Isovector observables in nuclear mean-field theories

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Acknowledgements

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Outline

1. Parameters and observables
2. Optimization of model parameters and subsequent variation
3. The influence of $J = a_{\rm sym}$ on observables
4. The weak-charge formfactor and correlation with r_n
5. A few words about low-lying dipole strength (“pygmy region”)

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  SHF/RMF are adjusted to data by χ^2 fits.
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- **Weak-charge formfactor F_W:**
  χ^2 uncertainties depend on q, maximal at $q_{\mathrm{PREX}} \leftrightarrow$ PREX data most informative
- **Low E dipole strenth:**
  Dominated by isoscalar $L = 1$ modes, spread over several distinct modes.
  Integrated dipole strength yields combined information on a_{sym} and κ.

Parameters and observables

The nuclear matter parameters (NMP)

given $E/A(\rho)$ = energy per particle in symmetric nuclear matter (function of density ρ)
this allows to define basic properties near equilibrium:

E/A_{eq}	binding energy per particle at equilibrium point
ρ_0	equilibrium density
$K = 9\rho_0^2\partial_\rho^2 \frac{E}{A}$	incompressibility (isoscalar static response)
$\frac{m^*}{m}$	effective mass (isoscalar dynamic response)
$J = a_{\mathrm{sym}}$	symmetry energy (isovector static response)
κ	TRK sum rule enhancement $\leftrightarrow$ isovector $\frac{m_1^*}{m}$ (dynamic response)
$L = 3\rho_0\partial_\rho J$	slope of symmetry energy

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considered here as part of the model parameters
(e.g.: equivalent to t_0, x_0, t_1, x_1, t_3, x_3, α in case of SHF)

Observables in the pool of fit data

E_B	binding energy	70 nuclei
r_C	charge r.m.s. radii	50 nuclei
$R_{\mathrm{diffr,C}}$	charge diffraction radii (first zero of charge formfactor)	27 nuclei
σ_C	charge surface thickness (first max. of charge formfactor)	26 nuclei
Δ_p	proton pairing gaps (third difference of E_B)	21 nuclei
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data have been scrutinized to have negligible effects from ground state correlation

"Predicted" observables in correlation study

- neutron r.m.s. radius r_n in ^{208}Pb
- neutron skin $r_n - r_p$ in ^{208}Pb
- weak-charge formfactor $F_W(q_{\mathrm{PREX}} = 0.475/\mathrm{fm})$ in ^{208}Pb (related to PREX)

$$F_W(q) = G_n^Z(q) F_n(q) + G_p^Z(q) F_p(q)$$

$G^Z_{p/n}$ = weak-charge formfactor for nucleon

$F_{p/n}$ = formfactor of nuclear distributions (Fourier transform of $\rho(\mathbf{r})$)

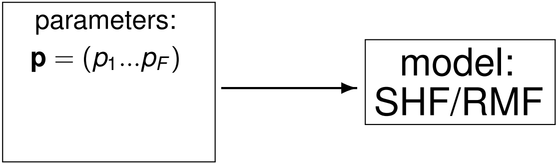
- dipole polarizability $\alpha_D = \int_0^\infty dE\, \sigma_{photoabs}(E)\, E^{-2}$ in ^{208}Pb
- binding energy E_B for extremely exotic nuclei (super heavy, very neutron rich)
- Q_α value for super-heavy element
- B_f = fission barrier in ^{266}Hs
- surface energy a_{surf} (computed from semi-infinite matter)
- low-lying dipole strength (from photo-absorption cross section $\sigma_{photoabs}(E)$)

Optimization of model parameters and subsequent variation

Optimization of model parameters and exploring its variation

model:
SHF/RMF

Optimization of model parameters and exploring its variation



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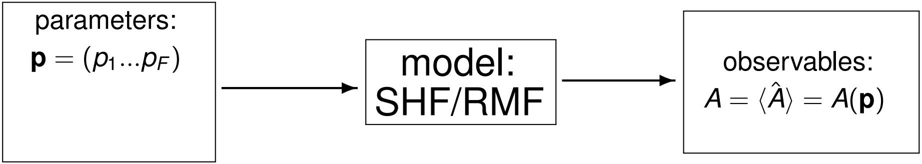


Optimization of model parameters and exploring its variation

parameters:
$\mathbf{p} = (p_1 ... p_F)$
partly NMP:
K, $\frac{m^*}{m}$, J, κ, L

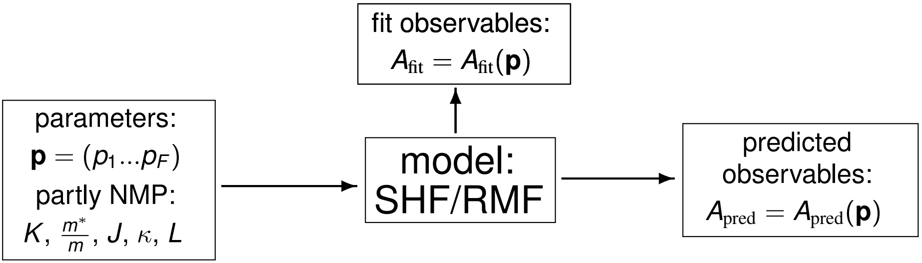
→

model:
SHF/RMF

→

observables:
$A = \langle \hat{A} \rangle = A(\mathbf{p})$

Optimization of model parameters and exploring its variation



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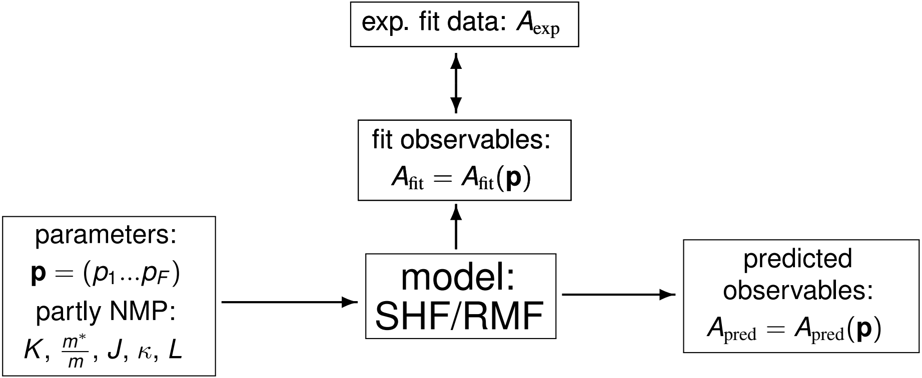

