Angular momenta from GPDs

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Outline:

- **Sum rules**
- **Analysis of nucleon form factors -** E_{val}
- **DVMP and DVCS - constraints on** E **for gluons and sea quarks**
- **Angular momenta**
- **Summary**

Sum rules

relation between $2^{\rm nd}$ moments of GPDs and angular momenta

Ji(96): $$J_{q+\bar q} = \frac{1}{2}\int_{-1}^{1} dx x \big[H_q(x,\xi,t=0) + E_q(x,\xi,t=0)\big]$$
$$J_g = \frac{1}{2}\int_{0}^{1} dx \big[H_g(x,\xi,t=0) + E_g(x,\xi,t=0)\big]$$

for any value of skewness $\xi \simeq x_B/(2-x_B)$

$H_q(x,\xi=0,t=0) = q(x)$ $\quad$ $H_q(-x,\xi=0,t=0) = -\bar q(x)$ $\quad$ $1 > x > 0$

$E_q(x,\xi=0,t=0) = e^q(x)$ $\quad$ $E_q(-x,\xi=0,t=0) = -e^{\bar q}(x)$

convenient choice to evaluate J_i: $\quad \xi = 0$

$$J_q = \frac{1}{2}\left[q_{20} + e^q_{20}\right] \qquad J_g = \frac{1}{2}\left[g_{20} + e^g_{20}\right]$$

$q_{20} = \int_0^1 dx x q(x)$ $q = u, d, s, \bar u, \bar d, \bar s$ $\quad$ $g_{20} = \int_0^1 dx x g(x)$ $\quad$ $e^{q,g}_{20}$ analogue

q_{20}, g_{20} from PDFs; forward limit of GPD E, not accesible in DIS

new information from hard exclusive reactions

sum rule $\quad 1/2 = \sum_q (q_{20} + e^q_{20}) + g_{20} + e^g_{20}$

momentum sum rule of DIS $\quad 1 = \sum_q q_{20} + g_{20}$

Teryaev(99) combining both sum rules: $\quad 0 = \sum_q e^q_{20} + e^g_{20}$

what do we know about E?

let us start with E for valence quarks

Sum rules for nucleon form factors

GPDs respect sum rules (hold for all ξ)   (work in frame with $\xi = 0$)

$$F_1^q(t) = \int_0^1 dx H_v^q(x, \xi = 0, t) \qquad F_2^q(t) = \int_0^1 dx E_v^q(x, \xi = 0, t)$$

only difference $K^q - K^{\bar{q}}$ $(K = H, E)$ contribute to form factors (C-invariance)

$$F_i^p = e_u F_i^u + e_d F_i^d + e_s F_i^s \qquad F_i^n = e_u F_i^d + e_d F_i^u + e_s F_i^s \qquad i = 1, 2$$

plenty of data on $G_M^p, G_M^n, G_E^p, G_E^n$ $(\Rightarrow F_1^p, F_1^n, F_2^p, F_2^n)$ are available
used to extract the GPDs H_v^q and E_v^q with help of a suitable parametrization
strange FFs negligible, contribution smaller than exp. error

previous work: DFJK hep-ph/0408173 and Guidal et al hep-ph/0410251
update of DFJK(04):   Diehl-K(1302.4604)
use new form factor data: extend to larger $-t$ (G_E^p, G_M^n, G_E^n)
new accurate data at low $-t$
(occasionally in conflict with old data)

Parametrization of the $\xi = 0$ GPDs

ANSATZ: $$H_v^q(x,\xi=0,t) \;=\; q_v(x)\,\exp\left[t f_q(x)\right]$$
$$E_v^q(x,\xi=0,t) \;=\; e_v^q(x)\,\exp\left[t g_q(x)\right]$$

$f_q = \left[\alpha_q' \log(1/x) + B_q\right](1-x)^3 + A_q\, x(1-x)^2$ $\qquad$ g_q analogous

$q_v(x)$ known PDFs

$e_v^q = \kappa_q N_q x^{-\alpha_q}(1-x)^{\beta_q}(1+\gamma_q\sqrt{x})$ $\qquad$ $\kappa_q = \int_0^1 dx e_v^q(x)$

Motivation: overlap of Gaussian LC wave fcts

Brodsky et al(81): $\Psi_N \sim \exp\left[-a^2 \sum_i \frac{k_{\perp i}^2}{x_i}\right]$ but profile fct $\sim (1-x)$

small x (and small $-t$) Regge-like term dominates: $K_v^q \sim x^{-\alpha_q - t\alpha_q'}$

large x (and large $-t$) last term dominates

$$\text{Fourier transform}: \qquad k_v^q(x,b_\perp^2) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2}\,\mathrm{e}^{-i\mathbf{b}_\perp\mathbf{\Delta}_\perp}\,K_v^q(x,t=\Delta_\perp)$$

Burkardt(02): $q_v(x,b_\perp^2)$ difference of densities for quarks and antiquarks with x and transverse distance, $\mathrm{b}_\perp$, from the center of the proton

average impact parameter associated with q_v: $\langle b_\perp^2\rangle_x^q = 4 f_q(x)$

factor $(1-x)^2$ leads to $\langle b_\perp^2\rangle_x^q \sim (1-x)^2$ in the limit $x \to 1$

guarantees finite size of proton in that limit

The fits

not all parameters can be varied independently (but more than in DFJK(04))
otherwise large uncertainties of parameters and violations of positivity bound:

$$\frac{\left[e_v^q(x)\right]^2}{8m^2} \leq \exp(1)\left[\frac{g_q(x)}{f_q(x)}\right]^3\left[f_q(x)-g_q(x)\right]\left\{\left[q_v(x)\right]^2-\left[\Delta q_v(x)\right]^2\right\}$$

we fix: $\alpha_d = \alpha_u = \alpha$ $\qquad \alpha'_u - \alpha'_d = 0.1\,\mathrm{GeV}^2$ $\qquad \gamma_u = 4$ $\qquad \gamma_d = 0$

