Playing with few-nucleon systems

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Future Directions in the Physics of Nuclei at Low Energies
ECT*, May 21-23, 2014

Introduction: "strict definition" of *ab-initio*

A-nucleon system $\leftrightarrow$ $H = T + V_{NN} + V_{NNN} + \cdots$

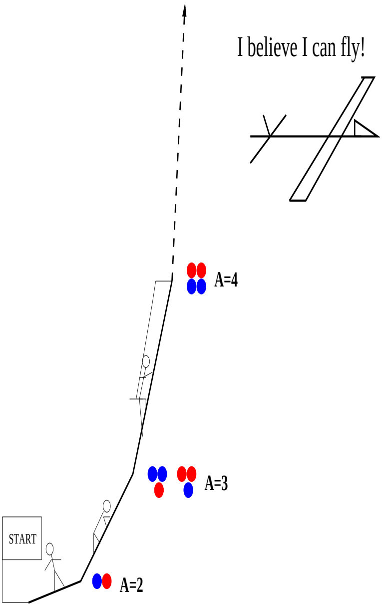
Observable X

Ab-initio method and *Ab-initio* results[1]

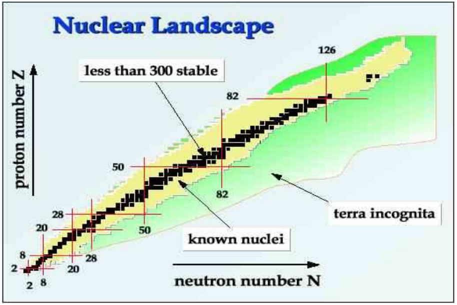
- *Ab-initio* method $\rightarrow$ obtain X by solving the relevant quantum many-body equations, without any uncontrolled approximation
- controlled approximations are allowed (expansion on a certain basis) $\rightarrow$ converged results = *ab-initio* results
- comparison of *ab-initio* results obtained with different *ab-initio* methods $\rightarrow$ benchmark calculations
- comparison of *ab-initio* results with data $\rightarrow$ test of H

[1] W. Leidemann and G. Orlandini, Progr. Part. Nucl. Phys. **68**, 158 (2013)

Few-nucleon systems



Few-nucleon systems ↔ *ab-initio* methods
$A \leq 12 \rightarrow A \leq 4$



- ideal "laboratory" to test H
- nuclear reactions for astrophysics
- test fundamental symmetries

The traditional potential model approach (PMA)

Nuclear interaction: $V_{NN} + V_{NNN}$

- Until $\simeq$ 15–20 years ago:
  phenomenological potentials
  - $V_{NN} + V_{NNN}$ semi-phenomenological
  - V_{NN} with $\simeq$ 40 parameters fitted to $A = 2$ data $\rightarrow \chi^2$/datum $\simeq 1$
  - V_{NNN} with 2 parameters fitted to $B(A = 3, 4)$

Nuclear electroweak currents

Meson-Exchange Currents (MEC) + $\Delta \Rightarrow$ 1b+2b+3b operators
2b/3b $\rightarrow V_{NN}/V_{NNN}$ (CCR satisfied)
$\Delta \rightarrow$ model-dependent

$\Rightarrow$ **large success**
but **no simple connection to QCD**

Marcucci *et al.*, Phys. Rev. C **72**, 014001 (2005)

Chiral Effective Field Theory (χEFT): a short summary

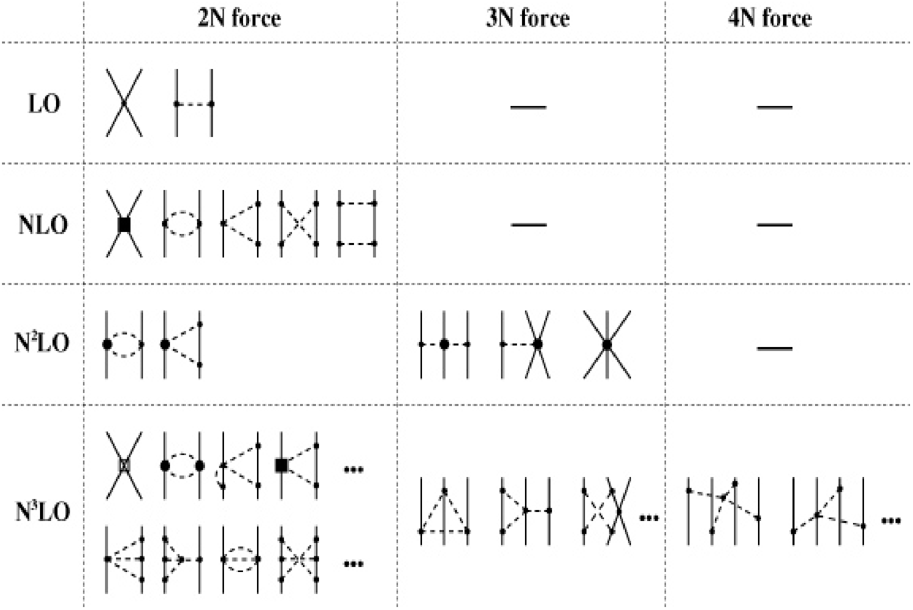
Apologies to the experts in the audience!

