

${}^3_{\Lambda}\text{n}$ & other neutron-rich hypernuclei

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- 1st sound calculation showing that ${}^3_{\Lambda}\text{n}$ is **unbound** is due to Downs-Dalitz, PR 114 (1959) 593.
However, the HypHI Collaboration, Rappold et al. PRC 88 (2013) 041001(R), argued recently by observing $\pi^- + {}^3\text{H}$ weak decay that ${}^3_{\Lambda}\text{n}$ is **bound**.
- **Recent calculations agree on unbound ${}^3_{\Lambda}\text{n}$:**
 - (i) Garcilazo-Valcarce, PRC 89 (2014) 057001
 - (ii) Hiyama-Ohnishi-Gibson-Rijken, *ibid* 061302(R)
 - (iii) Gal-Garcilazo, PLB 736 (2014) 93
- **We derived constraints from several hypernuclear systems to rule out a bound ${}^3_{\Lambda}\text{n}$.**

Simple considerations

s-shell Λ hypernuclei

Hypernucleus	J^π (g.s.)	B_Λ MeV	J^π	E_x MeV
${}^3_\Lambda\text{H}$	$1/2^+$	0.13(5)		
${}^4_\Lambda\text{H}-{}^4_\Lambda\text{He}$	0^+	2.04(4)–2.39(3)	1^+	1.04(5)–1.15(4)
${}^5_\Lambda\text{He}$	$1/2^+$	3.12(2)		

Past “Exact” Calculations

- $A = 3$ K. Miyagawa et al., PRC 51 (1995) 2905 Faddeev.
- $A = 3, 4$ A. Nogga et al., PRL 88 (2002) 172501 Faddeev & Fad-Yak. A. Nogga, NPA 914 (2013) 140 (chiral).
- $A = 4$ E. Hiyama et al., PRC 65 (2002) 011301(R) Jacobi, Gaussian basis PRC 89 (2014) 061302(R).
- $A = 3, 4, 5$ H. Nemura et al., PRL 89 (2002) 142504 Stochastic variation with correlated Gaussians.
- $A = 3, 4$ R. Wirth et al., arXiv:1403.3067 (ab-initio).

Incompatibility of bound-state ${}^3_{\Lambda}\text{n}$ with Λp scattering

	$B_{\Lambda}^{T=0}=0$		$B_{\Lambda}^{T=0}=0.13 \text{ MeV}$		$B_{\Lambda}^{T=1}=0$ (${}^3_{\Lambda}\text{n}$ just bound)		
r_{eff}	a	$\sigma_{\Lambda p}^{\text{tot}}$	a	$\sigma_{\Lambda p}^{\text{tot}}$	a	$\sigma_{\Lambda p}^{\text{tot}}$	$B_{\Lambda}^{T=0}$
(fm)	(fm)	(mb)	(fm)	(mb)	(fm)	(mb)	(MeV)
2.5	-1.185	129.7	-1.498	192.5	-4.491	953.8	2.59
3.5	-1.405	152.4	-1.895	239.7	-5.930	943.1	1.74

- Solve ΛNN Faddeev equations. Separable NN , ΛN interactions fitted to low-energy scattering, neglecting ΛN spin dependence for simplicity.
- **If ${}^3_{\Lambda}\text{n}$ is bound, ${}^3_{\Lambda}\text{H}$ is substantially overbound. Only $B_{\Lambda}^{T=0}=0.13\pm 0.05 \text{ MeV}$ is consistent with $\sigma_{\Lambda p}^{\text{tot}}(p_{\Lambda}=145\pm 25 \text{ MeV}/c)=180\pm 22 \text{ mb}$.**

- In next step relax ΛN spin independence:
 $a_s = -2.03$, $r_s = 3.66$, $a_t = -1.39$, $r_t = 3.32$ fm,
and scale up $V_t(\Lambda N) \rightarrow xV_t(\Lambda N)$, $x \geq 1$, until ${}^3_\Lambda n$
binds (Fredholm determinant vanishes at $E = 0$).
- This allows us to also study ${}^3_\Lambda H(\frac{3}{2}^+)_{\text{exc.}}$ vs ${}^3_\Lambda H(\frac{1}{2}^+)_{\text{g.s.}}$.

x	${}^3_\Lambda n$ FD($E = 0$)	$B_\Lambda[{}^3_\Lambda H^{T=0}(\frac{1}{2}^+)]$	$B_\Lambda[{}^3_\Lambda H^{T=0}(\frac{3}{2}^+)]$
1.00	0.55	0.096	unbound
1.10	0.47	0.147	0.124
1.20	0.39	0.211	0.448
...
1.61	+0.004	0.625	3.772
1.62	-0.006	0.638	3.890

- ${}^3_\Lambda H(\frac{3}{2}^+)$ becomes g.s. well before ${}^3_\Lambda n$ binds.

