Universität Regensburg

Lattice study of continuity and finite-temperature transition in SU(N) x SU(N) Principal Chiral Model

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arXiv:1706.08954

Simulating QCD on Lefschetz thimbles
Trento, Italy 2017

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Another related work to Lefschetz thimbles

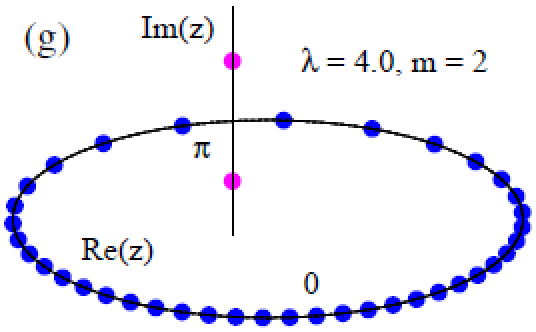
Complex Saddles in Two-dimensional Gauge Theory

P.V. Buividovich, Gerald V. Dunne, S.N. Valgushev

Phys. Rev. Lett. 116, 132001 (2016) arXiv:1512.09021

We study numerically the saddle point structure of **two-dimensional (2D) lattice gauge theory**, represented by the **Gross-Witten-Wadia unitary matrix model.** The saddle points are in general complex-valued, even though the original integration variables and action are real. We confirm the trans-series/instanton gas structure in the weak-coupling phase, and **identify a new complex-saddle interpretation of non-perturbative effects in the strong-coupling phase**. In both phases, eigenvalue tunneling refers to eigenvalues moving off the real interval, into the complex plane, and the weak-to-strong coupling phase transition is driven by saddle condensation.

$$\mathcal{Z} = \int_{U(N)} dW \exp\left[\frac{N}{\lambda}\mathrm{Tr}(W + W^{-1})\right]$$



Leading complex saddle point

See also: G. Álvarez, L.M. Alonso, E. Medina, Phys.Rev. D94 (2016) no.10, 105010

2d Principal Chiral Model

$$S = \frac{1}{g^2}\int \mathrm{d}^2x \mathrm{Tr}\left[\partial_\mu U(x)\partial^\mu U^\dagger(x)\right] \quad U(x) \in SU(N)$$

A toy model for Yang-Mills theory:

- **Asymptotically free theory**
- **Integrable model**

$$M_r = M\frac{\sin(r\pi/N)}{\sin(\pi/N)}$$

- **Dynamically generated mass gap**

$$\Lambda^{\beta_0} = \mu^{\beta_0} e^{-\frac{4\pi}{g^2(\mu)}} \quad \beta_0 = N$$

- **Matrix-like large N limit**
- **IR renormalon ambiguities**

(Fateev, Kazakov, Wiegmann)

$$\underbrace{e^{-\frac{8\pi}{g^2 N(Q)}}}_{1^{\mathrm{st}}\ \mathrm{IR-renormalon}} \gg \underbrace{e^{-\frac{16\pi}{g^2 N(Q)}}}_{2^{\mathrm{nd}}\ \mathrm{IR-renormalon}} \gg \dots$$

"Drawback": no topologically protected non-perturbative saddle points $\pi_2[SU(N)] = 0$

Compactification of PCM

PCM on $\mathbb{R}^2$: strongly coupled theory.

Resurgence: what saturates IR renormalons?

Idea: $\mathbb{R}^2 \to \mathbb{R}^1 \times S^1$

L

$L \to 0$

Thanks to asymptotic freedom, at small L theory should be weakly coupled. Beware of "deconfinement" phase transition!

$$\mathcal{F} \sim (N^2 - 1)T^2 \xrightarrow{T \to 0} \mathcal{F} \to NT^2 \left(\frac{m}{2\pi T}\right)^{1/2} e^{-m/T}$$

Evidence from lattice: hadrons at low temperature

A generic phenomena:
- **SU(N) Yang-Mills**
  - **Breaking of Z(N) => failure of Eguchi-Kawai reduction**
- **CP(N), O(N) sigma models**

For PCM order parameter is not known

Compactification of PCM

How to avoid phase transition?

Twisted boundary conditions!