- QCD $\longrightarrow$ quarks and gluons (“high-energy” d.o.f.)
- Nuclear physics $\longrightarrow$ nucleons and pions (“low-energy” d.o.f.)
- EFT $\longrightarrow$ processes with $E \simeq p \simeq m_\pi \ll \Lambda_{\rm QCD} \sim 1$ GeV
  - ⋆ “h-e” d.o.f. integrated out $\longrightarrow$ contact interactions with “l-e” d.o.f. and low-energy constants (LECs) obtained from experiment
  - ⋆ perturbative theory: matrix elements $\propto O(p/\Lambda_{\rm QCD})^\nu$
- **χEFT $\longrightarrow$ EFT with spontaneous breaking of QCD’s χ-symmetry**
- Regularization with cutoff function $\longrightarrow \Lambda \simeq 414, 450, 500, 600$ MeV

Disadvantage: limited to processes with $E \sim 1 - 2\, m_\pi$

Advantages

- nuclear force “hierarchy” $\longrightarrow$ accurate $V_{NN} + V_{NNN}$
- consistent framework for interactions + currents (just add EW field among the d.o.f.)


2N force
3N force
4N force
LO
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NLO
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N²LO
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N³LO
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Nuclear EW currents in χEFT

EW operators: $\rho^\gamma, \mathbf{j}^\gamma$; $\rho^{V/A}, \mathbf{j}^{V/A}$

CVC $\Rightarrow \rho^V/\mathbf{j}^V \rightarrow \rho^\gamma/\mathbf{j}^\gamma$

History

$\mathbf{j}^\gamma$

- Park *et al.* in heavy-baryon χPT (HBχPT) $\rightarrow$ since $\simeq$ 1995
- Pastore *et al.* in time-ordered perturbation theory (TOPT) $\rightarrow$ since 2009
- Kölling *et al.* with the unitary transform method $\rightarrow$ in parallel since 2009

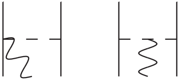
$\mathbf{j}^A$

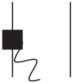
- Park *et al.* in HBχPT $\rightarrow$ since $\simeq$ 2000
- Baroni *et al.* in TOPT $\rightarrow$ work in progress

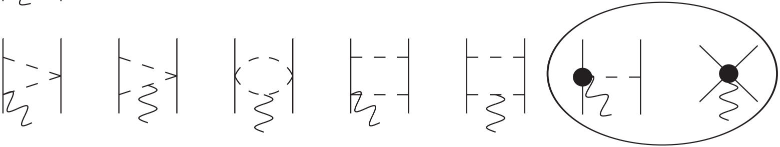
To be remarked:

- Park *et al.* currents ready BEFORE the χEFT potentials $\Rightarrow$ "hybrid" χEFT

$\mathcal{O}(Q^{-2})$  $\mathbf{j}^{(-2)} \propto [e_N(1)(\mathbf{p}_1' + \mathbf{p}_1) + \mathrm{i}\mu_N(1)\sigma_1 \times \mathbf{q}] \times \delta(\mathbf{p}_2' - \mathbf{p}_2) + 1 \leftrightarrow 2$

$\mathcal{O}(Q^{-1})$  "standard" one – pion – exchange

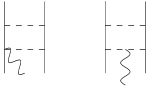
$\mathcal{O}(Q^{0})$  ■ = relativistic corrections