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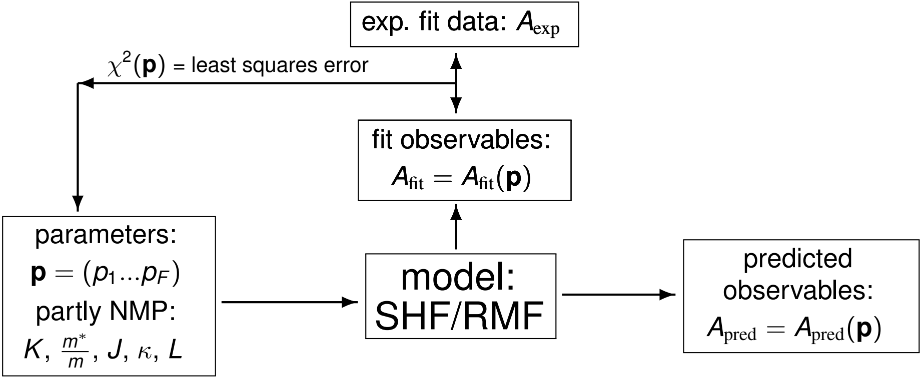

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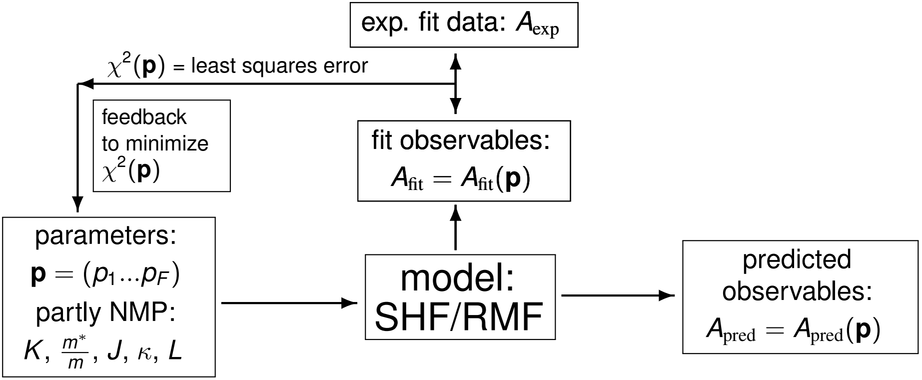

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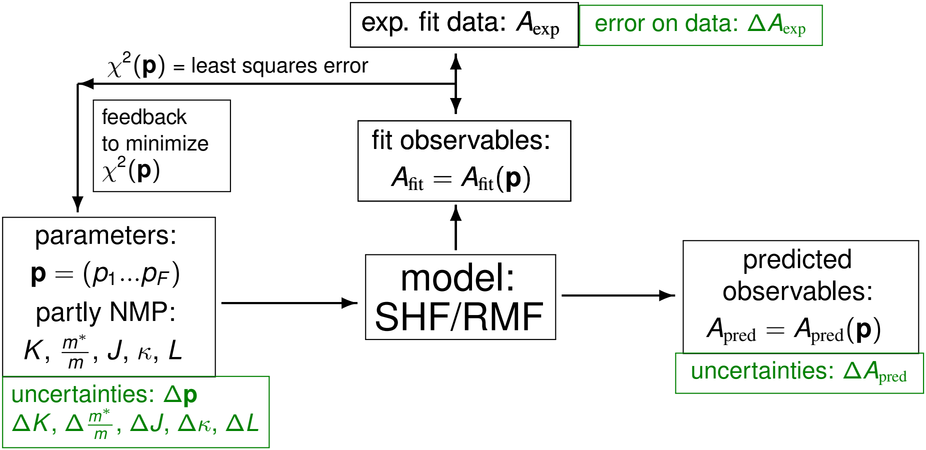

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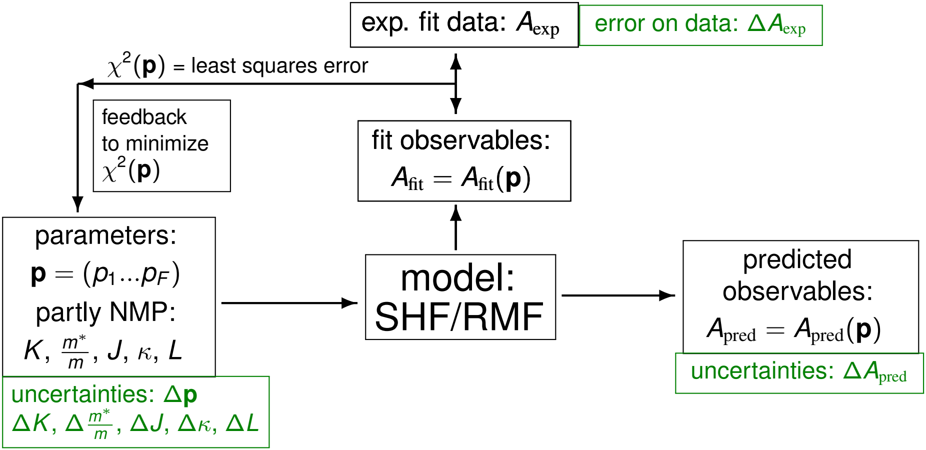

particularly large are
ΔJ, $\Delta\kappa$, ΔL (=isovector NMP)