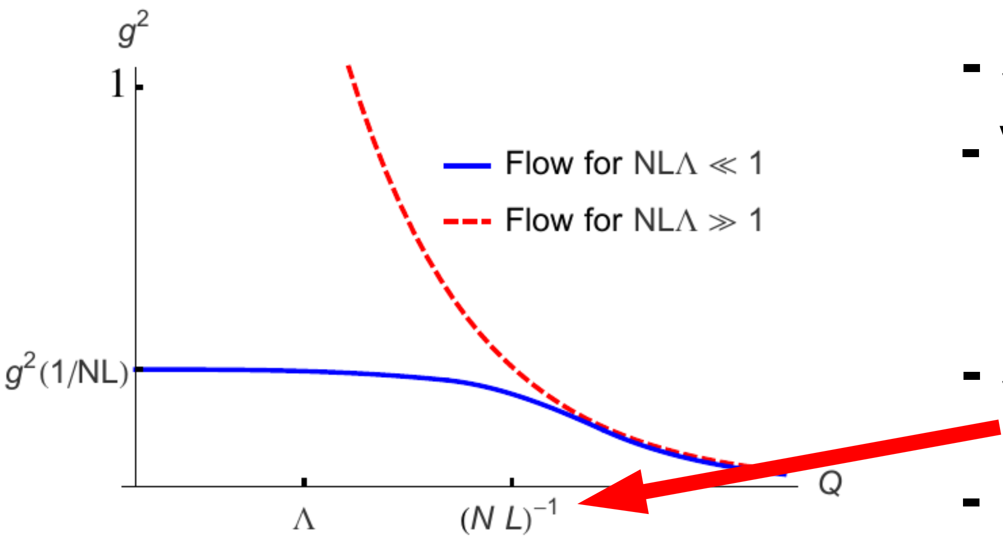
$$U(x_0 + L, x_1) = \Omega U(x_0, x_1) \Omega^\dagger$$

$$\Omega = \mathrm{diag}\left\{1, e^{i\frac{2\pi}{N}}, \ldots, e^{i\frac{2\pi(N-1)}{N}}\right\} \qquad \mathrm{Tr}\Omega^n = \begin{cases} N, & n \equiv 0 \bmod N \\ 0, & \text{otherwise} \end{cases}$$

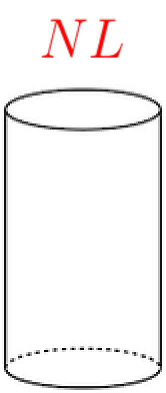
"Maximal" destructive interference => many excited states eliminated

Explicit demonstration: exactly solvable $\mathbb{CP}^{N-1}$ model

T. Sulejmanpasic, Phys. Rev. Lett. 118, 011601 (2017)



- **Stabilize center group**
- **Volume effectively increased N times** **(~ Twisted EK)**
- **Sliding scale ΛN**
  M.Unsal, Phys. Rev. Lett. 102, 182002
- **Does magic to saddle points**



A.Cherman, D.Dorigoni, M.Unsal, Phys. Rev. Lett. 112, 021601 (2014)

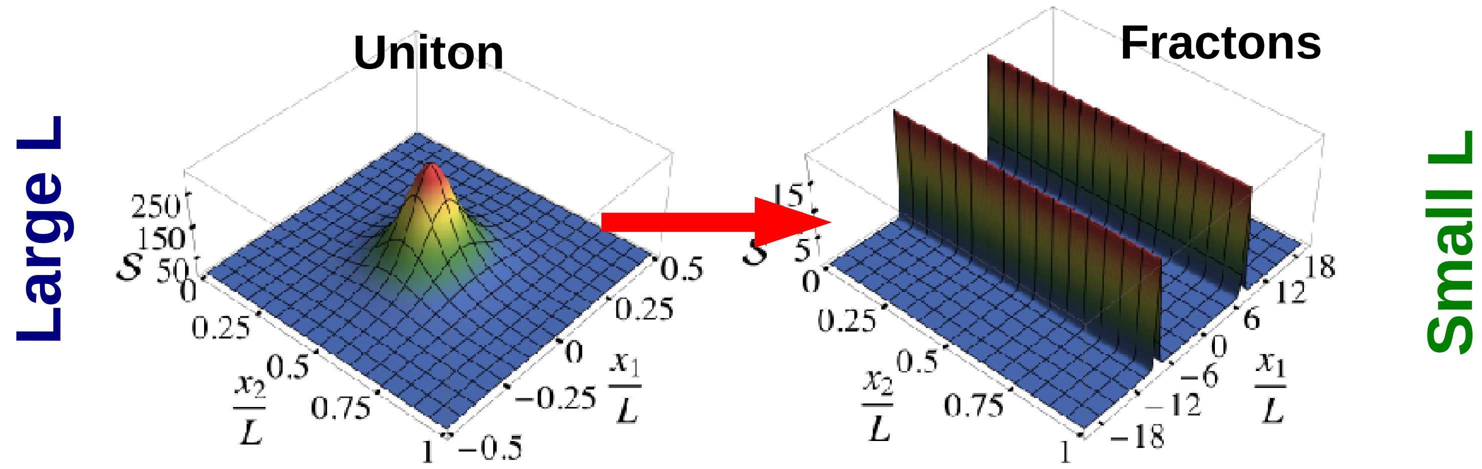
Non-perturbative saddle points

PCM on $\mathbb{R}^2$: unstable uniton saddle points

Harmonic maps $S^2 \to SU(N)$

$S_u = 8\pi/g^2$

Non-trivial effect of the twist in the small L limit:



