_c^{-\frac{1}{2}} \lambda M_{\rm KK}$

Holographic glueball decay rates:

- narrow partial width $G_D \to \pi\pi$,
  quite compatible with experimental data for $f_0(1710)$
- much stronger decay of $f_0(1710)$ into $K\bar{K}$ need not be indicative of $s\bar{s}$ nature – well reproduced if (so far unobserved) decay into $\eta\eta'$ small
- tensor glueball broad if at $\gtrsim 2$ GeV