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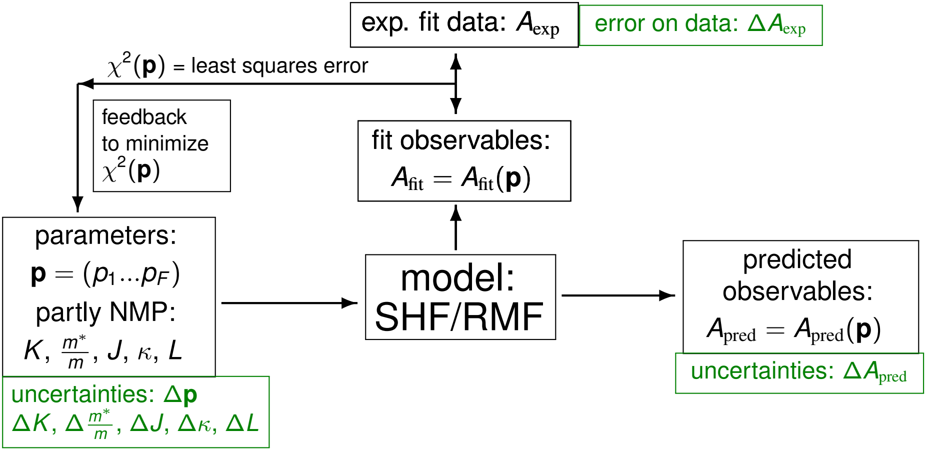
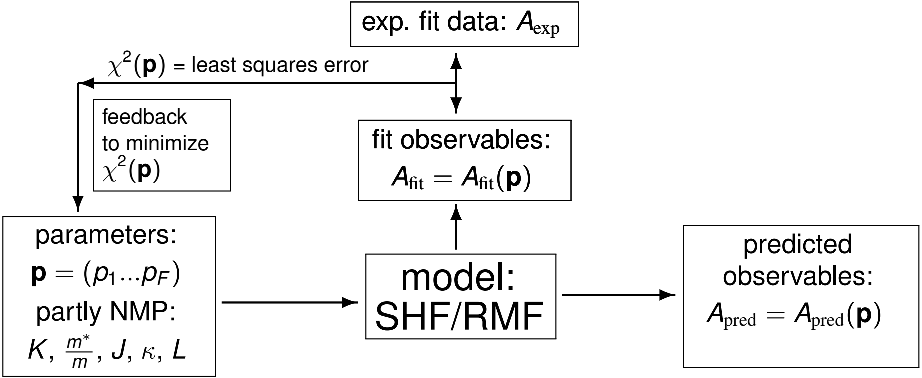


particularly large are
ΔJ, $\Delta\kappa$, ΔL (=isovector NMP)

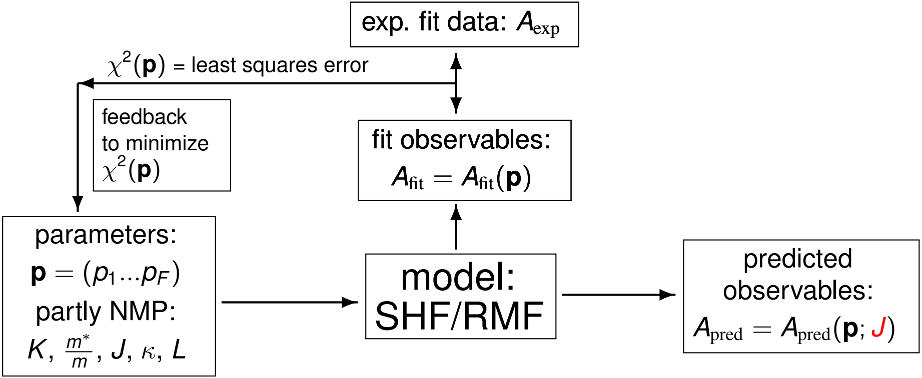
$\Longrightarrow$ study effect of variation ΔJ

Trend analysis: dedicated variation of J



fix J during fit $\Longrightarrow$
produce parameter set $\mathbf{p}(J)$
for a series of J

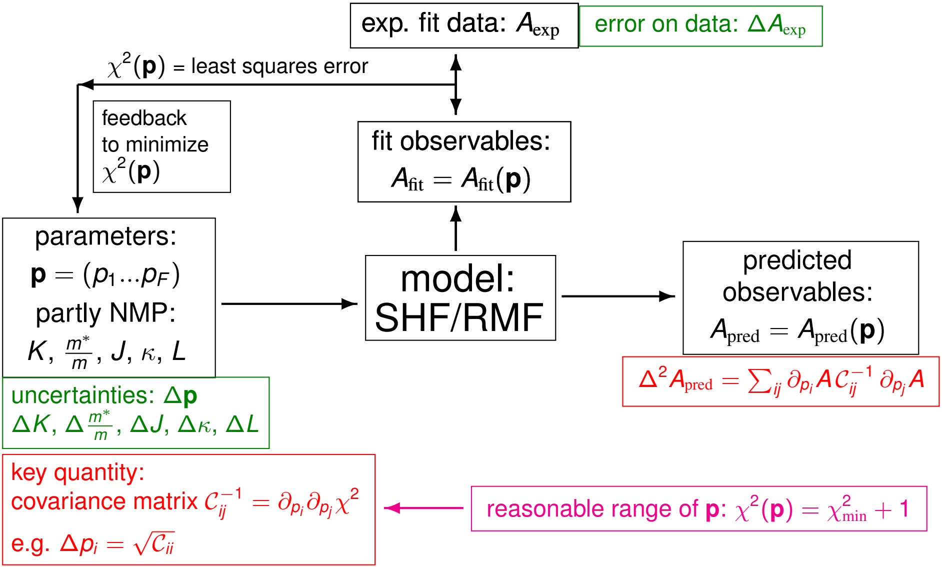
Trend analysis: dedicated variation of J



fix J during fit $\Longrightarrow$
produce parameter set $\mathbf{p}(J)$
for a series of J

trends $A = A(J)$
reveal dependences

Correlation analysis: propagate $\Delta\mathbf{p}$ to mixed variances $\Delta^2(A_1, A_2)$