$\Lambda - \Sigma$ coupling considerations

$\Lambda - \Sigma$ coupling for ${}^4_{\Lambda}\text{H}$ and ${}^4_{\Lambda}\text{He}$

Y. Akaishi et al., PRL 84 (2000) 3539

$$|{}^4_{\Lambda}\text{He}(T = 1/2)\rangle = \alpha s^3 s_{\Lambda} + \beta s^3 s_{\Sigma}$$

From $\Lambda N - \Sigma N$ g matrix for $0s$ orbits

$$v = \langle s^3 s_{\Lambda} | g | s^3 s_{\Sigma} \rangle, \quad \Delta E \sim 80 \text{ MeV} \quad {}^3g_{ss} = 4.8 \quad {}^1g_{ss} = -1.0$$

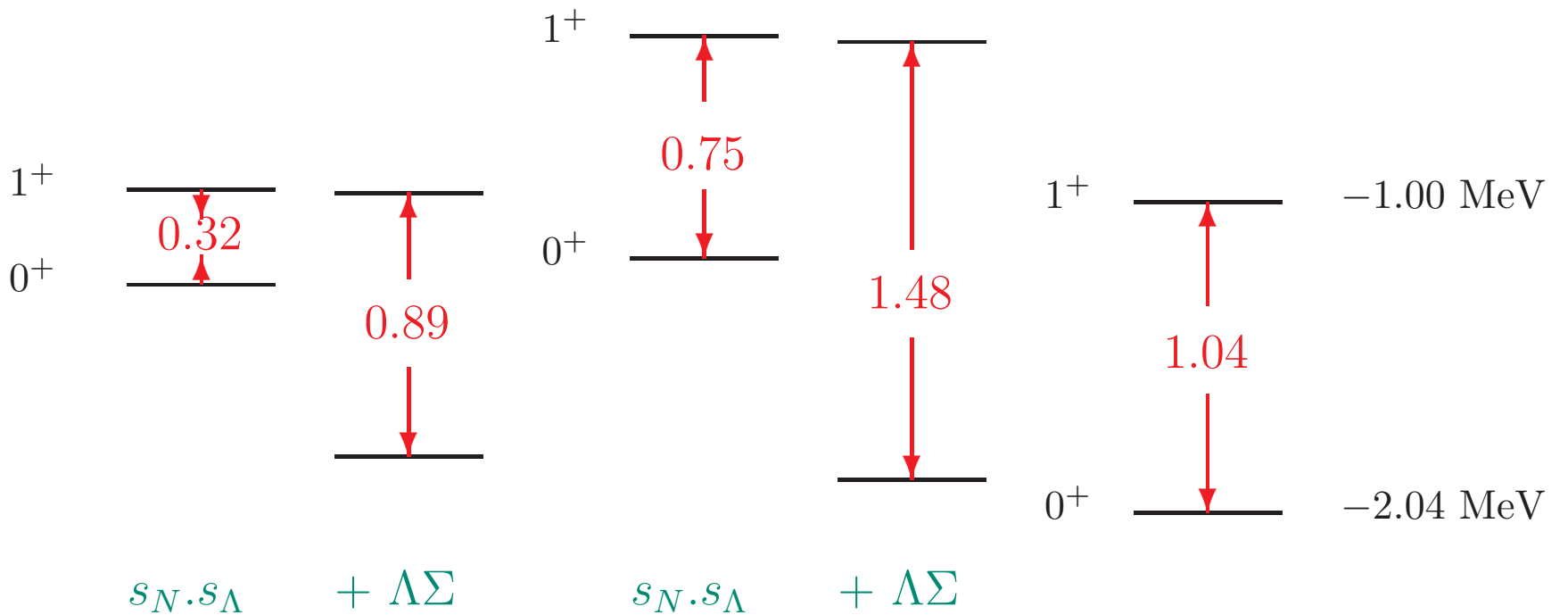
$$\begin{array}{ll} 0^+ & v = \frac{3}{2} {}^3g_{ss} - \frac{1}{2} {}^1g_{ss} & \text{Admixture} \sim -v/\Delta E \\ 1^+ & v = \frac{1}{2} {}^3g_{ss} + \frac{1}{2} {}^1g_{ss} & E^{\text{shift}} \sim v^2/\Delta E \end{array}$$

NSC97f: for 0^+ $v \sim 7.6 \text{ MeV} \Rightarrow E^{\text{shift}} \sim 0.72 \text{ MeV}$

comparable to genuine ΛN splitting $\Delta_s = 0.75 \text{ MeV}$

(s -shell ΛN interaction: $\bar{V}_s + \Delta_s s_N \cdot s_{\Lambda}$)

$\Lambda - \Sigma$ Nijmegen model dependence



NSC97e

NSC97f

${}^4_\Lambda\text{H}$

$\Lambda - \Sigma$ Coherent Coupling

($1s_\Lambda \rightarrow 1s_\Sigma$ & same nucleon orbital wavefunction)

- $\Lambda\Sigma$ coupling: $\sqrt{4/3} (t_N \cdot t_Y \bar{V}' + s_N \cdot s_Y t_N \cdot t_Y \Delta')$
($\sqrt{4/3}$ arises from t_Y changing Λ to Σ) leading to **Fermi & Gamow-Teller (GT)** nuclear matrix elements.
- The important $\Lambda\Sigma$ coupling matrix elements involve Σ and Λ hyperons coupled to the same nuclear core, and nuclear states connected by a large GT matrix element to the dominant core state.
- Sizable $\Lambda\Sigma$ matrix elements arise in realistic models, for example in models NSC97e(f):

$$\bar{V}_{\Lambda\Sigma} = 2.96 \text{ (3.35)}, \quad \Delta_{\Lambda\Sigma} = 5.09 \text{ (5.76) MeV.}$$