$\mathcal{O}(Q^{1})$ 

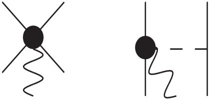
Comments

- Similar results between Pastore *et al.* and Kölling *et al.*
- Differences with Park *et al.*

  for the box-diagrams

  for the terms

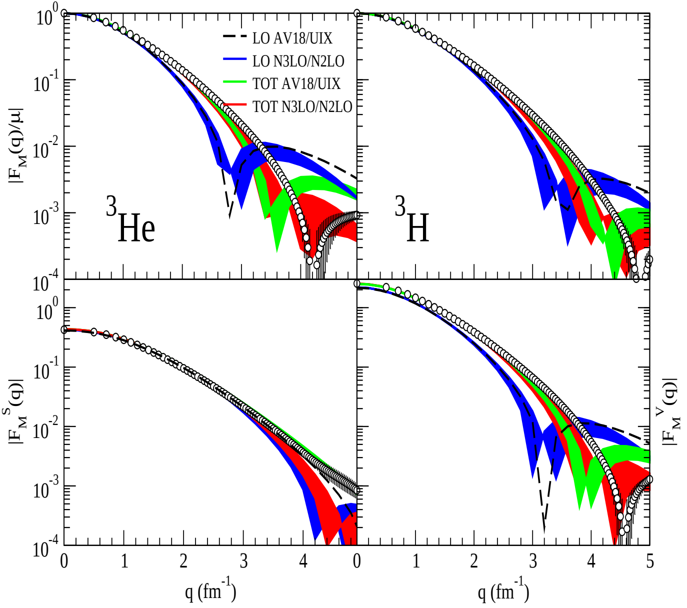
Park *et al.* χEFT EM currents should be reconsidered

- LECs fitted to selected EM observables [σ_{np}, μ_d, $\mu^{S/V}(A=3)$]

Overall nice description of $A = 2, 3$ EM structure

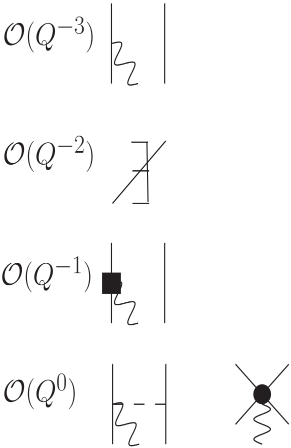
	Theory	Exp.
$r_c(d)$ [fm]	1.972 $\pm$0.004	1.9733 $\pm$0.0044
$Q(d)$ [fm^2]	**0.2836 $\pm$ 0.0016**	**0.2859 $\pm$ 0.0003**
$Q(d)$ [fm^2] (PMA-AV18)	**0.275**	
$r_c(^3\mathrm{He})$ [fm]	1.962 $\pm$0.004	1.959 $\pm$0.030
$r_c(^3\mathrm{H})$ [fm]	1.756 $\pm$0.006	1.755 $\pm$0.086
$r_m(^3\mathrm{He})$ [fm]	1.905 $\pm$0.022	1.965 $\pm$0.153
$r_m(^3\mathrm{H})$ [fm]	1.791 $\pm$0.018	1.840 $\pm$0.181

Piarulli *et al.*, PRC **87**, 014006 (2013)



Piarulli *et al.*, PRC **87**, 014006 (2013)

Power counting for $\mathbf{j}^A$



Note:

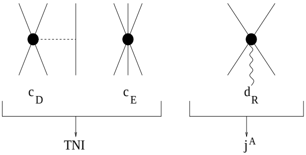
- $\mathcal{O}(Q^1)$: loop and two-pion-exchange contributions (not yet calculated)
- Park *et al.* only available model at $\mathcal{O}(Q^0)$ $\rightarrow$ one LEC - d_R

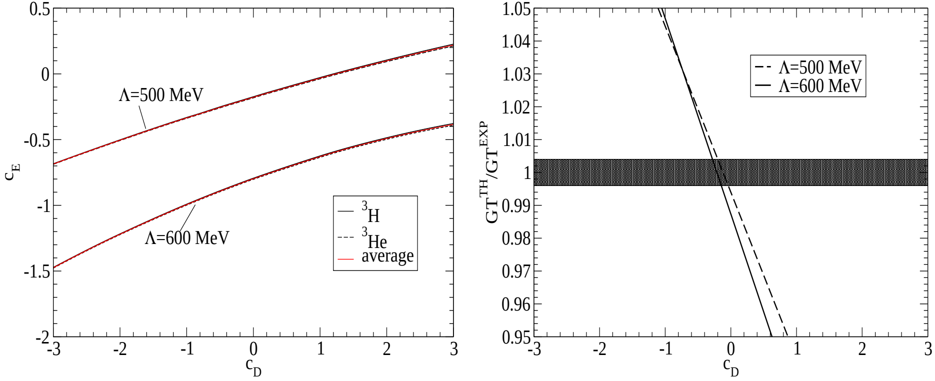
$$d_R = \frac{M_N}{\Lambda_\chi g_A} c_D + \frac{1}{3} M_N (c_3 + 2c_4) + \frac{1}{6}$$