β_q: various fits; look for solution with minimal χ^2 not violating positivity

default fit (ABM1): PDFs from ABM(12) at scale $\mu = 2\,\mathrm{GeV}$

q	u	d
A_q	1.264 ± 0.050	4.198 ± 0.231
B_q	0.545 ± 0.062	0.206 ± 0.073
C_q	1.187 ± 0.087	3.106 ± 0.249
D_q	0.333 ± 0.065	-0.635 ± 0.076

$\beta_u = 4.65 \qquad \beta_d = 5.25$

$\alpha = 0.603 \pm 0.020$

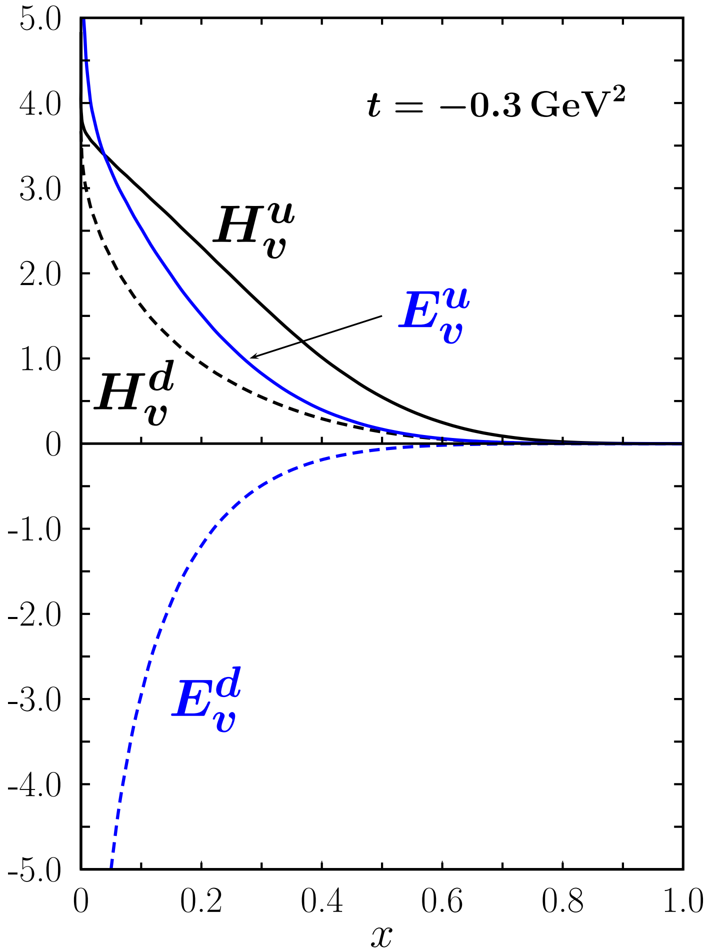
$\alpha'_d = (0.861 \pm 0.026)\,\mathrm{GeV}^2$

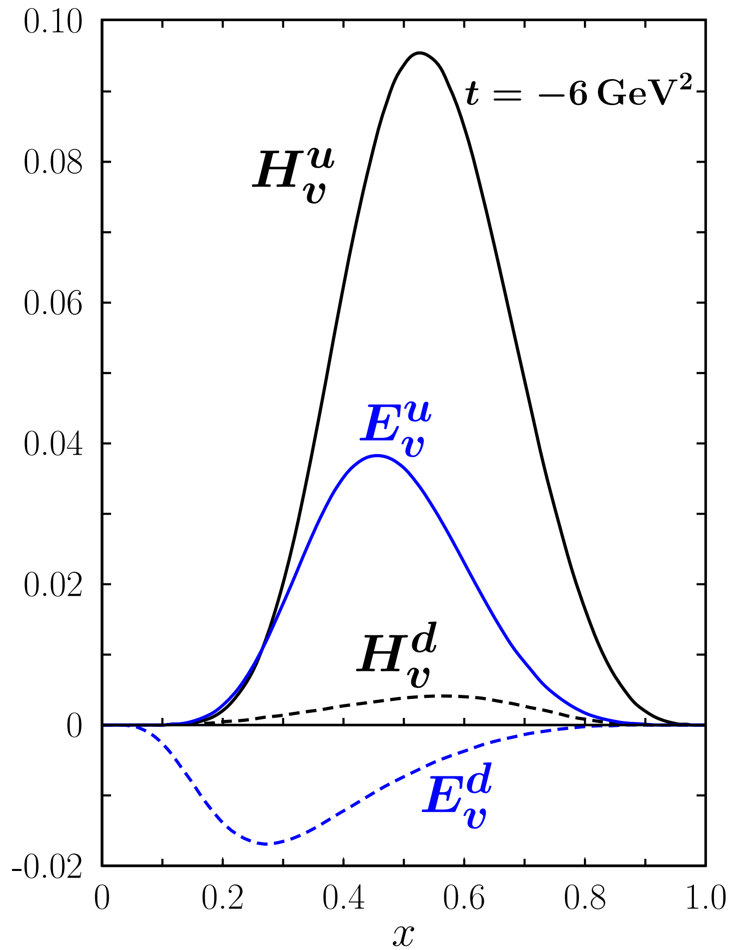
$\chi^2 \approx 220$ for 178 data points