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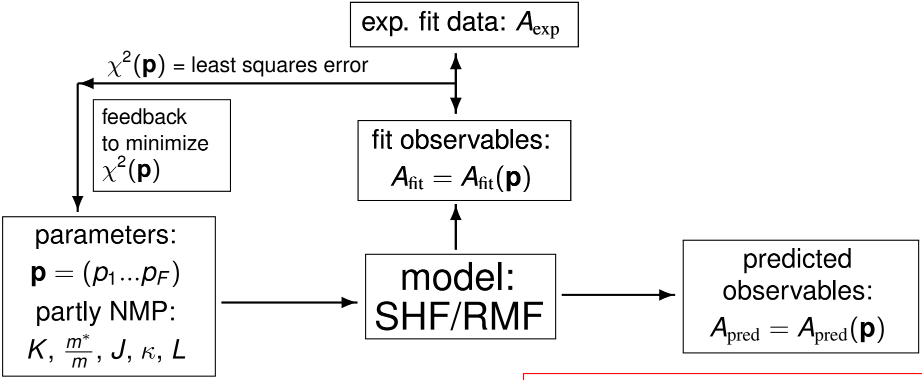


mixed variance:
$\Delta^2(AB) = \sum_{ij} \partial_{p_i} A\, \mathcal{C}^{-1}_{ij}\, \partial_{p_j} B$

covariance: $c_{AB} = \dfrac{\Delta^2(AB)}{\sqrt{\Delta^2(AA)\Delta^2(BB)}}$

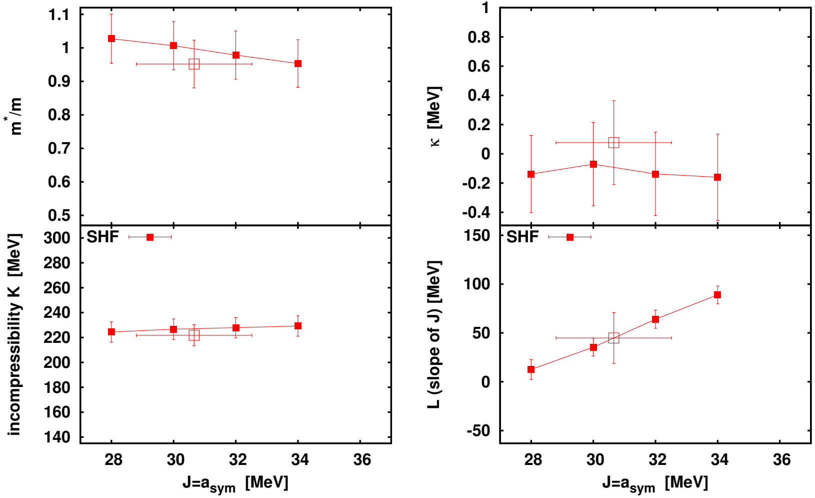
$c_{AB} = 1 \;\leftrightarrow\;$ highly correlated
$c_{AB} = 0 \;\leftrightarrow\;$ uncorrelated

key quantity:
covariance matrix $\mathcal{C}^{-1}_{ij} = \partial_{p_i}\partial_{p_j}\chi^2$

e.g. $\Delta p_i = \sqrt{\mathcal{C}_{ii}}$

The influence of $J = a_{\mathrm{sym}}$ on observables

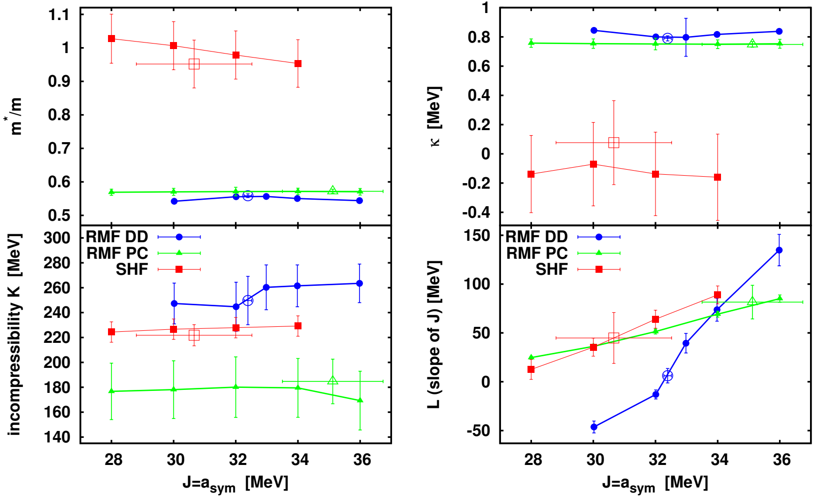
Trends of nuclear matter properties (NMP) with $a_{\rm sym}$ for the case of SHF



K, m^*/m, κ independent of J $(= a_{\rm sym})$ $\Longrightarrow$ four independent model parameters

L = slope of J strongly linked with J $\longleftrightarrow$ hidden correlation in data (and model)

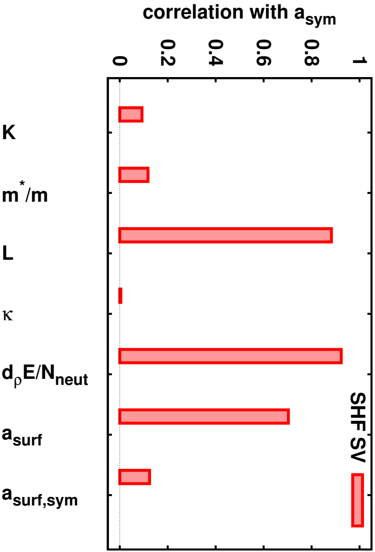
Trends of nuclear matter properties (NMP) with $a_{\rm sym}$ – SHF and RMF



for all cases: K, m^*/m, κ independent of $a_{\rm sym}$, and L strongly linked with J

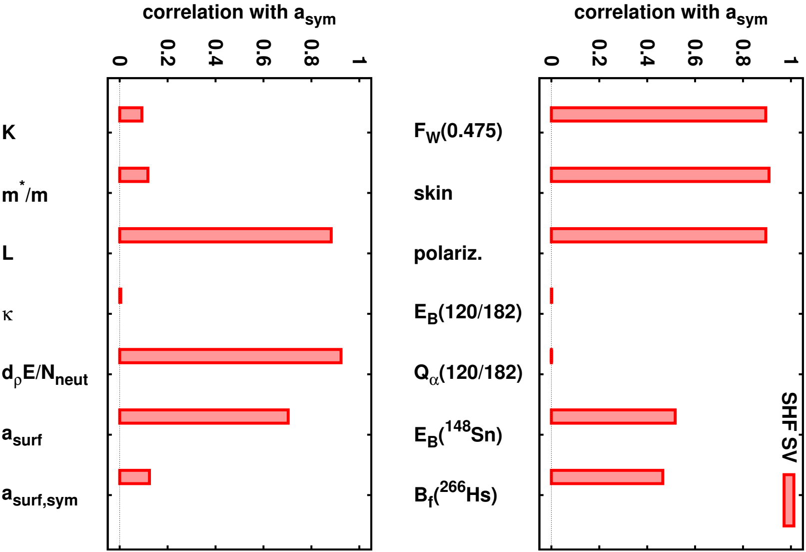
large differences SHF $\leftrightarrow$ RMF: mass parameters m^*/m and κ