A. Cherman, D. Dorigoni, M. Unsal, arXiv:1403.1277

Emergent topology => N stable fracton constituents at small L

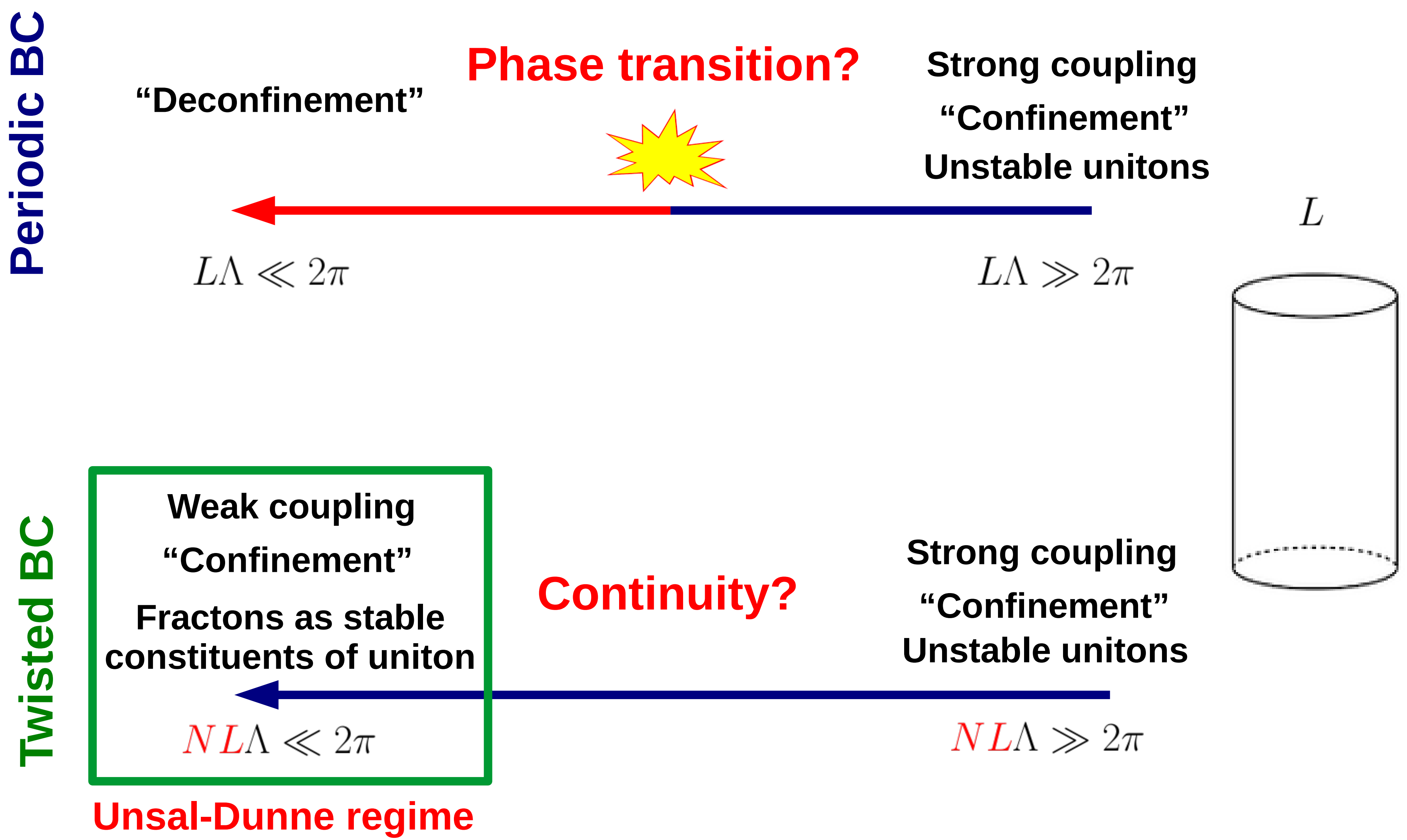
$SU(N) \to U(1)^{N-1}$ **at energies smaller than** $1/(NL)$

$S_f = S_u/N$

Fractons are responsible for mass gap generation and IR renormalon ambiguity regularization via resurgence theory

Phys. Rev. Lett. 112, 021601 (2014) arXiv:1403.1277

Continuity conjecture



Lattice PCM

$$S = -2\beta N \mathrm{Tr} \sum_{x,\mu} \left[U(x) U^{\dagger}(x+e_{\mu}) \right] \qquad \beta = 1/\lambda = 1/(g^2 N)$$

Simulation setup

Cabbibo-Marinari algorithm

- $N = 6, 9, 12, 18$ **+ fits** $O(L_0, N) = \tilde{O}(L_0) + c_1/N^2$
- **Lattice size:** $L_1 = 108$, $L_1 = 200, N = 18$