Gårdestig and Phillips, PRL **96**, 232301 (2006)

Gazit *et al.*, PRL **103**, 102502 (2009)

- fit c_D and c_E (in TNI at N2LO) to $B(A=3)$ and GT_{Exp}





$\Rightarrow \{c_D; c_E\}_{\mathrm{MAX}}$ and $\{c_D; c_E\}_{\mathrm{MIN}}$

Model	Λ [MeV]	c_D	c_E	B(^{4}He) [MeV]	$^2a_{nd}$ [fm]
N3LO/N2LO*	500	1.0	−0.029	28.36	0.675
N3LO/N2LO	500	−0.12	−0.196	28.49	0.666
N3LO/N2LO	600	−0.26	−0.846	28.64	0.696
Exp.				28.30	0.645(10)

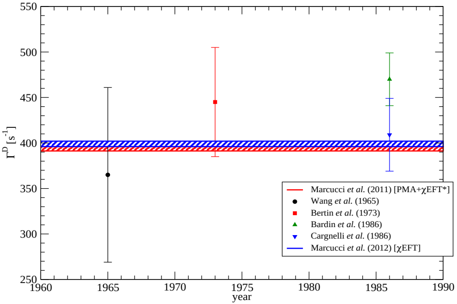
Marcucci *et al.*, PRL **108**, 052502 (2012); Viviani *et al.*, PRL **111**, 172302 (2013)

Nice description of muon capture rates on $A = 2, 3$ nuclei

- $\mu^- + d \rightarrow n + n + \nu_\mu$ $\longrightarrow$ capture rate in the doublet hyperfine state Γ^D
- $\mu^- + {}^3\mathrm{He} \rightarrow {}^3\mathrm{H} + \nu_\mu$ $\longrightarrow$ total capture rate Γ_0

$$\Gamma^D = \mathbf{399(3)\ s^{-1}}\ \&\ \Gamma_0 = \mathbf{1494(21)\ s^{-1}}$$

$$\text{vs. } \Gamma^D(\text{exp}) \cdots \quad \& \ \Gamma_0(\text{exp}) = 1496(4)\ \text{s}^{-1}$$



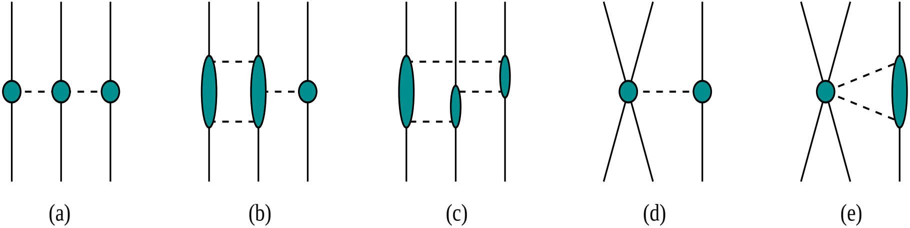
$\Rightarrow$ MuSun $\rightarrow$ 1.5 % accuracy

Marcucci *et al.*, PRL **108**, 052502 (2012)

Open issues (many and not all!)