Correlations of $a_{\rm sym}$ with NMP, for the case of SHF



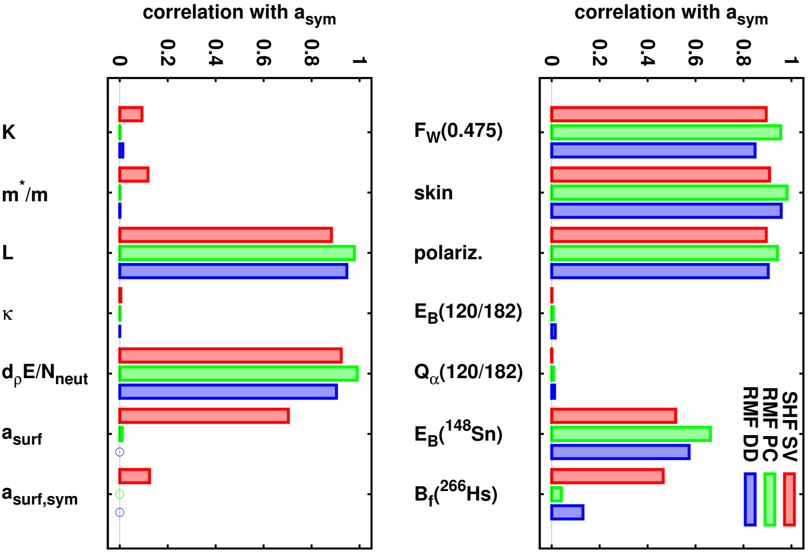
Again: K, m^*/m, κ perfectly independent; but strong correlation with L, $\partial_\rho E_{\rm neut}/N$
surface properties $a_{\rm surf}$, $a_{\rm surf,sym}$ show a mixed picture

Correlations of a_{sym} with NMP and observables in finite nuclei – SHF



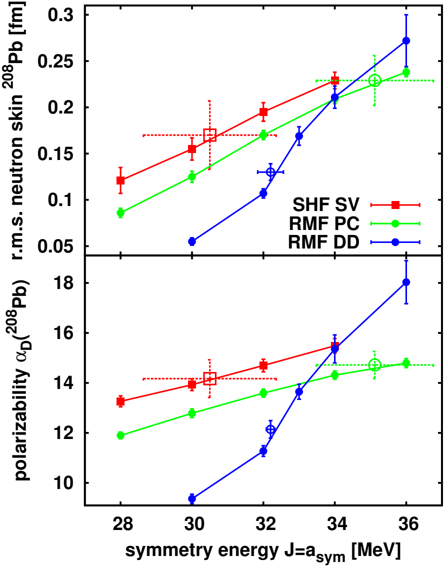
Good isovector observables (in ^{208}Pb): weak-charge formfactor F_W, neutron skin $r_n - r_p$, p
superheavy elements uncorrelated, neutron rich nuclei & fission somewhat correlated

Correlations of a_{sym} with NMP and obs. in finite nuclei – SHF & RMF



RMF models shows nearly the same correlations as SHF
except for surface properties and fission

Trends of isovector observables in ^{208}Pb with a_{sym}



for each model: clear trends with a_{sym}

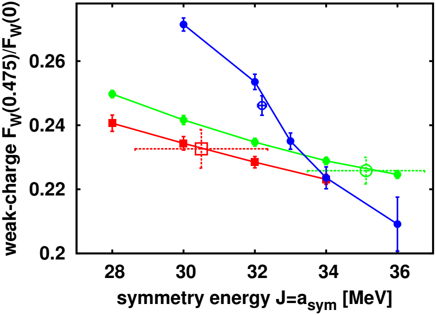
SHF $\leftrightarrow$ RMF-PC: same slope,

different offset

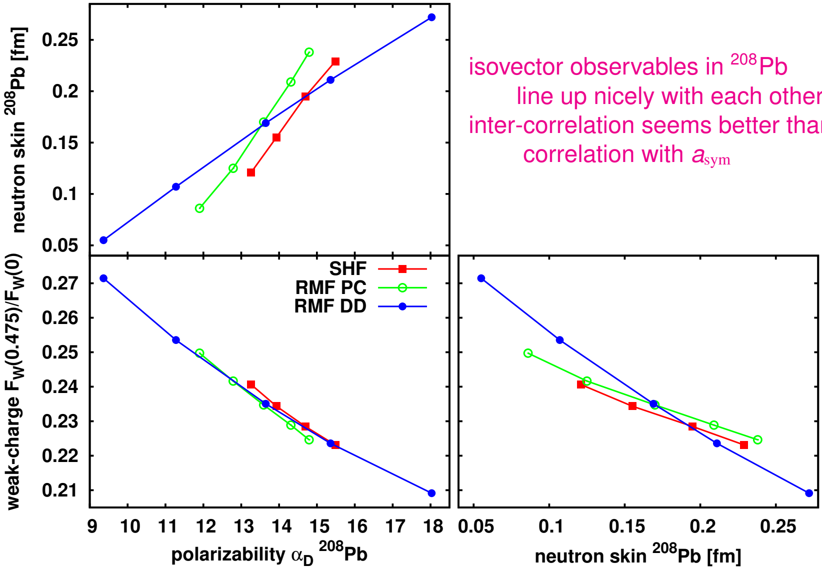
RMF-DD: even different slope

$\leftrightarrow$ different ρ-dependence

(but small variance of a_{sym})



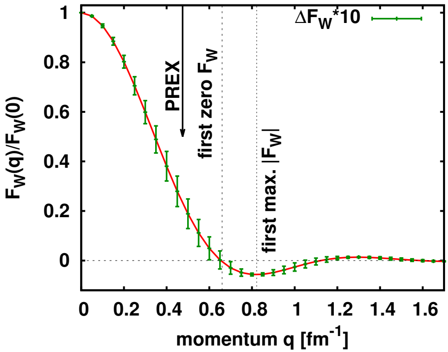
Trends of observables in ^{208}Pb against each other



isovector observables in ^{208}Pb
line up nicely with each other
inter-correlation seems better than
correlation with $a_{\rm sym}$