  $1 \leq L_0 \leq L_1$
  compact direction
- **Boundary conditions: periodic and twisted**
- **Coupling:** $\beta = 0.332$ $\beta > \beta_c = 0.305$ **(weak coupling)**

Observables

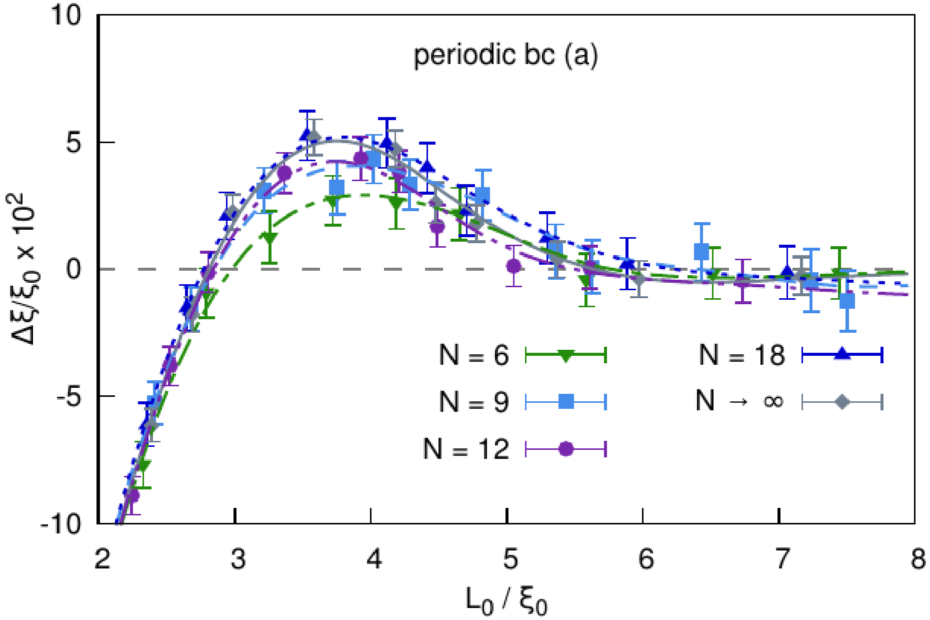
- **Static correlation length** $\xi(L_0)$
- **Mean energy** $E(L_0)$
- **Specific heat** $C(L_0)$

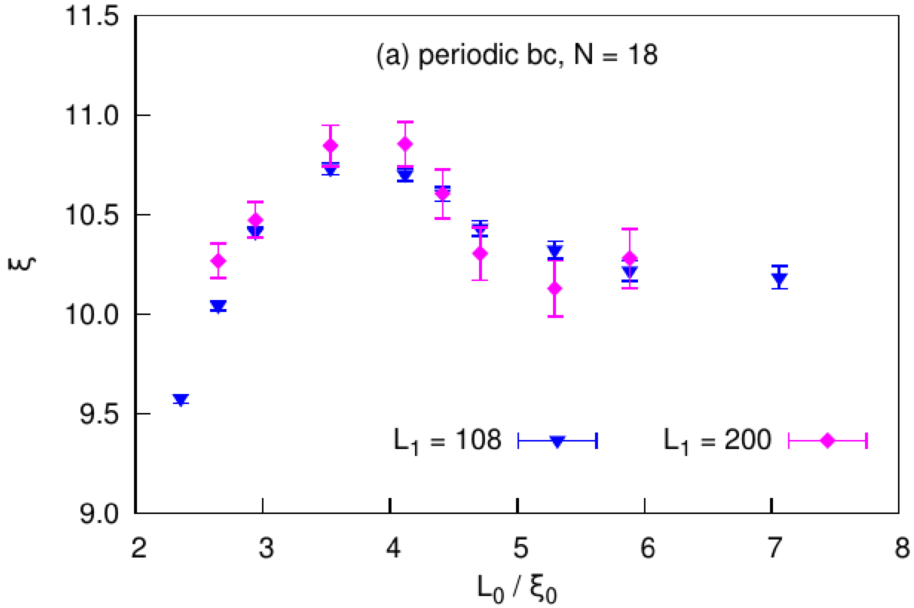
$\xi_0 \equiv \xi(L_0 = L_1, PBC)$
scale