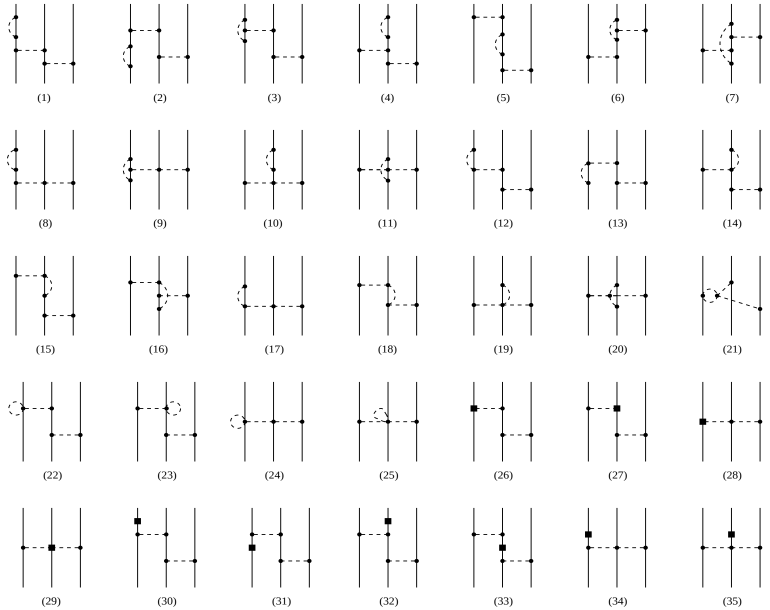
- Are we at convergence both for potential and currents?
  - N3LO TNI a **big mess of terms**
  - still open problems: "A_y-puzzle"
  - gauge-invariance
- Weak sector still way behind
  $\rightarrow \chi$EFT-TOPT to calculate two-pion exchange terms of $\mathbf{j}^A$
- Fully consistency not yet reached:
  - different cutoff functions between nuclear interactions and currents
  - different chiral order between V_{NN} and V_{NNN} + currents
- Can the whole framework be further tested? Need more data?

Topologies in TNI at N3LO



By the Bochum-Bonn group

a-type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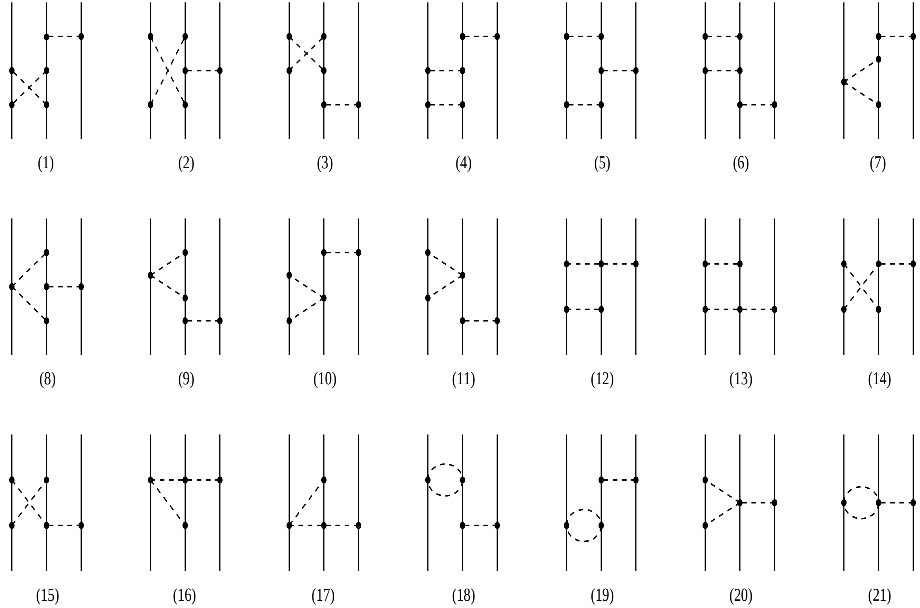
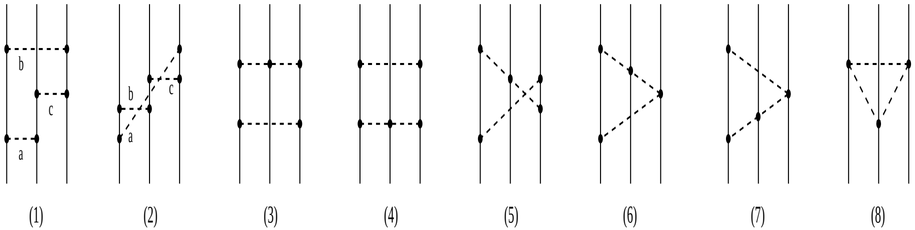
b-type



c-type



$\cdots$ and so on!