The weak-charge formfactor and correlation with r_n

Weak-charge formfactor with uncertainty, for ^{208}Pb with SV-min



$$F_W(q) = G_n^Z(q) F_n(q) + G_p^Z(q) F_p(q)$$

$G^Z_{p/n}$ = weak-charge FF nucleon

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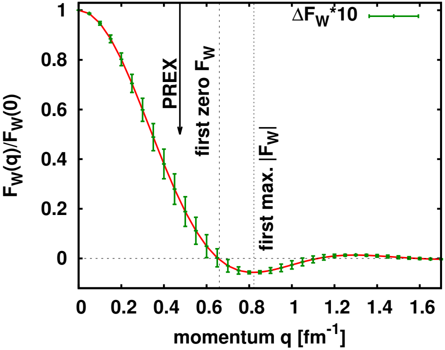

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$$F_{p/n} = \text{FF of } \rho_{p/n}(\mathbf{r})$$

dominated by neutron formfactor F_n

$\leftrightarrow$ measure neutron radius r_n

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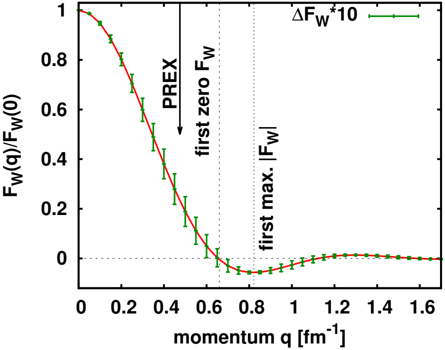

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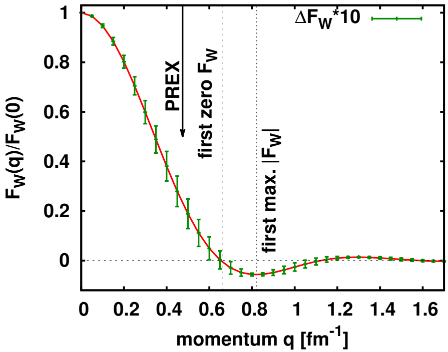
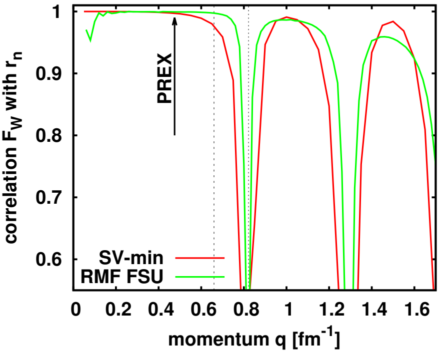
extrapolation uncertainties depend on q

large at PREX point $q_{\mathrm{PREX}} = 0.475$/fm

$\Longrightarrow$ PREX provides new information

Weak-charge formfactor and its correlation with neutron radius r_n



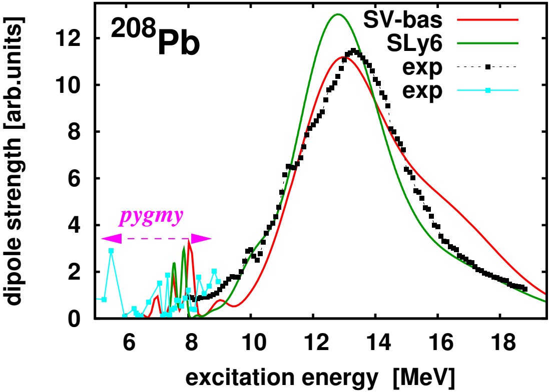


extrapolation uncertainties depend on q
large at PREX point $q_{\rm PREX} = 0.475$/fm
$\Longrightarrow$ PREX provides new information

F_W highly correlated with r_n at low q
lack of correlation near first zero of F_W
SHF and RMF very similar

A few words about low-lying dipole strength (“pygmy region”)

"Pygmy strengh" = low lying peaks in isovector dipole spectrum



pygmy "resonance"
= bunches of dipole strength
safely below GDR region

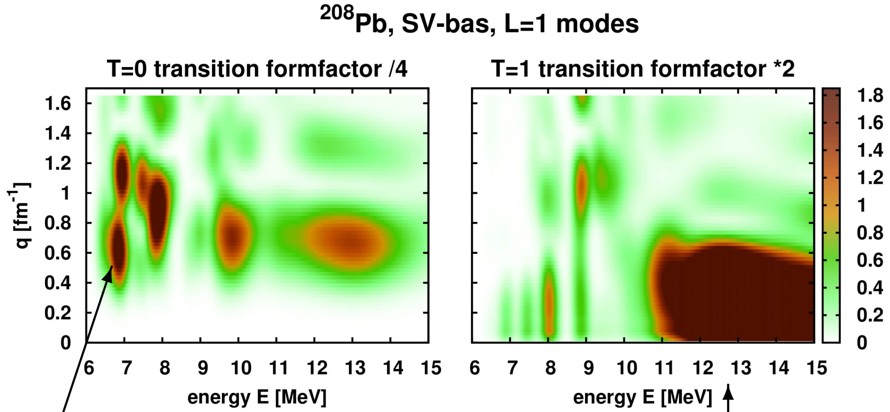
various interpretations:
1) 1ph structures
2) collective motion of neutron surface versus bulk