$p_N s_{\Lambda-\Sigma}$ coupling parameters from Nijmegen YN potentials

Source	Interaction	\bar{V}'_p	Δ'_p	S'_Λ	S'_N	T'
Akaishi (s-shell)	NSC97e/f	1.45	3.04	-0.09	-0.09	0.16
Yamamoto	NSC97f	0.96	3.62	-0.07	-0.07	0.31
Halderson	NSC97e	0.75	3.51	-0.45	-0.24	0.31
Halderson	NSC97f	1.10	3.73	-0.45	-0.23	0.30
Halderson	ESC08a	1.05	4.71	-0.07	0.02	0.32

Halderson, Phys. Rev. C 77, 034304 (2008)

- ${}^4_\Lambda\text{H}/{}^4_\Lambda\text{He}$ 0^+ : $\bar{V}'_s + 3/4 \Delta'_s$ 1^+ : $\bar{V}'_s - 1/4 \Delta'_s$
- Note: $\bar{V}'_p \approx 0.5 \bar{V}'_s$ & $\Delta'_p \approx 0.5 \Delta'_s$,
hence p -shell effect is roughly 1/4 of s -shell effect

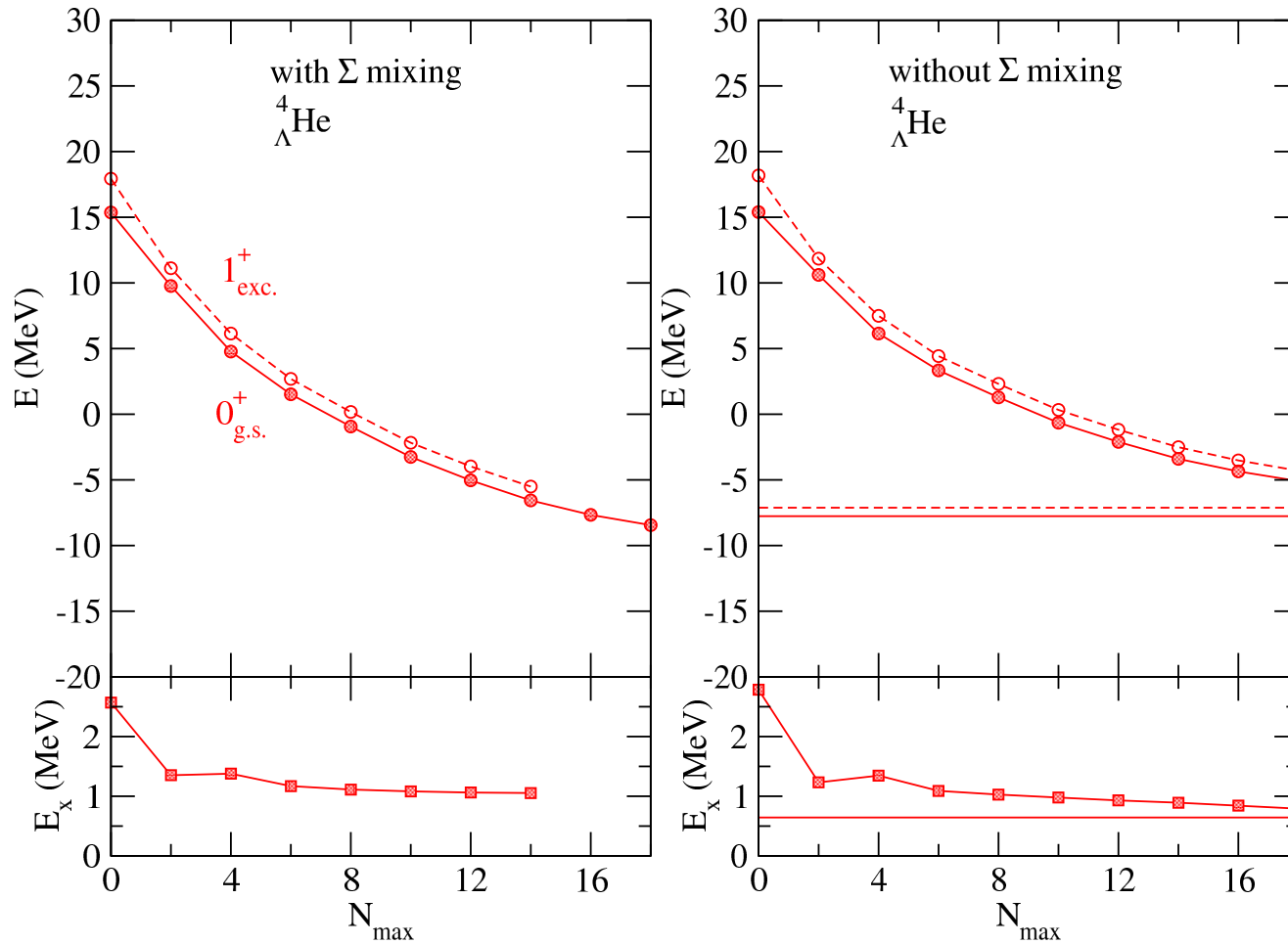
ΛΣ Fermi (F) & Gamow-Teller (GT) matrix elements and binding-energy contributions (MeV)