Notations:
$O_0 \equiv O(L_0 = L_1, PBC)$

$$\frac{\Delta O(L_0)}{O_0} \equiv \frac{O(L_0) - O_0}{O_0}$$

Periodic boundary conditions



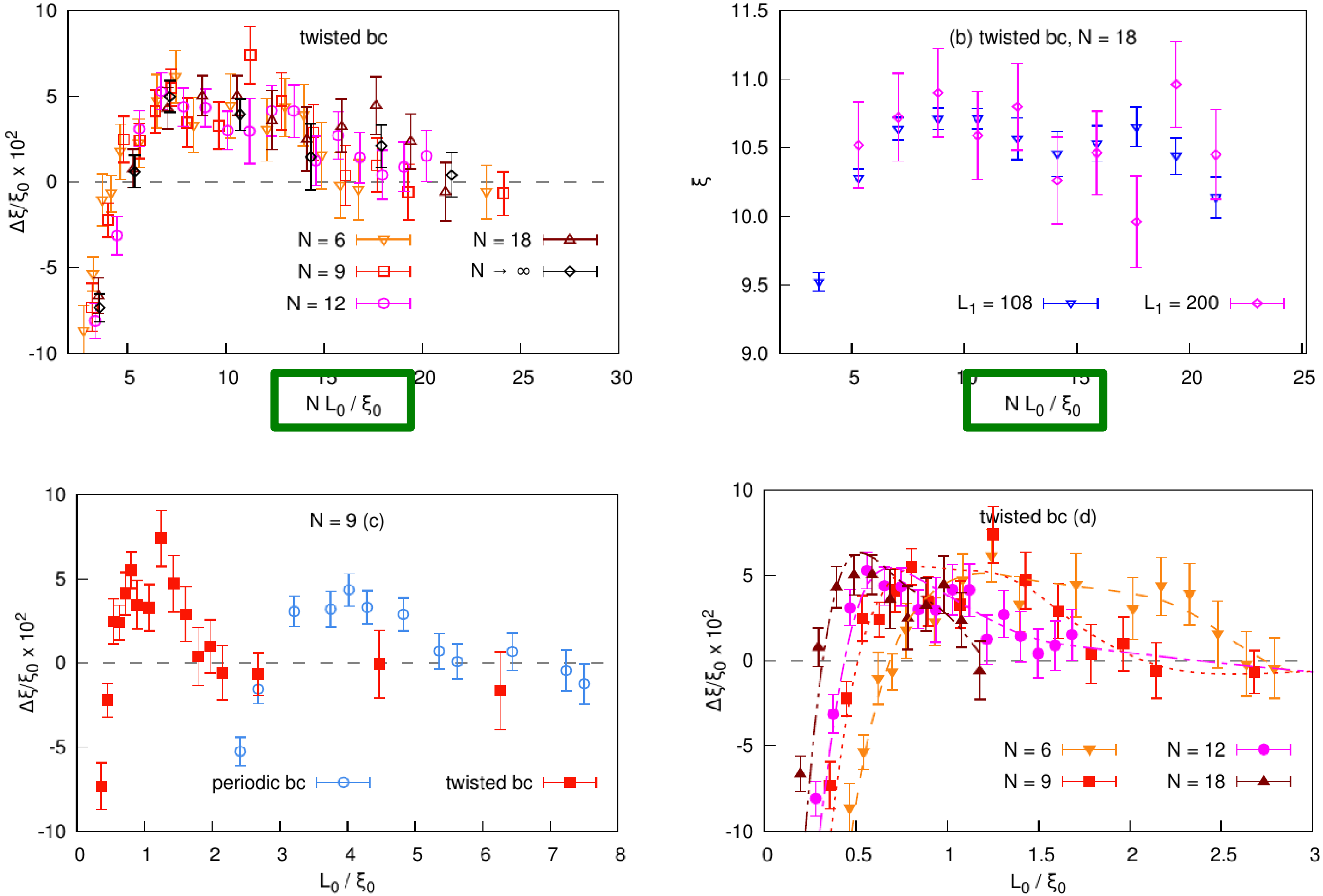


- **Weak enhancement in the region 3...5 (~ 5%)**
- **Slightly higher and narrower when N increases**
- **Infinite N extrapolation suggests that correlation length is finite**

