No-core shell model for hypernuclei

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Outline

- Motivation
- Methodology – no-core shell model with strangeness
- Results
  - *s*-shell hypernuclei
  - *p*-shell hypernuclei
- Conclusions & summary

Strangeness in nuclear many-body systems

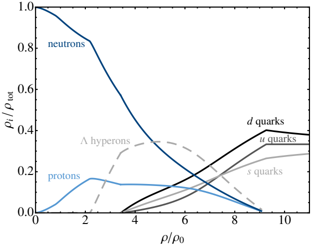
Interdisciplinary subject connecting particle physics, nuclear physics and astrophysics.

Related topical questions include:

- interaction of (anti)kaons with the nuclear medium
  - possible existence of deeply-bound K^-–nuclear states?
  - antikaons in dense matter?
- interaction of hyperons with the nuclear medium
  - S=-1 Λ hypernuclei, Σ-hypernuclei?
  - S=-2 $\Lambda\Lambda$-hypernuclei, Ξ hypernuclei
  - hyperons in dense nuclear matter and neutron stars?

Role of strangeness in dense nuclear matter?

- **admixtures of Λ and Σ hyperons in dense baryonic matter in neutron stars?**
- at baryon densities $\rho \gtrsim (2-3)\rho_0$,
  Λ hyperons can take role of neutrons if energetically favourable



(Hell, Weise, arXiv:1402:4098 [nucl-th])

- presence of hyperons results in considerable softening of EOS incompatible with recent astrophysical constraints – by reducing the maximum neutron star mass much below $2M_\odot$

Study of hypernuclei

- improve understanding of YN interaction
  - provide important constraints on YN interaction
  - precise experimental data on hypernuclear spectroscopy
  - supplement (very sparse) hyperon–nucleon scattering data base
- new precision experiments at J-PARC, J-Lab, FAIR, ...
- modern developments of YN interaction
  - based on SU(3) chiral EFT
  - require advanced many-body computational methods to confront with hypernuclear structure measurements

Ab initio calculations of light hypernuclei

- given microscopic NN (+NNN) and YN interactions, calculate the energy spectra of A-body hypernuclear system with controllable approximations
- calculations so far limited to A=3,4 hypernuclear systems (Faddeev, Faddeev–Yakubovsky equations)
- recent developments in computational many-body methods

Our aim:

- develop a method applicable to heavier A≥5 hypernuclei
- study available boson-exchange and chiral YN interaction models

No-core shell model for hypernuclei

Ab initio

- all particles are active (no rigid core)
- exact Pauli principle
- realistic 2- and 3-body interactions (accurate description of *NN* and *YN* data)
- controllable approximations

- Hamiltonian is diagonalized in a *finite* A-particle harmonic oscillator basis
- NCSM results converge to exact results

NCSM in relative Jacobi-coordinate HO basis

- Hamiltonian rewritten in Jacobi coordinates

$$\vec{\xi}_\alpha = \sqrt{\frac{m_L^\alpha m_R^\alpha}{m_L^\alpha + m_R^\alpha}} \left(\frac{1}{m_L^\alpha} \sum_{\{L\}} m_i \vec{x}_i - \frac{1}{m_R^\alpha} \sum_{\{R\}} m_i \vec{x}_i \right)$$

- Center of mass excitations decouple

$$H(\vec{x}_1, \cdots, \vec{x}_A) = H_{\rm cm}(\vec{\xi}_0) + H(\vec{\xi}_1, \cdots, \vec{\xi}_{A-1})$$

- Hamiltonian diagonalized in

$$| \underbrace{N i J_1 T_1}_{\text{antisymmetric (A-1)N state}}, \overbrace{n_Y l_Y j_Y t_Y}^{\text{Y state}}, J \pi T \rangle$$

$$N + 2n_Y + l_Y \leq N_{\rm max}$$

NCSM in Slater-determinant HO basis

- For larger number of particles ($A \gtrsim 5$) antisymmetrization of J-NCSM basis becomes inefficient
- Slater-determinant basis built from single-particle HO wavefunctions

$$\phi(\vec{x}, \sigma, \tau) = R_{nl} \left(Y_l(\hat{x} \chi(\sigma)) \right)_{jm_j} \chi(\tau)_{m_t}$$

$$\sum_{i=1}^{A} 2n_i + l_i \leq N_{\mathrm{totmax}}$$

- Importance truncation of the basis applied in large model spaces ($N_{\mathrm{max}} > 8$) (Roth, PRC 79 (2009) 064324)

Input V_{NN}, V_{NNN}, and V_{YN} potentials

NN+NNN interaction

- chiral N3LO NN potential
  (Entem, Machleidt, PRC 68 (2003) 041001)
- chiral N2LO NNN potential
  (Navrátil, FBS 41 (2007) 14)