	${}^3_{\Lambda}\text{H}$			${}^4_{\Lambda}\text{H}$	
	$0, \frac{1}{2}^+$	$0, \frac{3}{2}^+$	$1, \frac{1}{2}^+$	$\frac{1}{2}, 0^+$	$\frac{1}{2}, 1^+$
$\Lambda N (\times \Delta_{\Lambda\Lambda})$	1	-1/2	-	3/4	-1/4
F ($\times \bar{V}_{\Lambda\Sigma}$)	-	-	$2\sqrt{2/3}$	1	1
GT ($\times \Delta_{\Lambda\Sigma}$)	$\sqrt{3}/2$	-	-1/2	3/4	-1/4
$\frac{1}{80}(\text{F} ^2 + \text{GT} ^2)$	0.243	-	0.373	-	-
$\frac{1}{80} \text{F} + \text{GT} ^2$	-	-	-	0.574	0.036

- In SU(4) limit for L=0 NN core states:

$$B_{\Lambda}^{T=1}\left(\frac{1}{2}^+\right) - B_{\Lambda}^{T=0}\left(\frac{1}{2}^+\right) = (0.373 - 0.243) - \Delta_{\Lambda\Lambda}.$$

- Replace $\Delta_{\Lambda\Lambda}$ with increased $\Lambda\Sigma$ contribution by keeping $E(1^+) - E(0^+) \approx 1.1$ MeV in ${}^4_{\Lambda}\text{H}$.



LO chiral potentials, private communication Daniel Gazda,
 from joint work by R. Wirth et al., arXiv:1403.3067.

$B(^2n)$ & $B_\Lambda(^3_\Lambda n)$ solving Λnn Faddeev equations

$a_s(nn)$ (fm)	$r_s(nn)$ (fm)	$B(^2n)$ (MeV)	$B(^2n)_{\text{approx}}$	$B_\Lambda(^3_\Lambda n)$ (MeV)
5.4	1.75	2.23	2.24	0.39
5.4	2.25	2.79	2.87	0.27
5.4	2.881	4.98	–	0.16
6.0	2.881	2.86	3.20	0.11
9.0	2.881	0.80	0.80	0.01
13.0	2.881	0.32	0.32	0.003
17.612	2.881	0.16	0.16	–
–17.612	2.881	–	–	–

- $B(^2n)_{\text{approx}} = \frac{\hbar^2}{M_n r_s^2} \left(1 - \sqrt{1 - \frac{2r_s}{a_s}} \right)^2$, $B_\Lambda(^3_\Lambda n) \ll B(^2n)$.

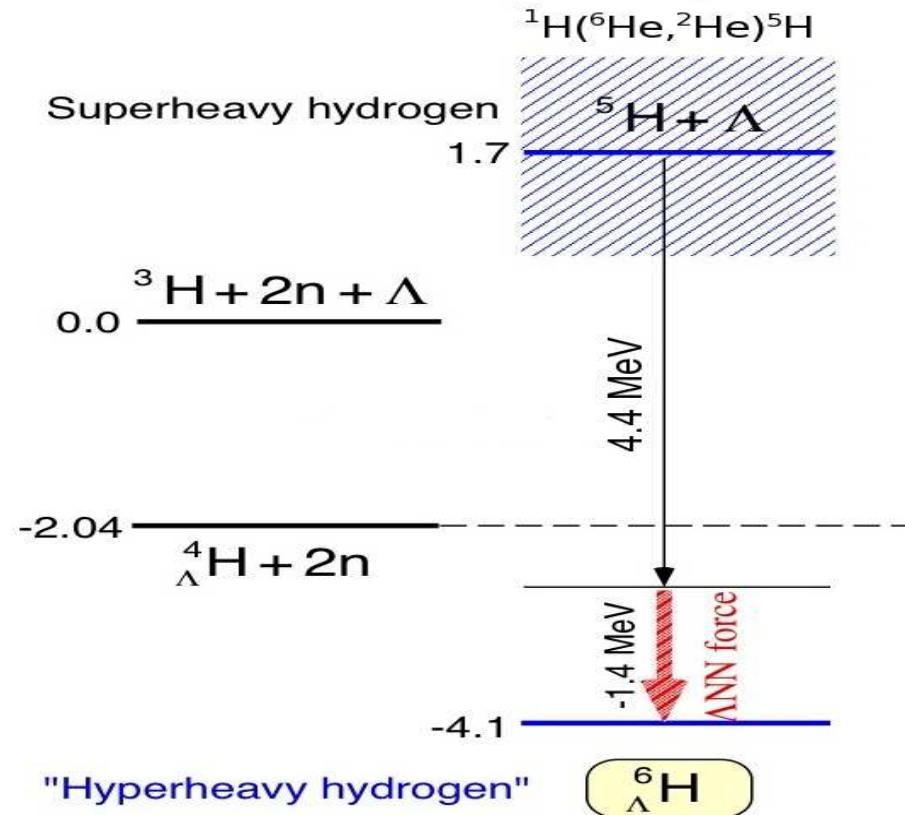
$^3_\Lambda n$ dissolves upon breaking SU(4), for fixed $V_{\Lambda N}$,
 from $B(^2n)=B(d)$ progressively to unbound nn .