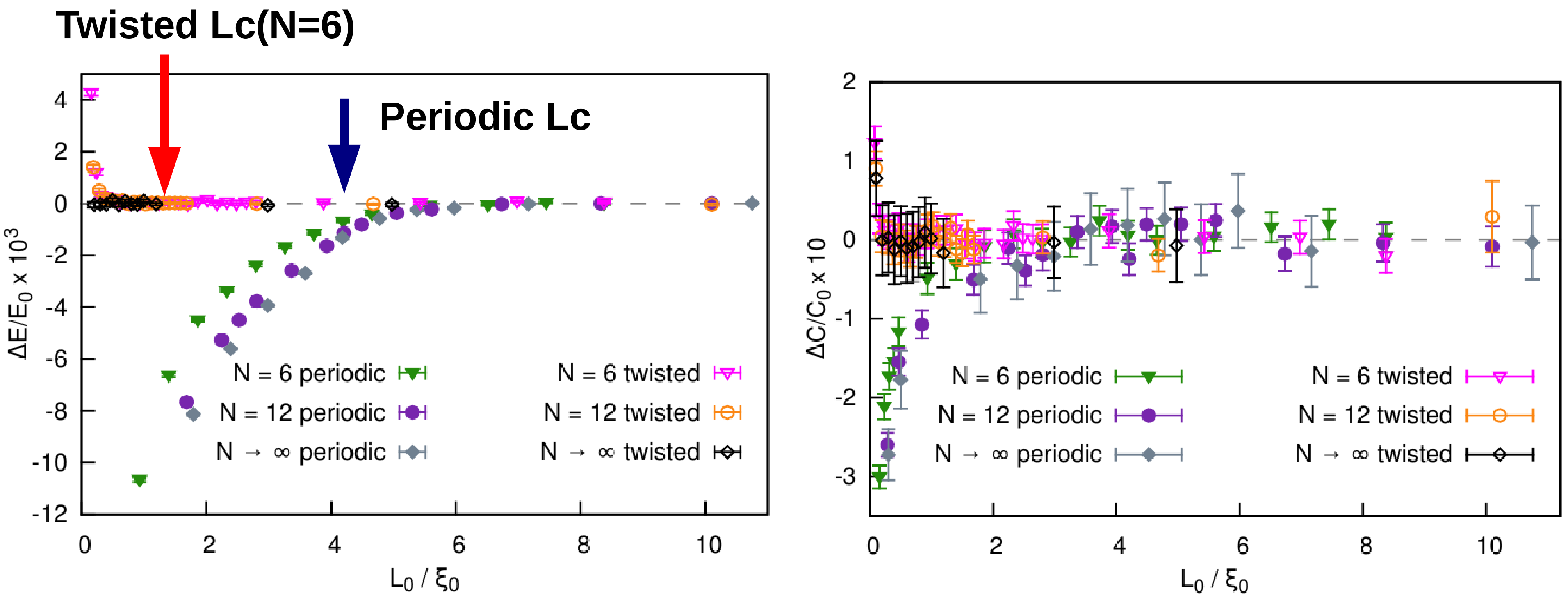
Compatible with DiagMC arXiv:1705.03368

- **Very mild volume dependence**
- **Large N volume independence in large volumes**

Twisted boundary conditions



Mean energy and specific heat



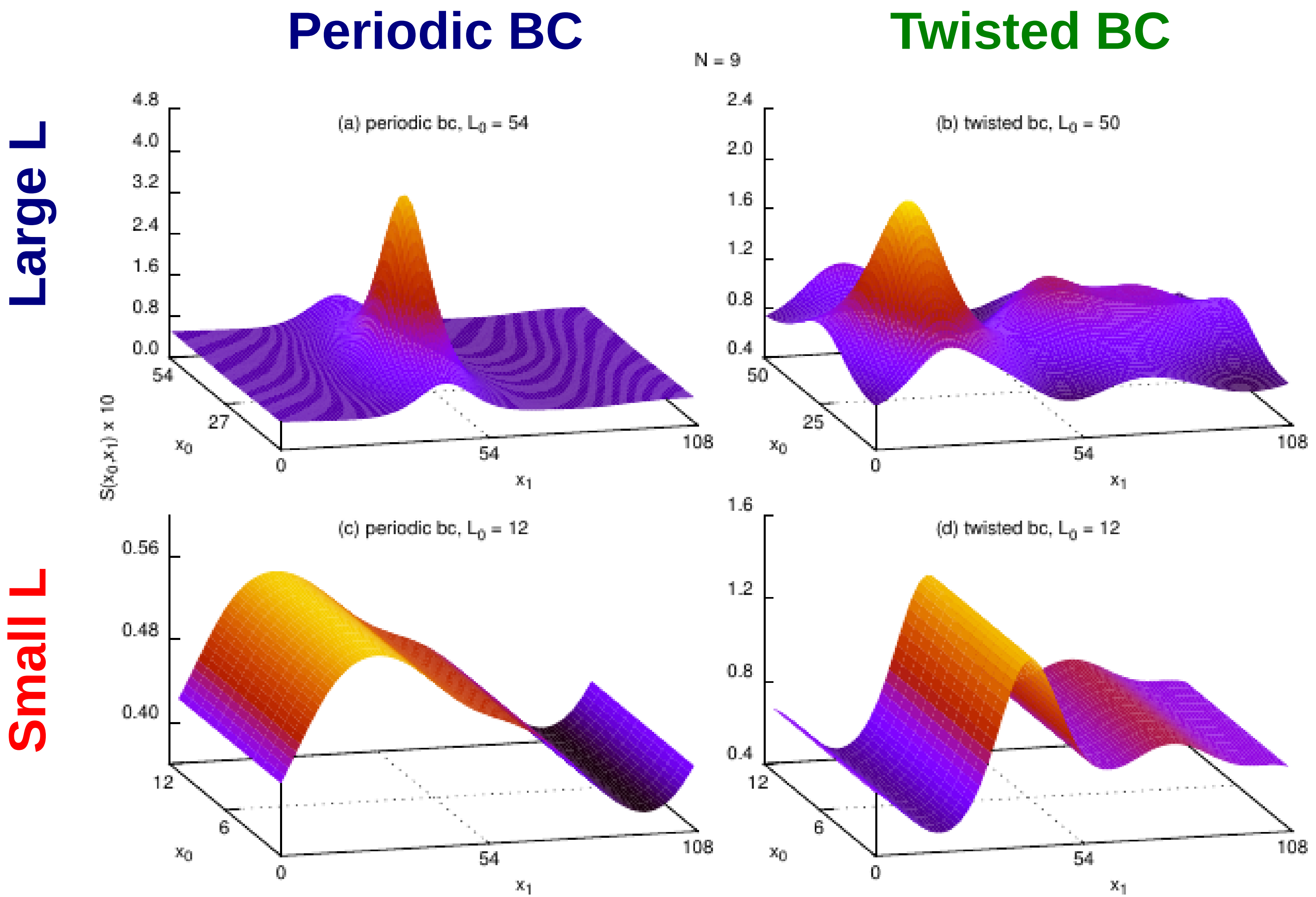
- **Volume independence in large volumes**
- **Different behavior in small L limit**
- **Transition points agree with those for correlation length**
- **No signatures for phase co-existence**