YN interaction

- phenomenological meson-exchange Jülich04 potential
  (Haidenbauer, Meißner, PRC 72 (2006) 044005)
- chiral LO potential
  NLO version recently developed (Haidenbauer *et al.*, NPA 915 (2013) 24)

$\Lambda N - \Sigma N$ mixing explicitly taken into account:

$$V_{YN} = \begin{pmatrix} V_{\Lambda N-\Lambda N} & V_{\Lambda N-\Sigma N} \\ V_{\Sigma N-\Lambda N} & V_{\Sigma N-\Sigma N} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & m_\Sigma - m_\Lambda \end{pmatrix}$$

Similarity renormalization group

- Series of unitary transformations of the original Hamiltonian to accelerate convergence of many-body calculations

$$H_\lambda = U_\lambda H U_\lambda^\dagger$$

implemented as a flow equation in λ

$$\frac{\mathrm{d}}{\mathrm{d}\lambda} H_\lambda = -\frac{4}{\lambda^5}[[T, H_\lambda], H_\lambda]$$

- Decoupling of low-momentum and high-momentum parts of the Hamiltonian
- Essential for *p*-shell hypernuclear calculations
- Induces 3- and higher-body forces

S-shell hypernuclei: ${}^{3}_{\Lambda}\mathrm{H}$

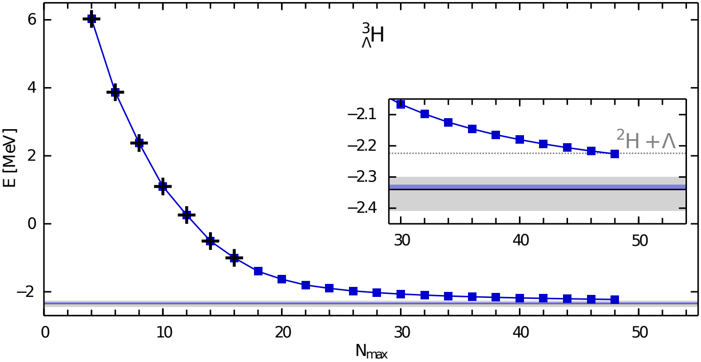


Figure: Ground state energy of ${}^{3}_{\Lambda}\mathrm{H}$ as a function of the size of the model space, with bare chiral LO @ 600 MeV interactions.

S-shell hypernuclei: $nn\Lambda$ bound state?

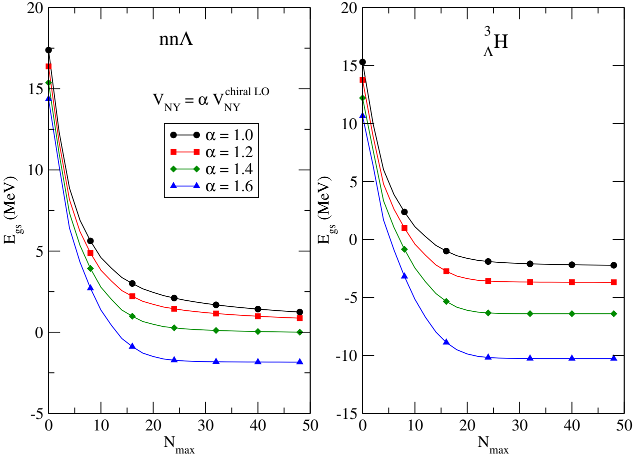


Figure: Ground state energy of $nn\Lambda$ and ${}^{3}_{\Lambda}$H as a function of the size of the model space, with scaled chiral LO @ 600 MeV interactions.

S-shell hypernuclei: ${}^4_\Lambda$H, ${}^4_\Lambda$He

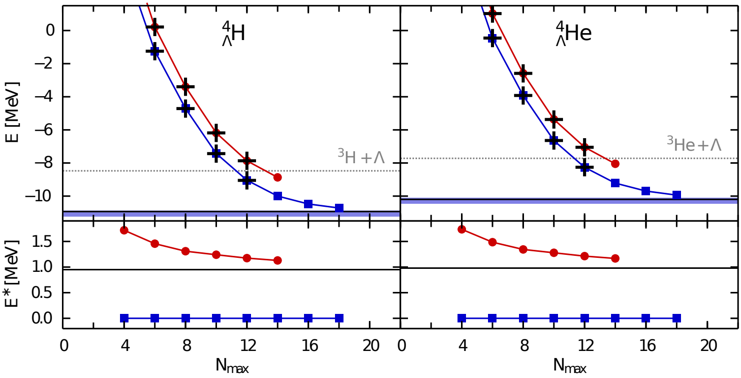


Figure: Ground state (blue) and excited state (red) energy of ${}^4_\Lambda$H and ${}^4_\Lambda$He as a function of the size of the model space, with chiral LO @ 600 MeV interactions.