(SHF surveys favor option 1)

choose a cut off at E_{cut}=9.5 MeV to define

$$\text{"pygmy strength"} = \int_0^{E_{cut}} dE\ \sigma_{photoabs}(E)/E$$

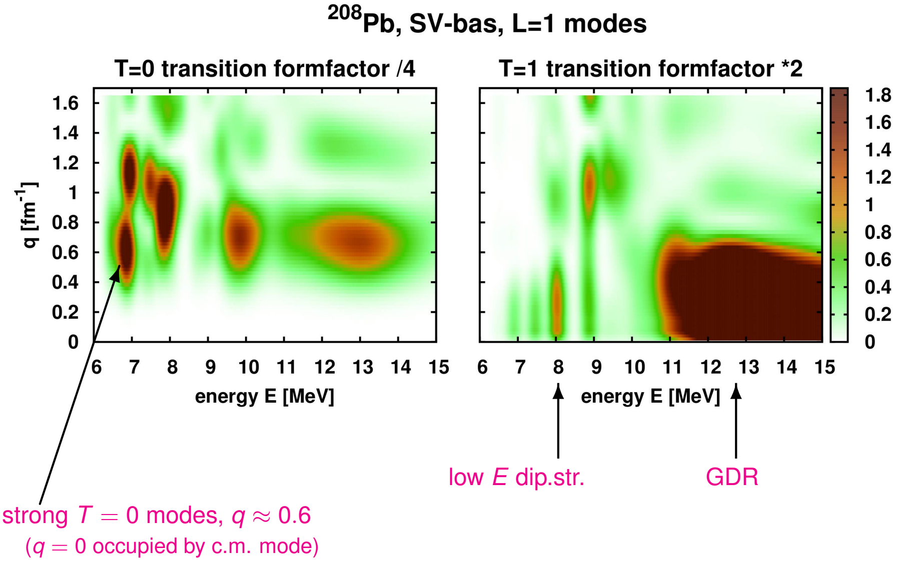
Transition formfactor $f_{\mathrm{trans}}(q, E)$ for $L = 1$ modes in ^{208}Pb



GDR

strong $T = 0$ modes, $q \approx 0.6$
($q = 0$ occupied by c.m. mode)

Transition formfactor $f_{\text{trans}}(q, E)$ for $L = 1$ modes in ^{208}Pb



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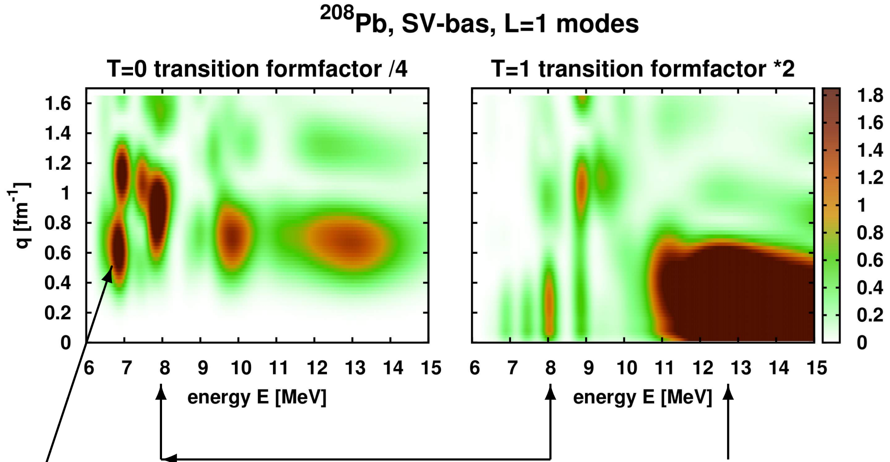

related to strong $T = 0$ mode $\leftarrow$ low E dip.str.

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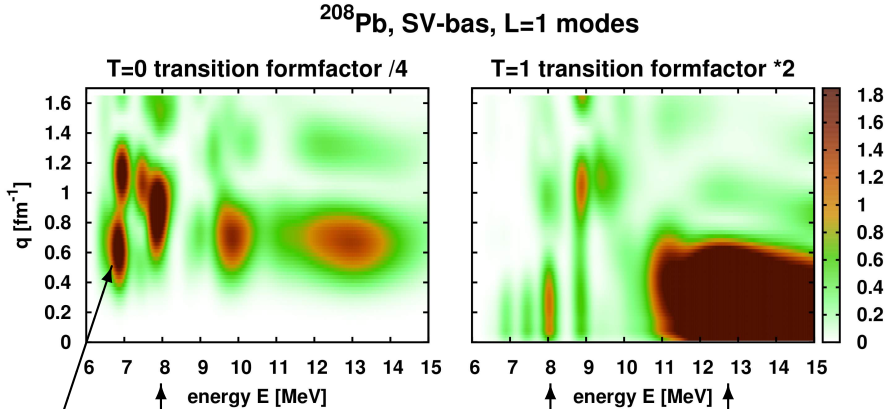

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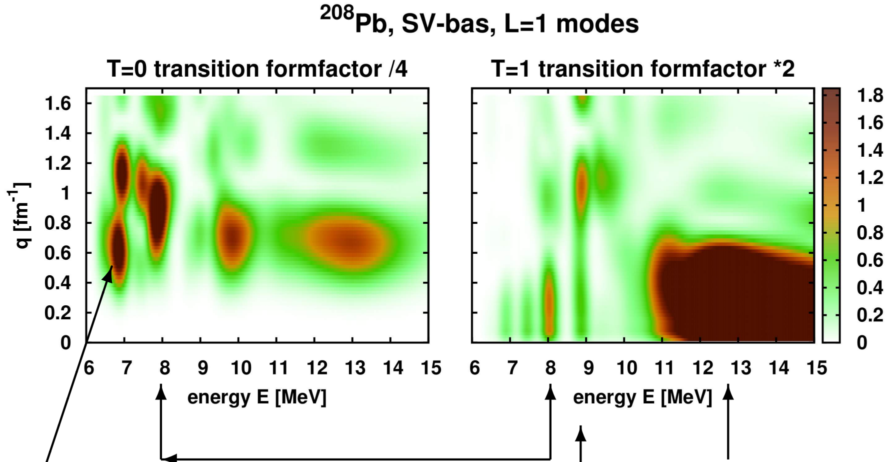

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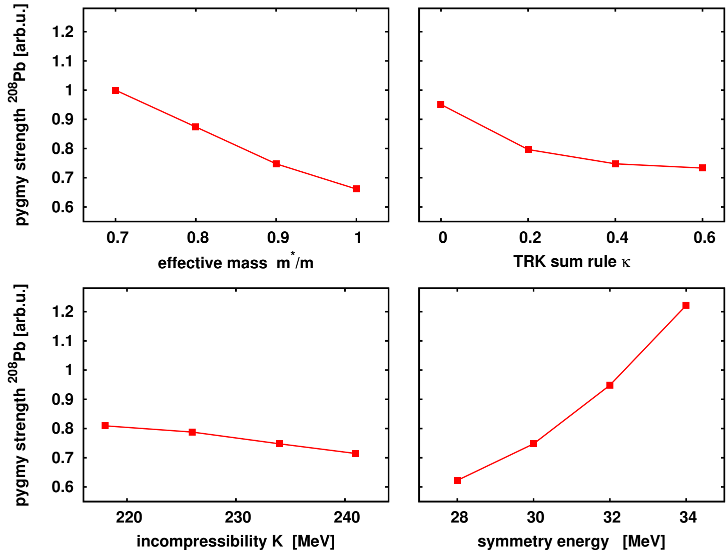
low E dipoles: mixed modes, predominantly isoscalar, nonetheless useful info on NMP