${}^3_{\Lambda}\text{n}$: conclusions

- The ΛN interactions required to bind ${}^3_{\Lambda}\text{n}$ are inconsistent with Λp scattering cross sections at low energies, with ${}^3_{\Lambda}\text{H}_{\text{g.s.}}$ binding energy, and with the $0_{\text{g.s.}}^+ - 1_{\text{exc}}^+$ excitation energy of the $A = 4$ Λ hypernuclei.
- The consequences of accepting a bound ${}^3_{\Lambda}\text{n}$ for Λ hypernuclear data are sufficiently strong that the use of more refined interactions is unlikely to modify any of the conclusions reached here.
- Could ΛNN interactions bind ${}^3_{\Lambda}\text{n}$? – unlikely.
- Could CSB bind ${}^3_{\Lambda}\text{n}$? – unlikely.

${}^6_{\Lambda}\text{H}$: experiment & theory

Akaishi's ${}^6_{\Lambda}\text{H}$ [Frascati Phys. Ser. XVI (1999) 59]

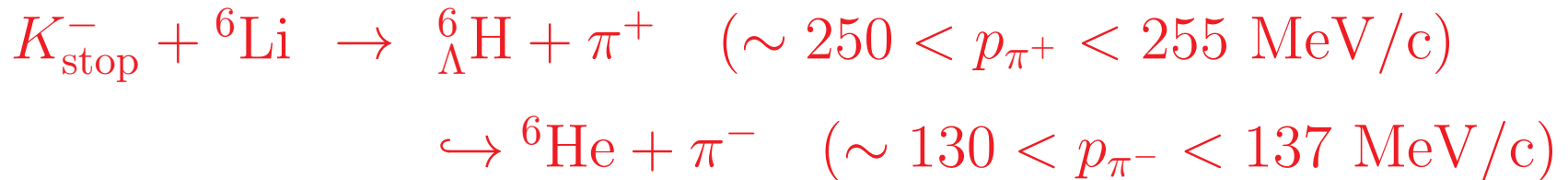


$\Lambda - \Sigma$ coupling, resulting in " ΛNN force", lowers $0_{\text{g.s.}}^+$ by 1.4 MeV and increases $\Delta E(0_{\text{g.s.}}^+ - 1^+)$ from 1 MeV in ${}^4_{\Lambda}\text{H}$ to 2.4 MeV in ${}^6_{\Lambda}\text{H}$

- If Akaishi is right, far-reaching consequences could follow for dense neutron-rich strange stellar matter.
- ${}^6_{\Lambda}\text{H}$ was predicted by Dalitz & Levi Setti,
NC 30 (1963) 489: $B_{\Lambda}^{\text{Dalitz}}({}^6_{\Lambda}\text{H})=4.2 \text{ MeV}$.
Akaishi (1999) predicted $B_{\Lambda}^{\text{Akaishi}}({}^6_{\Lambda}\text{H})=5.8 \text{ MeV}$.
- FINUDA found three ${}^6_{\Lambda}\text{H}$ events in ${}^6\text{Li}(K_{\text{stop}}^-, \pi^+)$
Production rate: $R(\pi^+) = (5.9 \pm 4.0) \cdot 10^{-6} / K_{\text{stop}}^-$
[PRL 108 (2012) 042501, NPA 881 (2012) 269].
 $B_{\Lambda}({}^6_{\Lambda}\text{H})=(4.0 \pm 1.1) \text{ MeV}$ vs $(3.9 \pm 0.1) \text{ MeV}$
calc. by Gal-Millener, PLB 725 (2013) 445.
- E. Hiyama et al., tnn Λ calc.: **unbound**,
NPA 908 (2013) 29.
- ${}^6_{\Lambda}\text{H}$ not seen in ${}^6\text{Li}(\pi^-, K^+)$, J-PARC E10.