All quantities in units GeV^{-2}

variations of fit: other PDFs, ABMx: $\gamma_d = \gamma_u = 4$, with strangeness,
alternative data ($\mu_p G_E^p/G_M^p$ or G_M^p Arrigton(05),instead (07))

The $\xi = 0$ GPDs H and E ($\mu = 2\,\mathrm{GeV}$)





$K_v^q \sim x^{-\alpha_q - t\alpha'_q}$ at small x

singular (zero) at small (large) $-t$

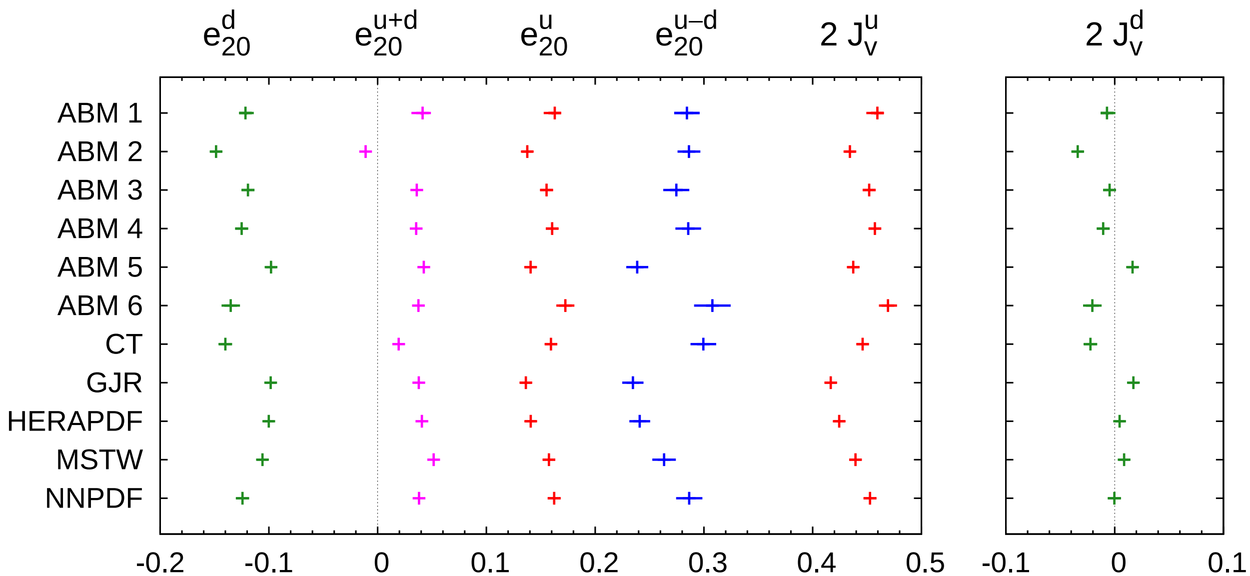
makes $x \leftrightarrow t$ correlation obvious

$K_v^q \sim (1-x)^{\beta_q}$ at large x

pronounced peak

position moves to larger x and becomes narrower with increasing $-t$

Angular momenta of valence quarks



fits	$2J_v^u$	$2J_v^d$
ABM 1	$0.460^{+0.006}_{-0.010}$	$-0.007^{+0.008}_{-0.006}$
all fits	$0.460^{+0.018}_{-0.048}$	$-0.007^{+0.021}_{-0.033}$
ABM 0	$0.560^{+0.009}_{-0.010}$	$-0.019^{+0.009}_{-0.009}$

Ji(96)

$2J_v^q = q_{20}^v + e_{20}^{q_v}$

(ABM 1: $\mu = 2\,\mathrm{GeV}$ ABM 0: $\mu = 1\,\mathrm{GeV}$)

Analysis of meson leptoproduction

Goloskokov-K. 05, 07, 08, 09, 10, 11, 13 small $\xi(\simeq x_{Bj}/2)$, small $-t$

amplitudes are given by convolutions of GPDs and hard subprocess amplitudes

double distribution ansatz for GPDs (Mueller *et al* (94), Radyushkin (99))

$$K^i(x,\xi,t) = \int_{-1}^{1} d\rho \int_{-1+|\rho|}^{1-|\rho|} d\eta\, \delta(\rho+\xi\eta-x)\, K^i(\rho,\xi=0,t) w_i(\rho,\eta) + D_i\, \Theta(\xi^2-\bar{x}^2)$$

weight fct $w_i(\rho,\eta) \sim [(1-|\rho|)^2-\eta^2]^{n_i}$ $(n_g = n_{\rm sea} = 2, n_{\rm val} = 1)$, generates ξ dep.

$K^i(\rho,\xi=0,t) = k^i(\rho) \exp[(b_{ki} + \alpha'_{ki} \ln(1/\rho))t]$ (for small $-t$ reasonable appr.)