S-shell hypernuclei: $\Lambda N - \Sigma N$ mixing in ${}^4_\Lambda$He

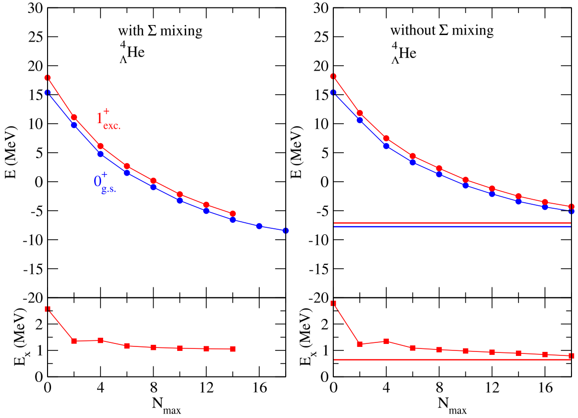


Figure: Ground state (blue) and excited state (red) energy of ${}^4_\Lambda$He with and without $\Lambda N - \Sigma N$ mixing as a function of the size of the model space, with chiral LO @ 600 MeV interactions.

P-shell hypernuclei: $^{7}_{\Lambda}$Li

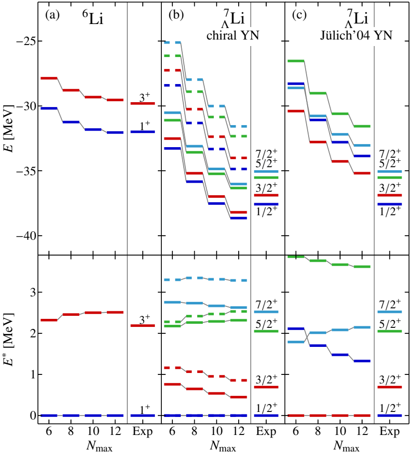


Figure: Calculations of $^{7}_{\Lambda}$Li with chiral LO @ 600@MeV (solid lines) and 700 MeV (dashed lines) and Jülich04 YN interactions.

P-shell hypernuclei: ${}^{9}_{\Lambda}$Be

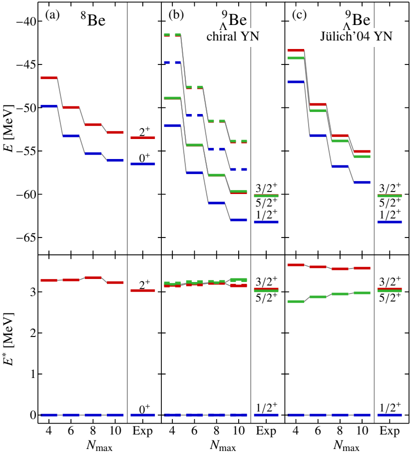


Figure: Calculations of ${}^{9}_{\Lambda}$Be with chiral LO @ 600@MeV (solid lines) and 700 MeV (dashed lines) and Jülich04 YN interactions.

P-shell hypernuclei: $^{13}_{\Lambda}$C

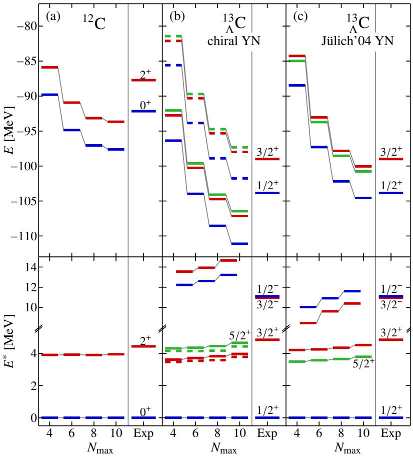


Figure: Calculations of $^{13}_{\Lambda}$C with chiral LO @ 600@MeV (solid lines) and 700 MeV (dashed lines) and Jülich04 YN interactions.

Summary

Calculations of light hypernuclei within NCSM

- reliable *ab initio* calculations of p-shell hypernuclei with microscopic interactions are now possible
- systematic study of p-shell hypernuclei improves understanding of YN interactions
- LO chiral YN interactions are consistent with measured low-lying energy levels of light hypernuclei
- indication of deficiencies for higher relative partial waves of LO chiral YN interactions – study of YN interaction at NLO desirable

Gazda, Mareš, Navrátil, Roth, Wirth; Few-Body Systems, 55 (2014) 857.
Wirth, Gazda, Navrátil, Calci, Langhammer; accepted in PRL, arXiv:1403.3067 [nucl-th].