FINUDA's innovation

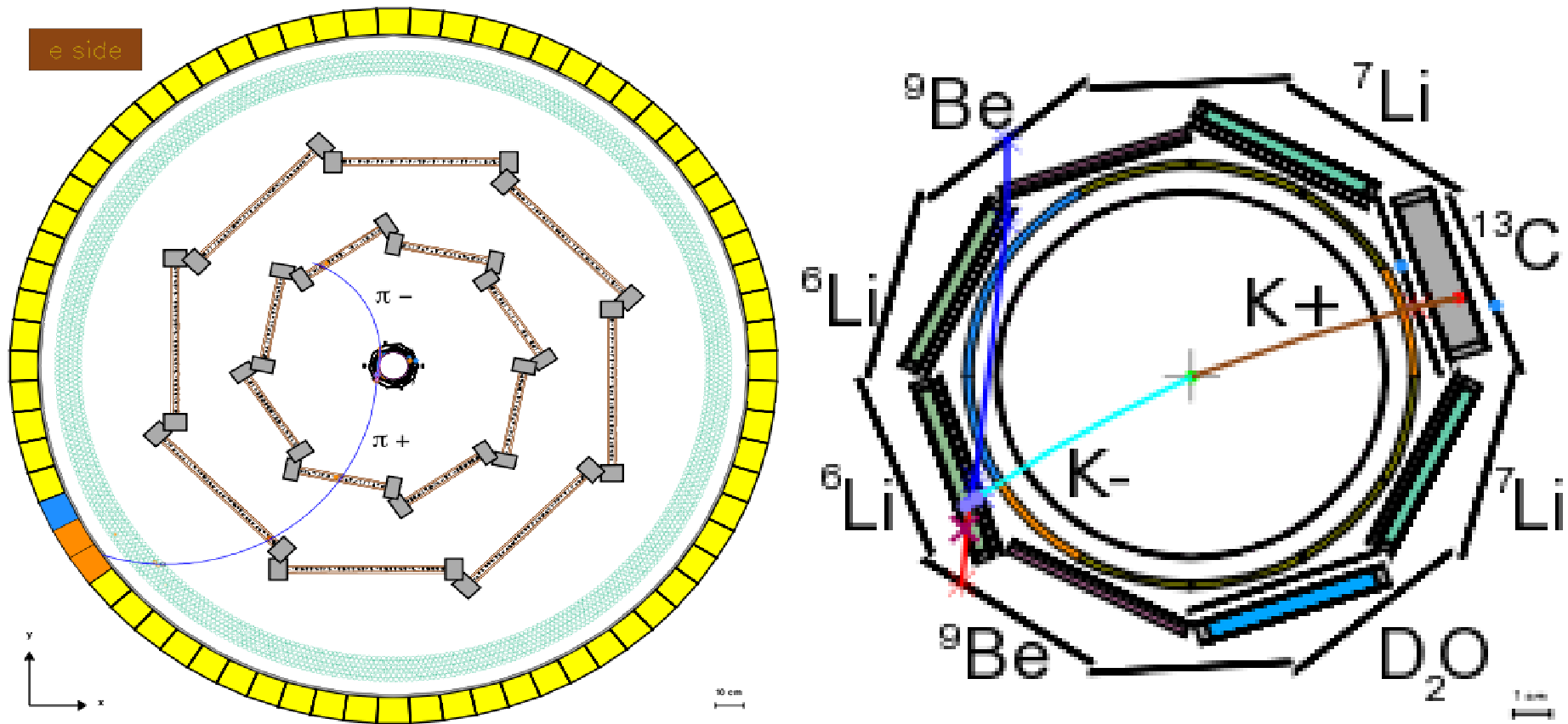
production & decay in coincidence



$$T(\pi^+) + T(\pi^-) = M(K^-) + M(p) - M(n) - 2M(\pi) \\ - B({}^6\text{Li}) + B({}^6\text{He}) - T({}^6\text{He}) - T({}^6_{\Lambda}\text{H})$$

- Recoil uncertainty is 0.2 MeV for 6 MeV B interval
- Pions kinetic energy uncertainty is 1.3 MeV
- Altogether $T(\pi^+) + T(\pi^-) = 203 \pm 1.3 \text{ MeV}$
- Three ${}^6_{\Lambda}\text{H}$ candidate events out of $2.7 \cdot 10^7 K_{\text{stop}}^-$

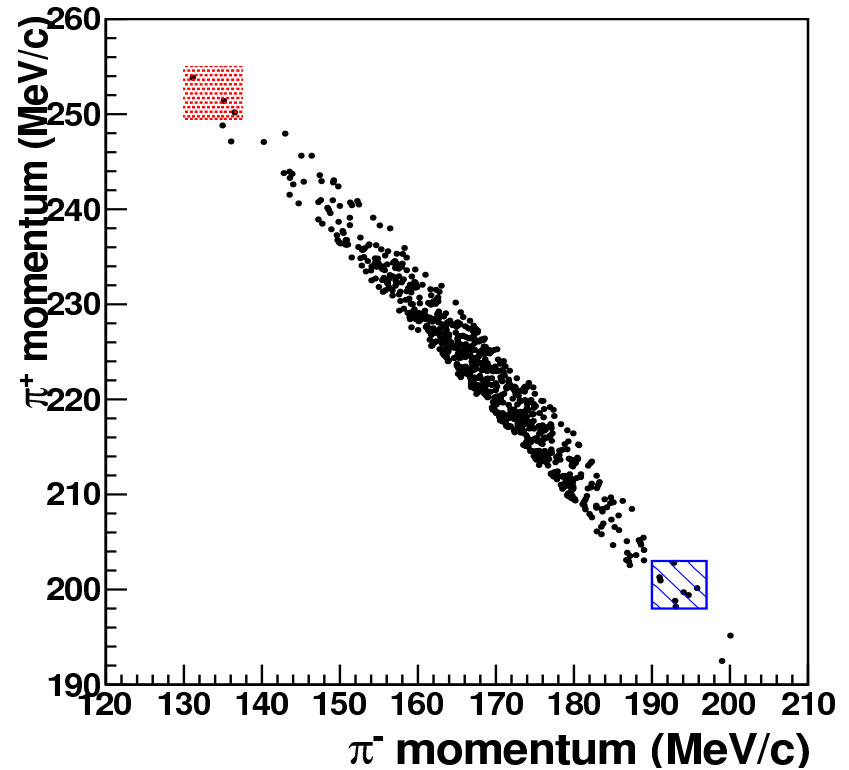
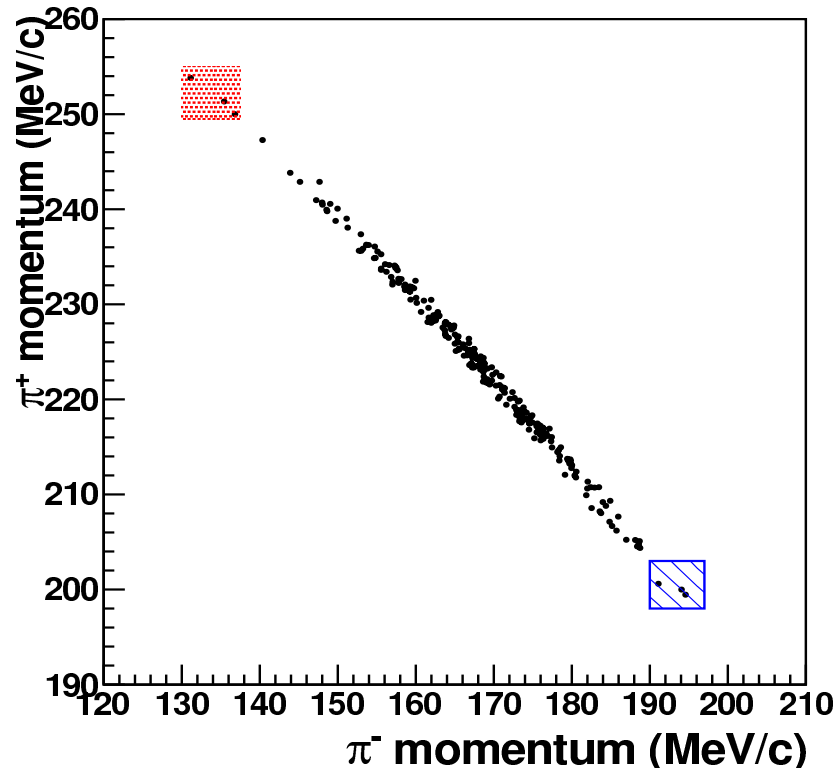
FINUDA's reconstructed ${}^6_{\Lambda}\text{H}$ candidate event in ${}^6\text{Li}$ target