for gluons and sea quarks same parametrization as for valence quarks

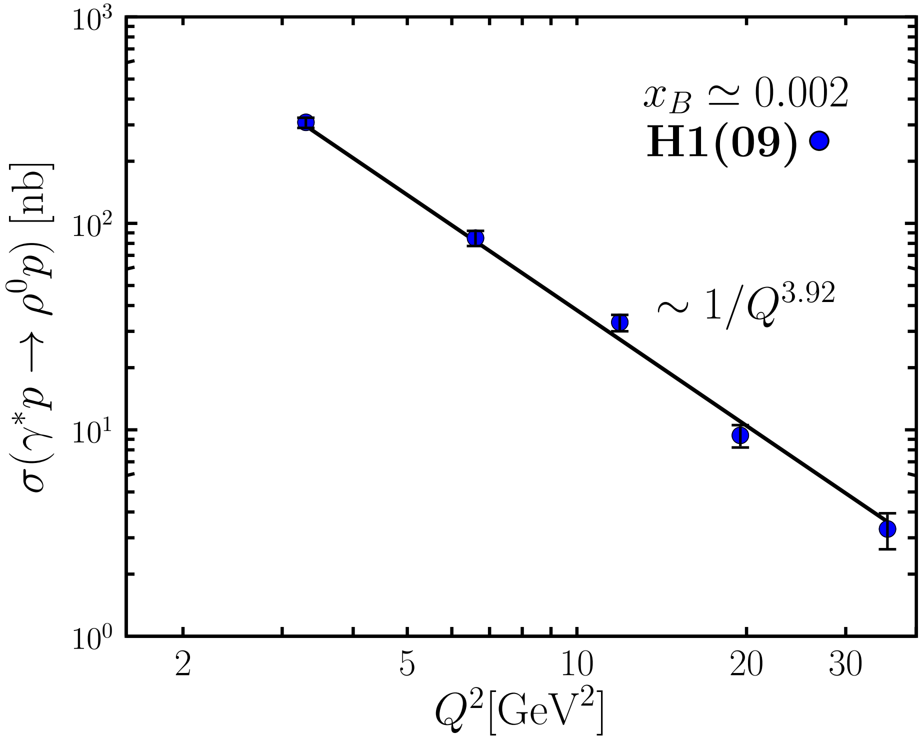
advantages: polynomiality and reduction formulas automatically satisfied

positivity bounds respected D-term neglected

cross sections probe $\langle H \rangle$ A_{UT} probes $\mathrm{Im}\big[\langle E\rangle^*\langle H\rangle\big]$

SDME, other asymmetries, π^+ data are also sensitive to $\widetilde{H}$ and transv. GPDs

Q^2 dependence of ρ^0 cross section



data: H1

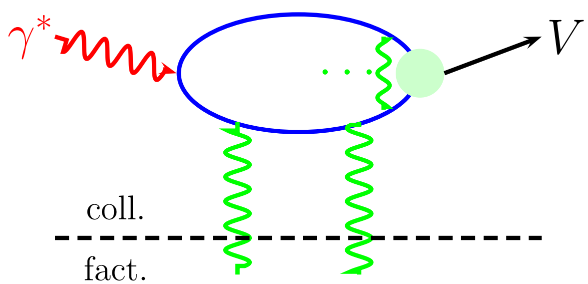
leading-twist scaling: $\sigma_L \sim 1/Q^6$ (modulo $\ln Q^2$) at fixed x_B

σ_L even flatter than σ

need for power corrections

The subprocess amplitude for DVMP

Goloskokov-K. (06) mod. pert. approach - quark trans. momenta in subprocess
(emission and absorption of partons from proton collinear to proton momenta)
transverse separation of color sources $\Longrightarrow$ gluon radiation



Sudakov factor Sterman et al(93)

$$S(\tau, \mathbf{b}, Q^2) \propto \ln \frac{\ln(\tau Q/\sqrt{2}\Lambda_{\rm QCD})}{-\ln(b\Lambda_{\rm QCD})} + \text{NLL}$$

resummed gluon radiation to NLL: e^{-S}
provides rather sharp cut-off at $b = 1/\Lambda_{\rm QCD}$

$$\mathcal{H}^M_{0\lambda,0\lambda} = \int d\tau d^2b\, \hat{\Psi}_M(\tau, -\vec{b})\, e^{-S} \hat{\mathcal{F}}_{0\lambda,0\lambda}(\bar{x}, \xi, \tau, Q^2, \vec{b})$$

LO pQCD
+ quark trans. mom.
+ Sudakov supp.
$\Rightarrow$ asymp. fact. formula (lead. twist) for $Q^2 \to \infty$

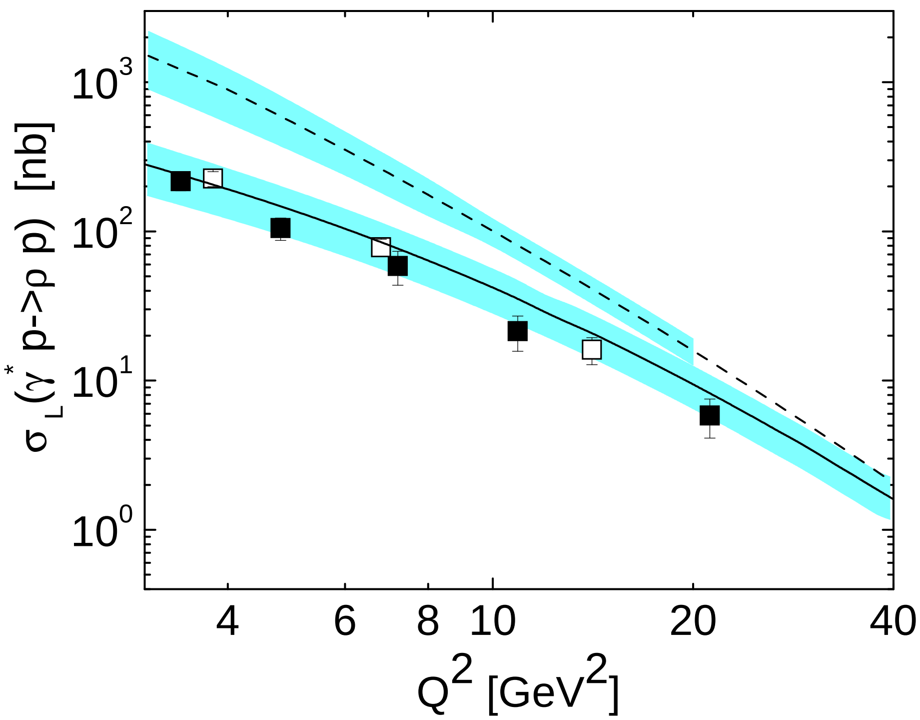
$\hat{\Psi}_M$ light-cone wave fct of meson
$\hat{\mathcal{F}}$ FT of hard scattering kernel
e.g. $\propto 1/[k_\perp^2 + \tau(\bar{x}+\xi)Q^2/(2\xi)] \Rightarrow$ Bessel fct