$\cdots$ But no NEW LECs

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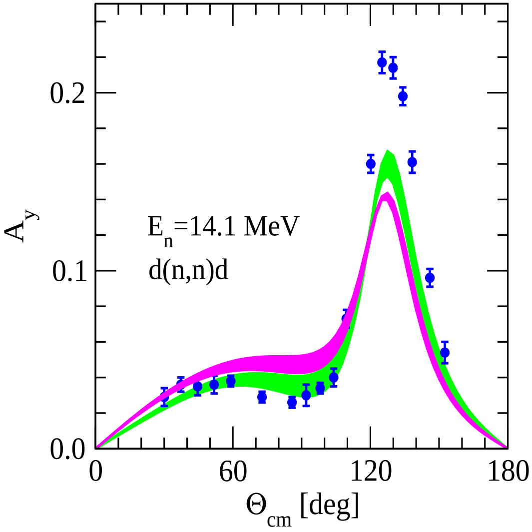

Witala *et al.*, `arXiv:1310.0198`

Some hope ... at N4LO there are interesting structures

$$
\begin{aligned}
V = \sum_{i \neq j \neq k} \quad & (E_1 + E_2 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j + E_3 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + E_4 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \\
& \times \left[Z_0''(r_{ij}) + 2 \frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik}) \\
& + (E_5 + E_6 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) S_{ij} \left[Z_0''(r_{ij}) - \frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik}) \\
& + (E_7 + E_8 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k)(\mathbf{L} \cdot \mathbf{S})_{ij} \frac{Z_0'(r_{ij})}{r_{ij}} Z_0(r_{ik}) \\
& + (E_9 + E_{10} \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k) \boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}_{ij} \boldsymbol{\sigma}_k \cdot \hat{\mathbf{r}}_{ik} Z_0'(r_{ij}) Z_0'(r_{ik})
\end{aligned}
$$

Girlanda *et al.*, PRC **84**, 014001 (2011); Girlanda *et al.*, work in progress

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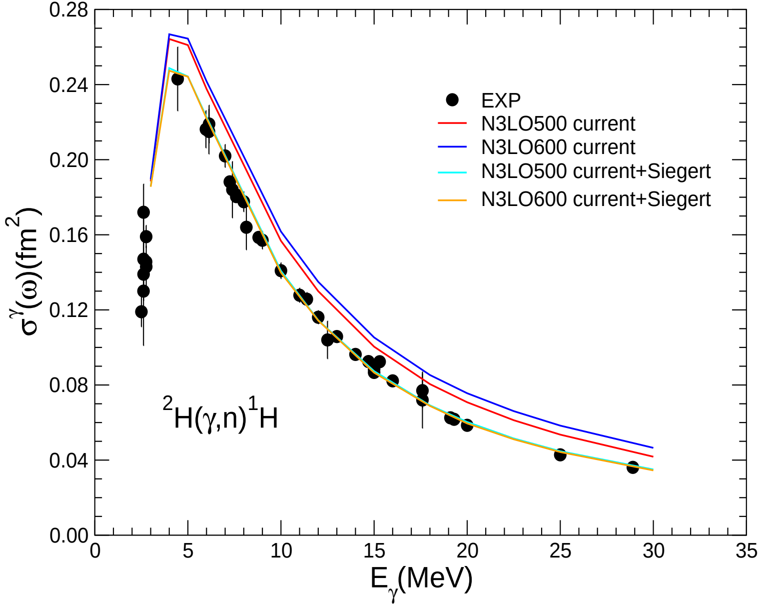

0.28
0.24
0.20
0.16
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0.04
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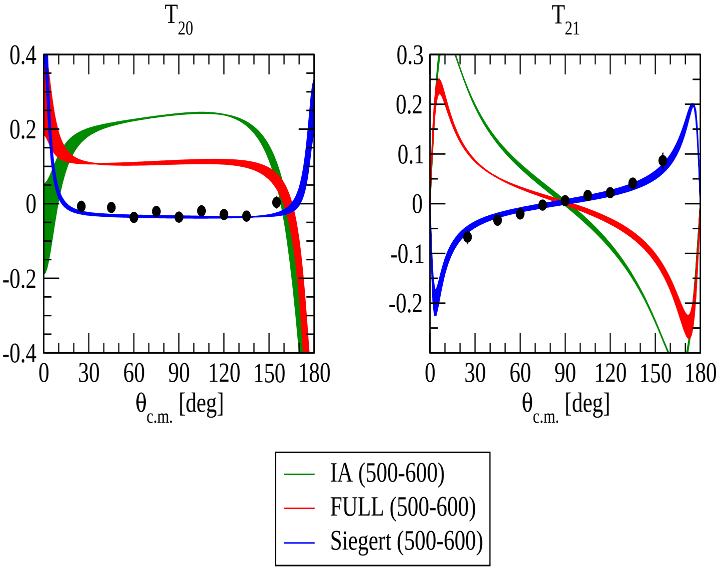
15
20
25
30
35
$\sigma^{\gamma}(\omega)(fm^2)$
E_γ(MeV)
EXP
N3LO500 current
N3LO600 current
N3LO500 current+Siegert
N3LO600 current+Siegert
$^2H(\gamma,n)^1H$