Trends of low-lying dipole strength with nuclear matter parameters



pygmy str.:
$\int_0^{E_{\rm cut}} dE\,\sigma(E)/E$

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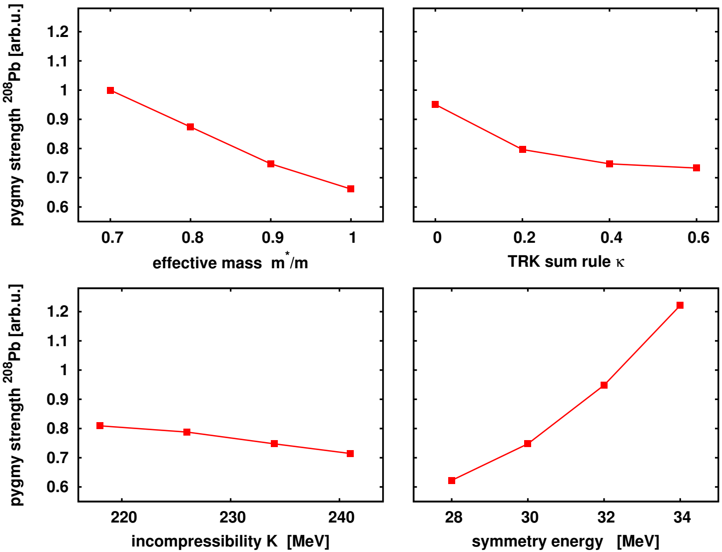

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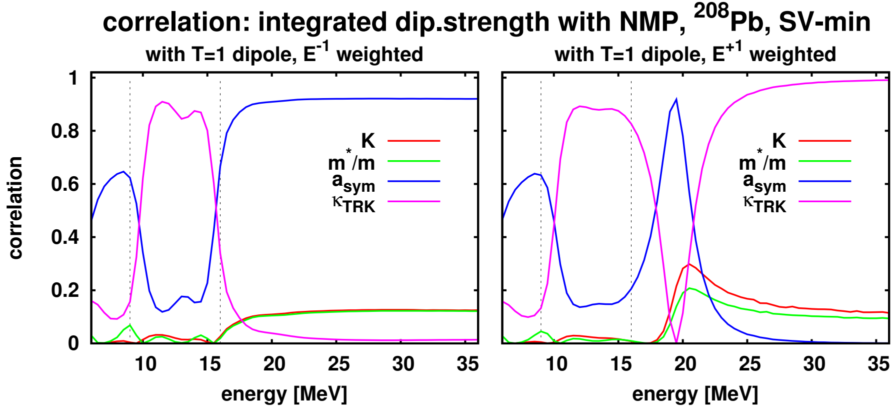
some dependence on each NMP

$\Longrightarrow$

mixed info on NMP

Correlation of $\int_0^E dE' \, \sigma(E') \, E'^n$ with nuclear matter parameters



E^{-1} weight favors $a_{\rm sym}$ for $E \to \infty$

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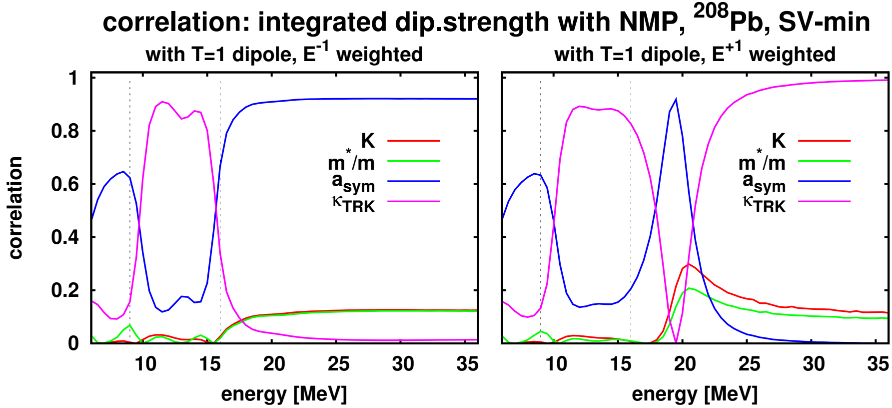

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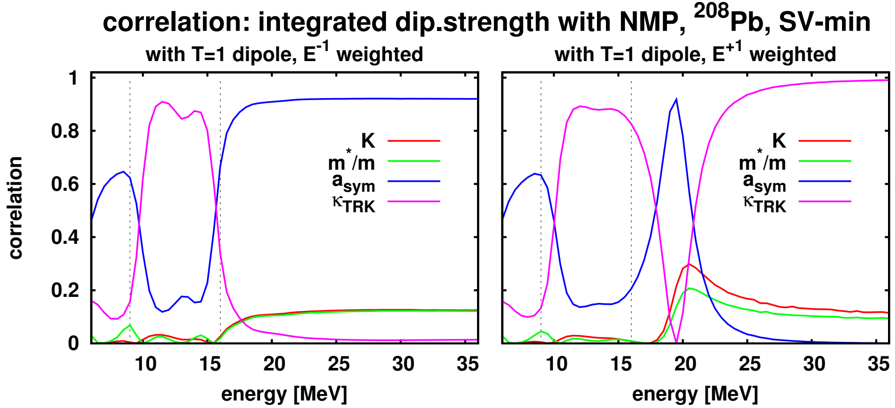

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GDR region favors κ for all E weights

pygmy region yields mixed info about $a_{\rm sym}$ and κ $\Longrightarrow$ useful data in combined analysis
(open problem: robust choice of cutoff energy $E_{\rm cut}$)

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  SHF/RMF are adjusted to data by χ^2 fits.
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- **Low E dipole strenth:**
  Dominated by isoscalar $L = 1$ modes, spread over several distinct modes.
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