Sudakov generates series of power corr. $\sim (\Lambda^2_{\rm QCD}/Q^2)^n$
(from region of soft quark momenta $\tau, \bar{\tau} \to 0$)
from intrinsic transv. momenta (wave fct) series $\sim (\langle k_\perp^2\rangle/Q^2)^n$ (from all τ)

Probing H

fit to all available ρ^0 and ϕ data from HERMES, COMPASS, E665, H1, ZEUS

cover large range of kinematics $Q^2 \simeq 3 - 100\,\mathrm{GeV}^2$ $W \simeq 4 - 180\,\mathrm{GeV}$



data on ρ^0 cross section from H1 and ZEUS $W = 75\,\mathrm{GeV}$

E for gluons and sea quarks

sum rule: $e^g_{20} = -\sum e^{q_v}_{20} - 2\sum e^{\bar{q}}_{20}$ valence term very small

$\Rightarrow$ 2nd moments of gluon and sea quarks cancel each other almost completely

(holds approximately for other moments too

provided GPDs don't possess nodes except at the end points)

positivity bound for FTs forbids large sea $\Longrightarrow$ gluon small too

$\frac{b_\perp^2}{m^2}\left(\frac{\partial e_s(x,b_\perp)}{\partial b_\perp^2}\right)^2 \le s^2(x,b_\perp) - \Delta s^2(x,b_\perp)$ Burkhardt (04)

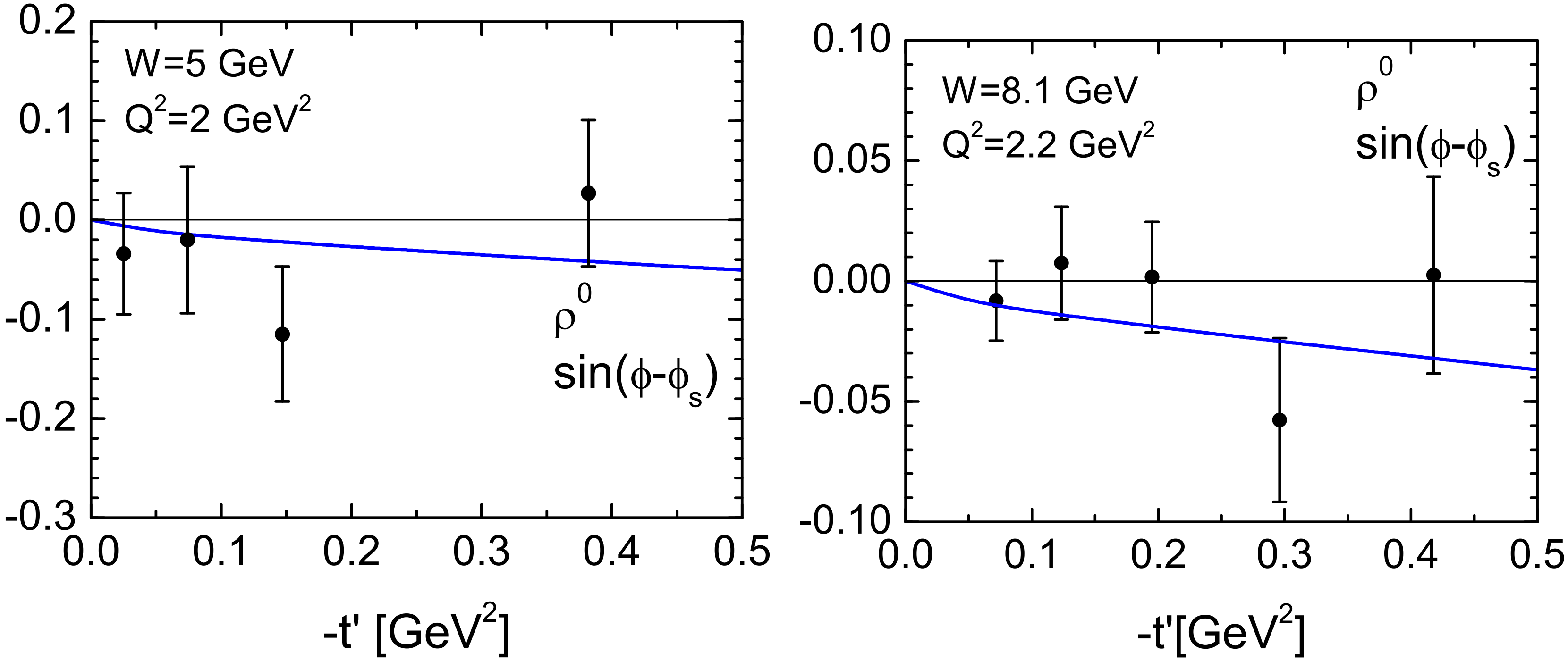
parameterization as described: $\beta_e^s = 7$, $\beta_e^g = 6$ Regge-like parameters as for H