Left: a π^\pm pair from ${}^6\text{Li}$ target crosses the spectrometer

Right: expanded view of a K^- track stopping in ${}^6\text{Li}$ target

p_{π^+} vs p_{π^-} in ${}^6\text{Li}$ for two $T(\pi^+) + T(\pi^-)$ cuts

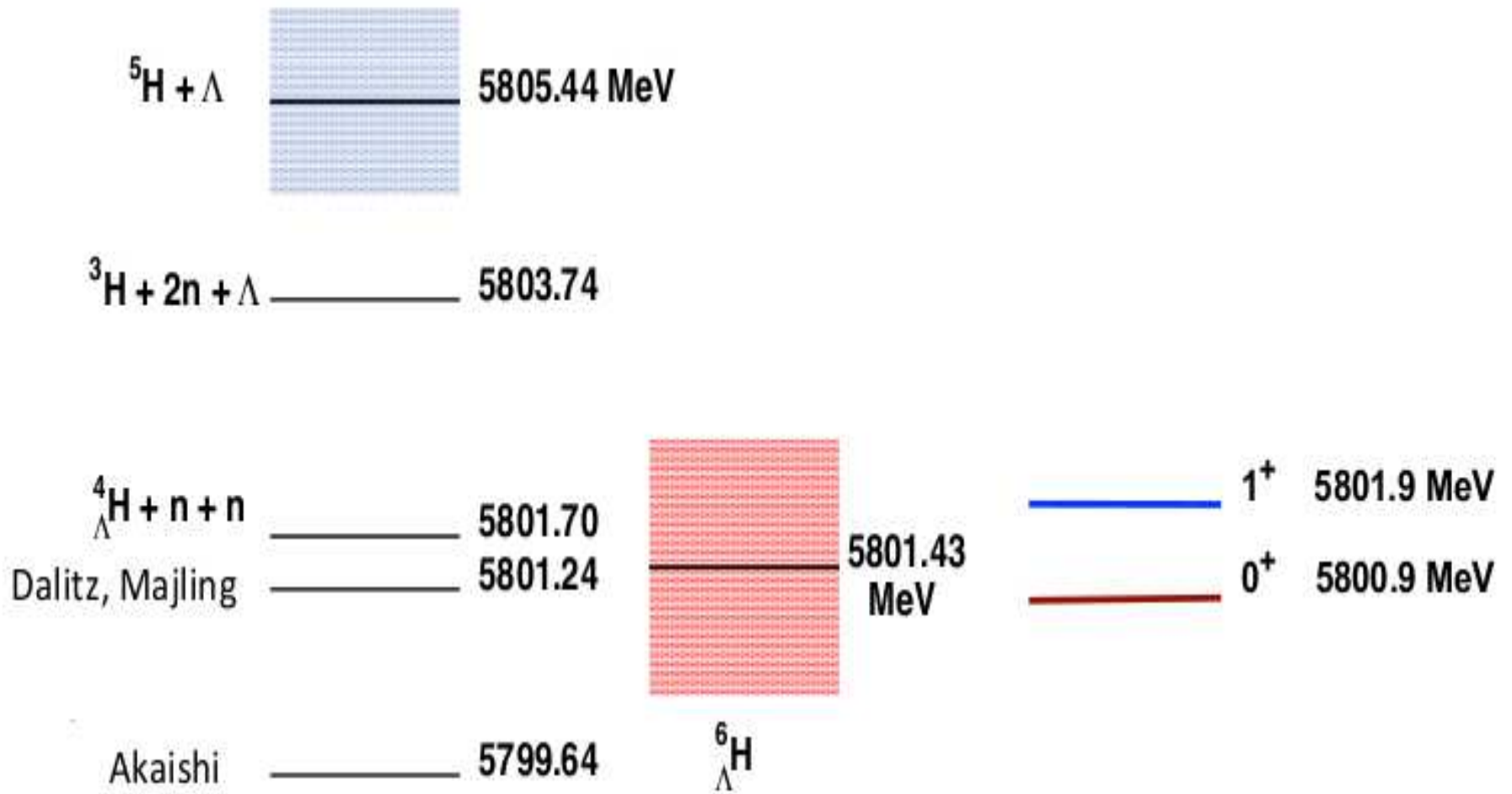


$T(\pi^+) + T(\pi^-) = 202\text{--}204$ MeV (l.h.s.) $200\text{--}206$ MeV (r.h.s.)

Shaded red rectangles: $p_{\pi^+} = 250\text{--}255$, $p_{\pi^-} = 130\text{--}137$ MeV/c

The 3 enclosed events are stable against $T(\pi^+) + T(\pi^-)$ cuts

FINUDA+Gal (2012) [PRL 108, 042501; NPA 881 269]



Note: $\Delta E(0^+_{\text{g.s.}} - 1^+) = 1.0 \pm 0.7 \text{ MeV}$ (2σ away from Akaishi)