$p + d \rightarrow {}^3\mathrm{He} + \gamma$ at $E_{c.m.} = 2$ MeV



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Within the χEFT framework, some more fun stuff (I)

Reactions of astrophysical interest: $p + p \rightarrow d + e^+ + \nu_e$

	$^1S_0$	$\cdots + {}^3P_0$	$\cdots + {}^3P_1$	$\cdots + {}^3P_2$
PMA	4.000(3)	4.003(3)	4.015(3)	4.033(3)
χEFT(500)	4.008(5)	4.011(5)	4.020(5)	4.030(5)
χEFT(600)	4.007(5)	4.010(5)	4.019(5)	4.029(5)

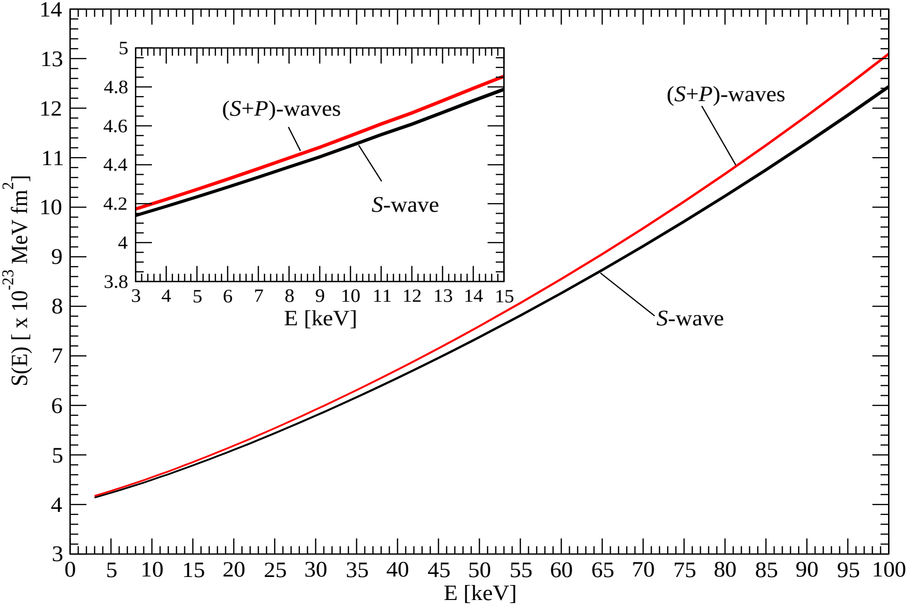
$$S(0) = \mathbf{4.03(1 \pm 0.006)} \times 10^{-23} \ \textbf{MeV fm}^2$$

vs.

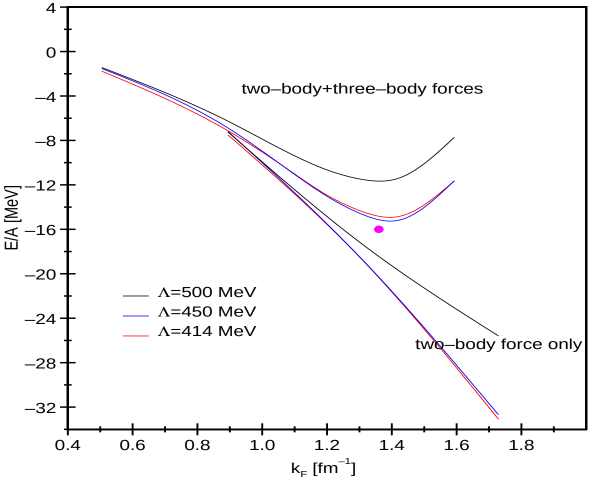
$$S(0)^{\rm SFII} = 4.01(1 \pm 0.009) \times 10^{-23} \ \text{MeV fm}^2$$

SFII: E.G. Adelberger *et al.*, RMP **83**, 195 (2011)