flavor symm. sea for E assumed

N_s fixed by saturating bound ($N_s = \pm 0.155$), N_g from sum rules

for $\xi \neq 0$ input to double distribution ansatz

$A_{UT}^{\sin(\phi-\phi_s)}$ for ρ^0 production



data: HERMES(08) COMPASS(12)

theor. result: Goloskokov-K(09)

$$A_{UT}^{\sin(\phi-\phi_s)} \sim \mathrm{Im}\big[\langle E\rangle^*\langle H\rangle\big]$$

gluon and sea contr. from E cancel to a large extent

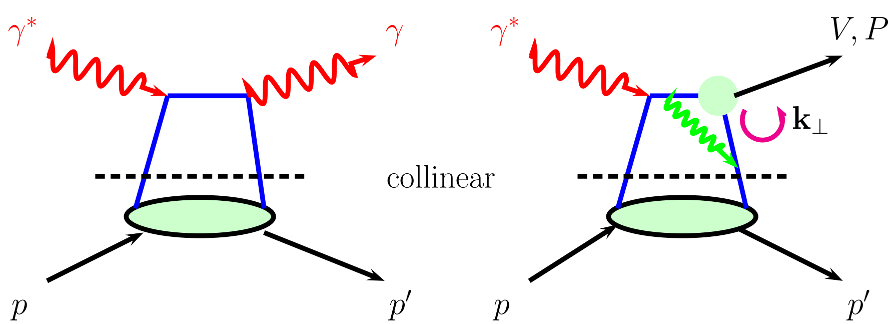
dominated by valence quark contr. from E

(ϕ azimuthal angle between lepton and hadron plane; ϕ_s orientation of target spin vector)

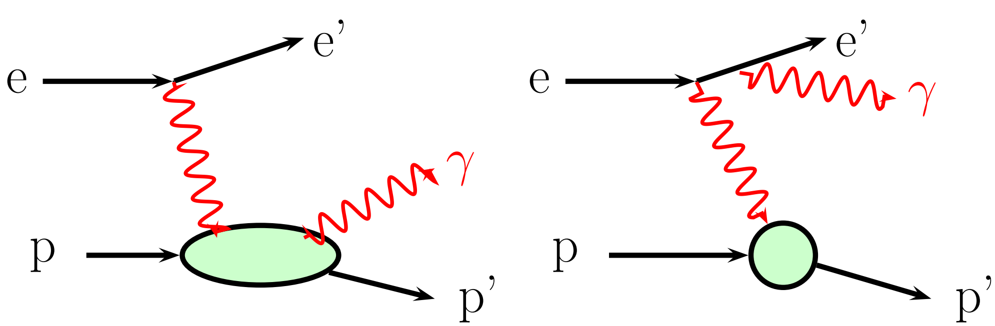
DVCS

Exploiting universality:

K.-Moutarde-Sabatié (12): use GK GPDs to compute DVCS free of parameters
to leading-twist, LO accuracy (collinear for consistency)



reasonable agreement with
HERMES, H1 and ZEUS data
less satisfactory description
of Jlab data (large skewness)



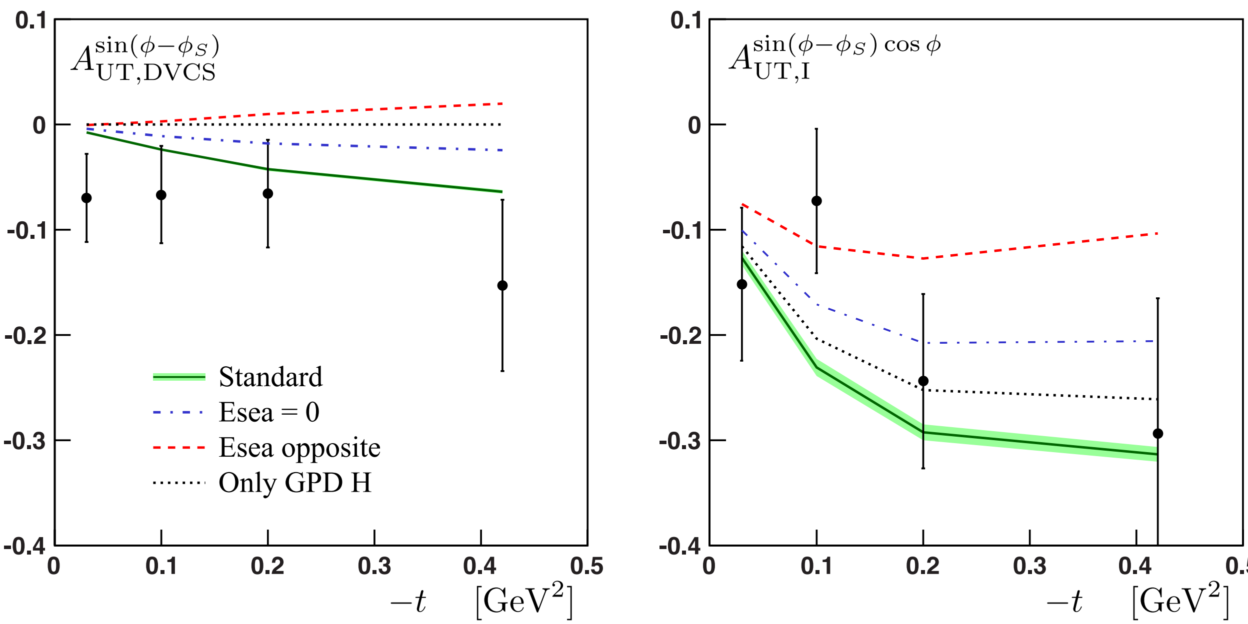
$$d\sigma(lp \to lp\gamma) = d\sigma_{BH} + d\sigma_I + d\sigma_{DVCS}$$

$$d\sigma_i \propto \sum_{n=0}^{3} \left[c_n^i \cos(n\phi) + s_n^i \sin(n\phi)\right]$$