Gradient flow: non-perturbative objects

$$\frac{\partial U(\mathbf{x},\tau)}{\partial \tau} = -\frac{i}{\beta N}\nabla^a_{\mathbf{x}} S\left[U(\mathbf{x},\tau)\right] T_a U(\mathbf{x},\tau)$$

$$U(\mathbf{x},\tau=0) \equiv U(\mathbf{x})$$

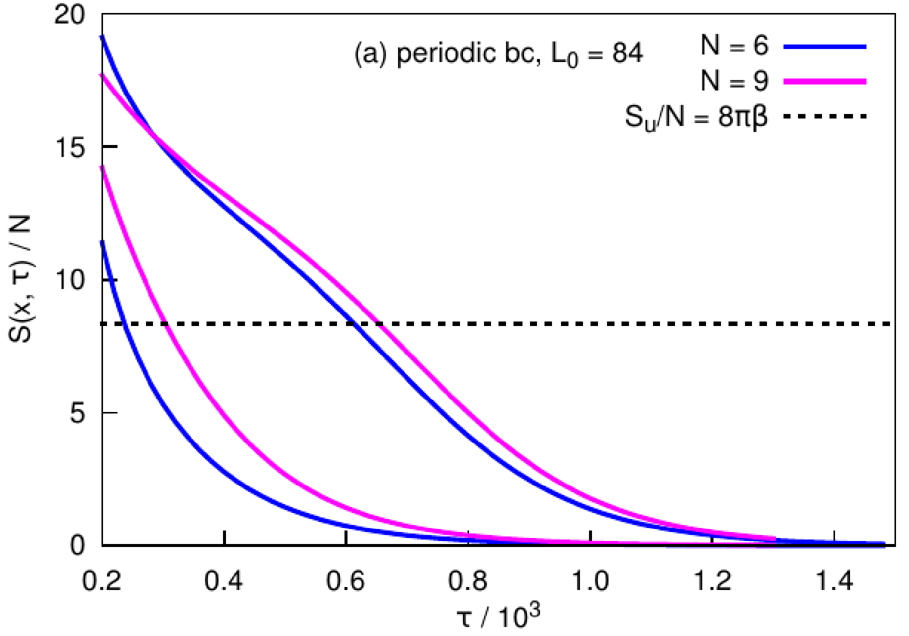
$$\tau = 0 \ldots 1.5 \times 10^3$$



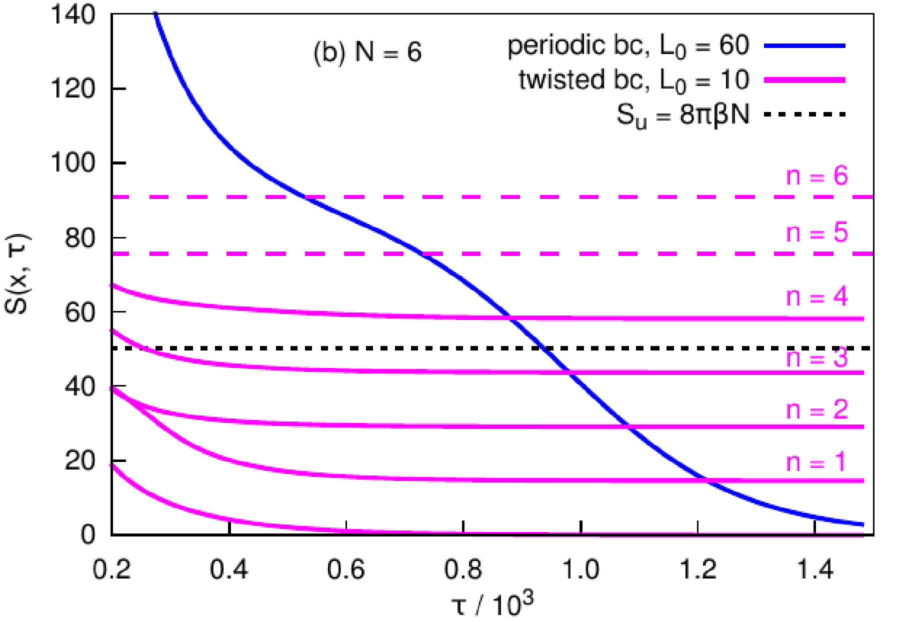
Gradient flow: the action

Uniton: $S_u = 8\pi\beta N$

Fracton: $S_f = 8\pi\beta$



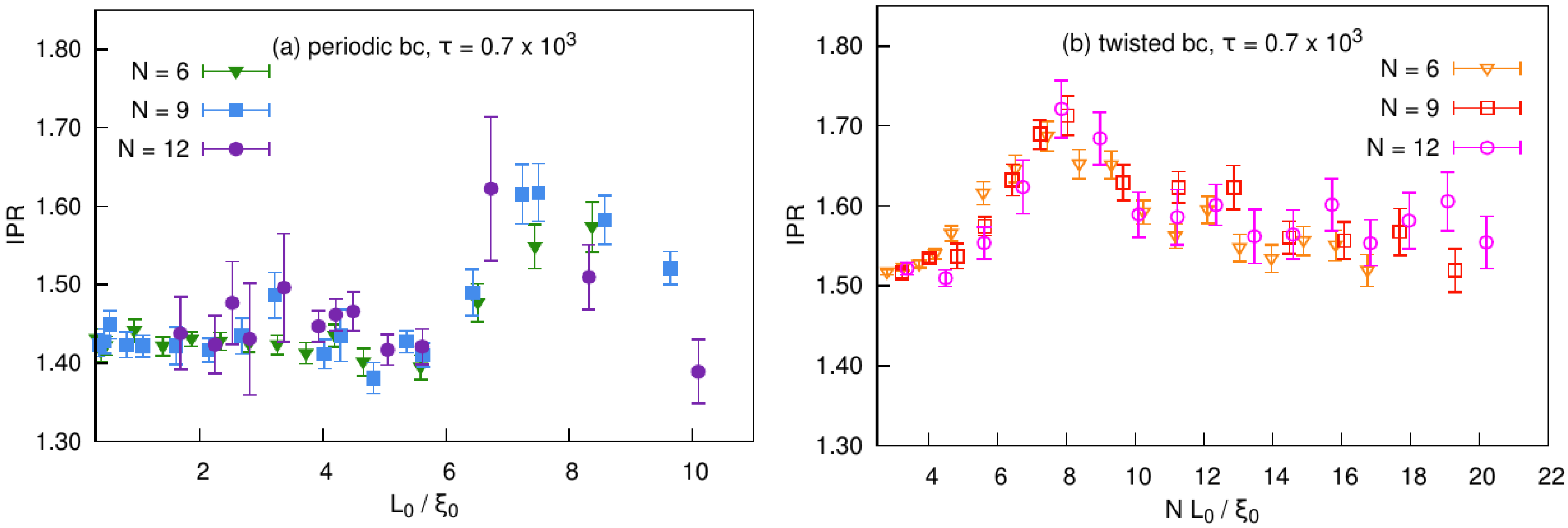
Unitons for N=6,9



Stable fractons compared to uniton with $N L^{\mathrm{TBC}} = L^{\mathrm{PBC}}$

- **Very stable saddle points with twist in Unsal-Dunne limit, evidence for emergent topology**
- **Presumably, the plateaus can be associated with unitons and fractons**

Inverse Participation Ratio (IPR)



$$\mathrm{IPR}\,(\tau) = V \left\langle \frac{\sum_{\mathbf{x}} \tilde{S}^2(\mathbf{x},\tau)}{\left(\sum_{\mathbf{x}} \tilde{S}(\mathbf{x},\tau)\right)^2} \right\rangle$$

$$\tilde{S}(\mathbf{x},\tau) = S(\mathbf{x},\tau) - \min_{\mathbf{x}} S(\mathbf{x},\tau)$$

We use IPR as a measure of action density localization

- **Interesting peak in twisted case which coincide with twisted NLc**

Conclusions

- We find evidences compatible with a weak crossover or phase transitions for both types of boundary conditions
- For periodic BC, correlation length enhancement become larger and narrower as N increases
- For twisted BC, correlation length enhancement is N independent if considered as a function NL
- Volume scaling seems to be very mild in both cases.
- Using Gradient flow equations, we find an evidence for emergent topology in Unsal-Dunne limit with twisted BC.
- More work is needed: combined study of volume and N scaling, continuum limit.
- Might be a challenge for resurgence theory if phase transition (possibly of infinite order) is confirmed