Marcucci *et al.*, PRL **110**, 192503 (2013)


S(E) [x 10^-23 MeV fm^2]
E [keV]
(S+P)-waves
S-wave
(S+P)-waves
S-wave
E [keV]
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
3
4
5
6
7
8
9
10
11
12
13
14
3.8
4
4.2
4.4
4.6
4.8
5

E/A in infinite symmetric nuclear matter (up to 3rd order in many-body PT)



Coraggio *et al.*, PRC **89**, 044321 (2014)

Within the χEFT framework, some more fun stuff (III)

Test of fundamental symmetries and their violations (C, P, T)
Example: P-violating hadronic interaction
(For T-violation $\rightarrow$ Bira)

A "historical" parenthesis on P-violating interaction

- First V_{PV} model in a PMA (DDH model)
- First hadronic P-violating data on heavy nuclei (^{18,19}F, ^{133}Cs, ^{235}Tl)
- Difficult to extract information on V_{PV} due to nuclear structure uncertainties
- Again **no connection to QCD**

Past, present and future experiments on P-violation

- Available experimental data
  - longitudinal asymmetry in $\vec{p}-p$ and $\vec{p}-\alpha$
- Ongoing/planned experiments (LANL, ORNL and NIST) to measure:
  - γ-asymmetry and γ circular polarization in ${}^1\mathrm{H}(\vec{n},\gamma)^2\mathrm{H}$ and ${}^1\mathrm{H}(n,\vec{\gamma})^2\mathrm{H}$
  - neutron spin rotation in $\vec{n}-p$, $\vec{n}-d$ and $\vec{n}-\alpha$
  - longitudinal asymmetry in ${}^3\mathrm{He}(\vec{n},p)^3\mathrm{H}$ at cold neutron energies

Motivations

- h_π^1 is a **fundamental coupling constant** (as g_π for P-conserving potential) and it mixes strong (PC) and weak (PV) interactions
- Ultracold neutron facility (ESS) at Lund (Sweden) $\Rightarrow$ lot of PV observables will be at reach $\rightarrow$ test of fundamental symmetries and interactions

P-violation in χEFT up to N2LO

$$\mathcal{L}^{PV} \text{ at } \mathcal{O}(Q^2) \rightarrow V_{PV} \rightarrow \text{LECs}(h_\pi^1, C_{i=1,5})$$

To get informations on the LECs

$\rightarrow$ few-nucleon systems $\leftarrow$ experimental challenge!

Sensitivity of the different observables to the LECs

- $\vec{p} - p$ longitudinal asymmetry $\rightarrow h_\pi^1$ and $C = C_1 + C_2 + 2C_4 + 2C_5$
- $\vec{n} - p$ spin rotation $\rightarrow C_i,\ i \neq 4$
- $\vec{n} - d$ spin rotation $\rightarrow h_\pi^1$ and $C_{2,3}$
- $\vec{n} - {}^3\text{He}$ longitudinal asymmetry $\rightarrow h_\pi^1$ and $C_{2,3,4}$

Viviani *et al.*, `arXiv:1403.226`, PRC in press

Back to muon capture

CAN be measured and accurately calculated (on few-nucleon systems)
Basic process:

$$\mu^- + p \rightarrow n + \nu_\mu$$

which involves

$$j^\mu = \overline{u_p}\left[F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N} - G_A(q^2)\gamma^\mu\gamma^5 - G_{PS}(q^2)\frac{q^\mu\gamma^5}{2M_N} + \bullet\bullet\bullet\right]u_n$$

- Theory vs. experiment → form factors

  $\longrightarrow \Gamma_0(\mu + {}^3\mathrm{He})$ G_{PS} =8.2 ±0.7 vs. $G_{PS}^{\chi\mathrm{PT}}$ =7.99±0.20
- Are there other observables sensitive to the "$\bullet\bullet\bullet$"?
- Other open issues: $G_A(q^2) = g_A/(1 + q^2/\Lambda_A^2)^2 \rightarrow g_A$ and Λ_A (see MiniBoone data, $\Lambda_A \simeq 20$ % larger)?

Conclusions

Desiderata

- Some open issues should be solved (A_y-puzzle and TNI)
  $\longrightarrow$ the framework should be consistent and well established
  $\longrightarrow$ solve the discrepancies (power counting & EW currents)
- Push the "few" to as "many" as possible (especially for scattering)
- Join our expertises

BUT

after this long journey ...

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