DVCS convolutions

$$\langle K \rangle = \langle e_u^2 K^u + e_d^2 K^d + e_s^2 K^s \rangle$$

Target asymmetry in DVCS



data: HERMES(08)

$\langle Q^2 \rangle \simeq 2.5\,\mathrm{GeV}^2$

$\langle x_{\rm Bj} \rangle \simeq 0.09$

theory: K-Moutarde-Sabatie(12)

$A_{UT,DVCS}^{\sin(\phi-\phi_s)} \sim \mathrm{Im}\big[\langle E\rangle^* \langle H\rangle\big]$

no cancellation between sea and gluon

$\Rightarrow \langle E^{\rm sea}\rangle$ seen

from BH-DVCS interference

separate contr. from $\mathrm{Im}\langle H\rangle$ and $\mathrm{Im}\langle E\rangle$

negative $\langle E^{\rm sea}\rangle$ favored in both cases

$\langle E^g\rangle \geq 0$ Koempel et al(12) transverse target polarisation in J/Ψ photoproduction, dominated by gluonic GPDs

Application: Angular momenta of partons

$$J^q = \frac{1}{2}\left[q_{20} + e^q_{20}\right] \qquad J^g = \frac{1}{2}\left[g_{20} + e^g_{20}\right] \qquad (\xi = t = 0)$$

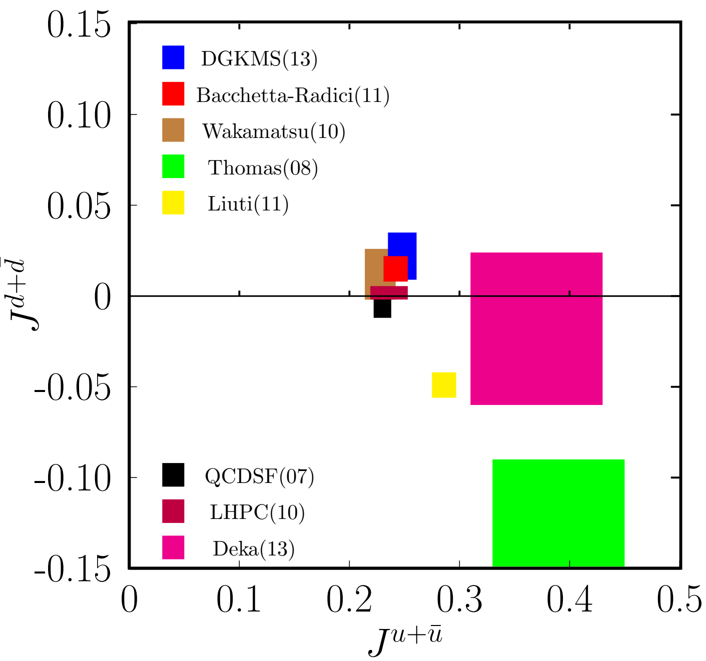
q_{20}, g_{20} from ABM11 (NLO) PDFs
$e^{q_v}_{20}$ from form factor analysis Diehl-K. (13)
$e^s_{20} \approx 0 \cdots - 0.026$ from A_{UT} in DVMP GK(09) and DVCS KMS(13)
and saturation of pos. bound (flavor symm. sea assumed for E)
e^g_{20} from sum rule for e_{20}

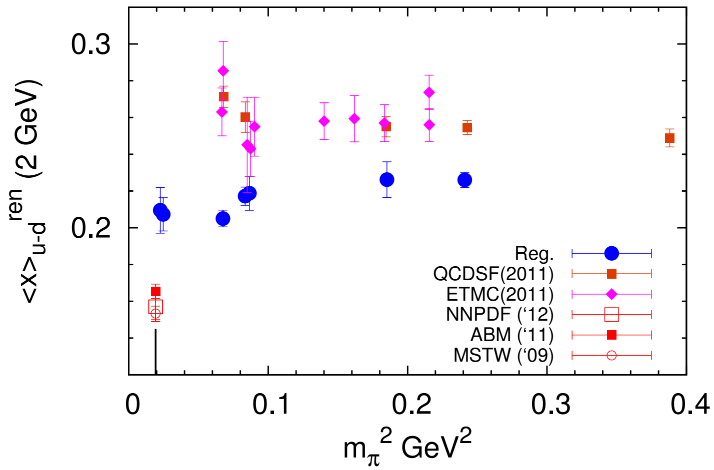
$$\begin{aligned} J^{u+\bar{u}} &= 0.261\,; J^{d+\bar{d}} = 0.035\,; J^{s+\bar{s}} = 0.017\,; J^g = 0.188 \quad (e^s_{20} = 0) \\ &= 0.235\,; \qquad\quad = 0.009\,; \qquad\quad = -0.009\,; \quad\; = 0.266 \quad (e^s_{20} = -0.026) \end{aligned}$$

J^i quoted at scale $2\,\mathrm{GeV}$ $\quad \sum J^i = 1/2$ spin of the proton

need better determ. of E^s (smaller errors of A_{UT} in DVCS)

Comparison with other results





from DVCS exp:

CLAS $J^{d+\bar{d}} + J^{u+\bar{u}}/5 = 0.18 \pm 0.14$

HERMES $J^{d+\bar{d}}/2.9 + J^{u+\bar{u}} = 0.42 \pm 0.22$

strongly model dependent

assump: $e_{q_v}(x) \sim q_v(x)$

in conflict with FF analysis and

with pert.QCD Yuan(04)

$\langle x \rangle_{u-d} \equiv u_{20} - d_{20}$

Bali et al, 1311.7041

estimate of excited state contributions

Deka et al (13) $\langle x \rangle_{u-d} = 0.265(45)$

$(m_\pi \geq 478 \text{ MeV})$

Orbital angular momentum

orbital angular momentum: $\tilde{h}^q_{10}(0) = \Delta q_{10} = \int_0^1 dx \Delta q(x)$ DSSV(09)

$$2L^q = q_{20} + e^q_{20} - \Delta q_{10}$$

$$L^u_v = -0.141^{+0.025}_{-0.033} \qquad L^d_v = 0.114^{+0.034}_{-0.035}$$

errors likely underestimated, systematic study of all pol. PDFs not done

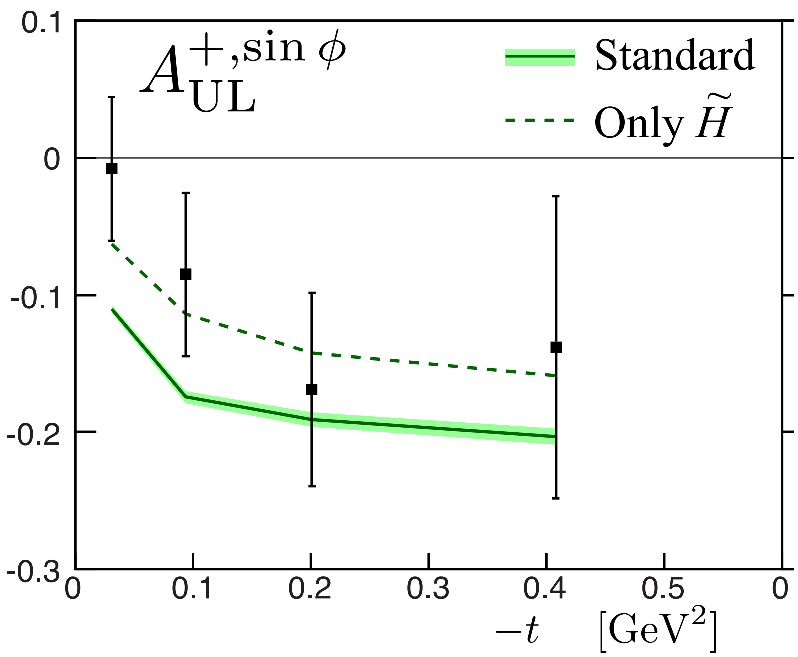
$$L^{u+\bar{u}} = -0.146\ldots-0.172 \qquad L^{d+\bar{d}} = 0.263\ldots0.237 \qquad L^{s+\bar{s}} = 0.073\ldots0.047$$

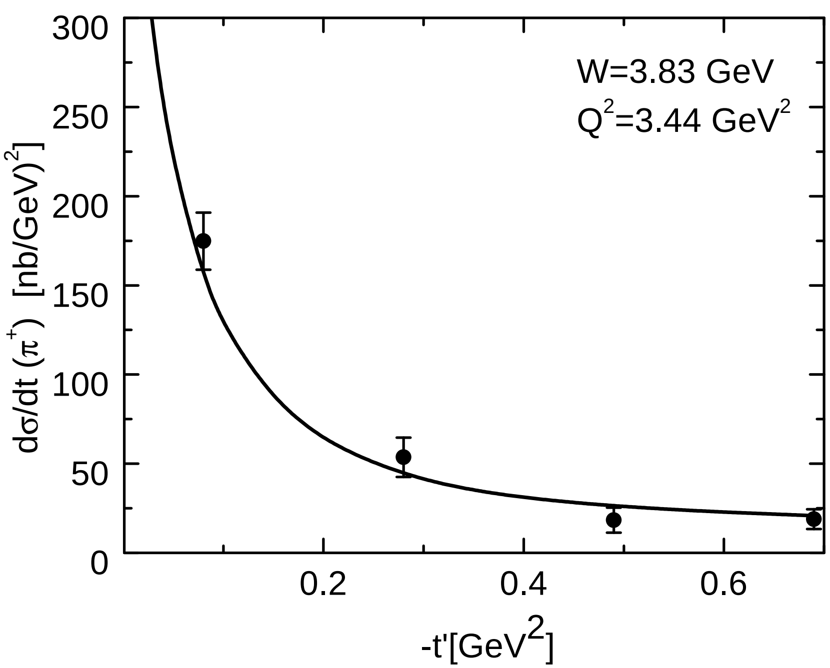
$$J^g - \Delta g/2 = 0.140\ldots0.218$$

$\Delta q \to \widetilde{H}$ and tests

$\widetilde{H}$ at $\xi = 0$ parametrized as the other GPDs and used as input to DD

Tests: axial form factor $\sim \widetilde{H}^u - \widetilde{H}^d$ poor data but agreement





HERMES

long. pol. target spin asymmetry

π^+ electroproduction

$+$ pion pole

Summary

- the analysis of data on electromagnetic form factors and exclusive meson leptoproduction at small skewness and small $-t$ fixed H (for gluons, valence- and sea quarks) and E (for valence quarks)
- estimate of E^g and E^{sea} from a sum rule and a positivity bound
- present data on A_{UT} for DVCS favor a negative E^{sea} and a positive E^g
- this information allows to evaluate the angular momenta the partons inside the proton carry with the help of Ji's sum rule
- for an improvement of the results better A_{UT} data are needed