${}^6_{\Lambda}\text{H}$ phenomenology

- Spin flip is forbidden in production at rest:



Here, $L_f = 0$, so only ${}^6_{\Lambda}\text{H}(1_{\text{exc.}}^+)$ is produced, followed by



- If so, $B_{\Lambda}({}^6_{\Lambda}\text{H}) = (4.5 \pm 1.2) \text{ MeV}$; Is $(1_{\text{exc.}}^+)$ particle stable?

- Shell-model estimate for Λ interaction with p-shell neutrons:

$$B_{\Lambda}({}^7_{\Lambda}\text{He}) - B_{\Lambda}({}^5_{\Lambda}\text{He}) = (5.36 \pm 0.09) - (3.12 \pm 0.02) = (2.24 \pm 0.09) \text{ MeV}$$

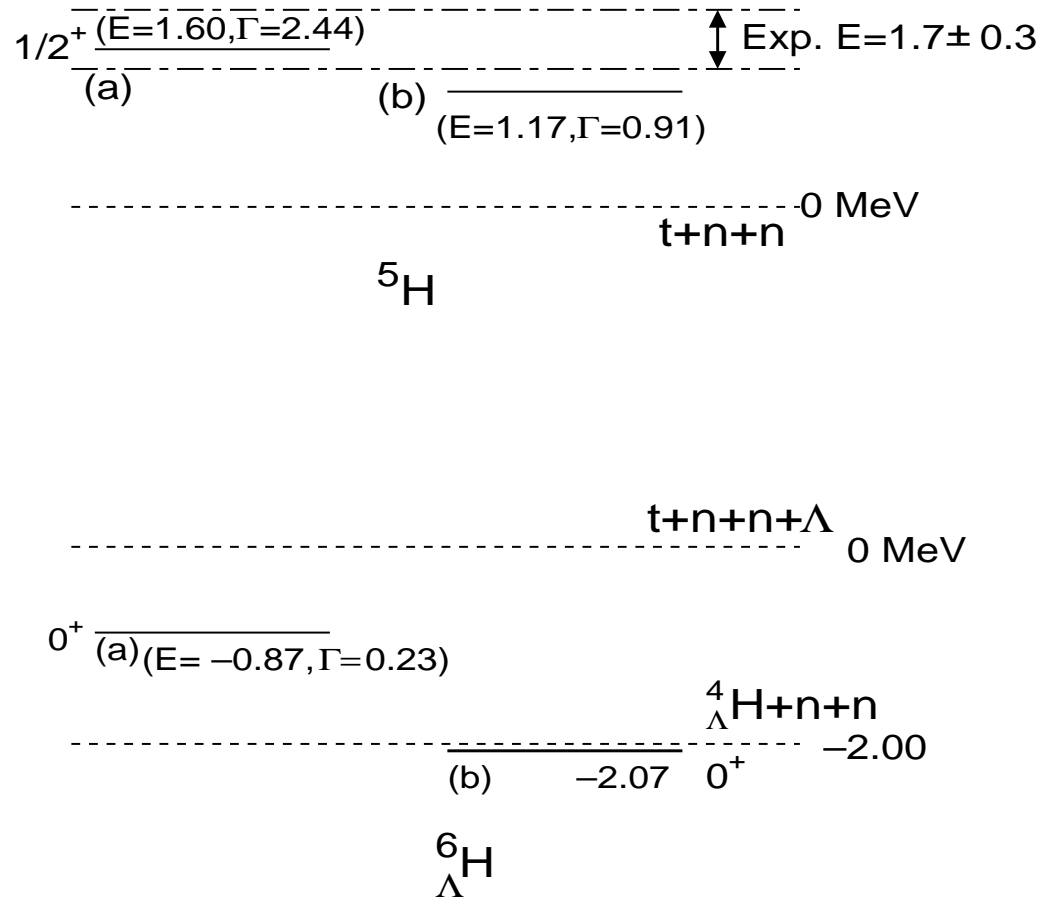
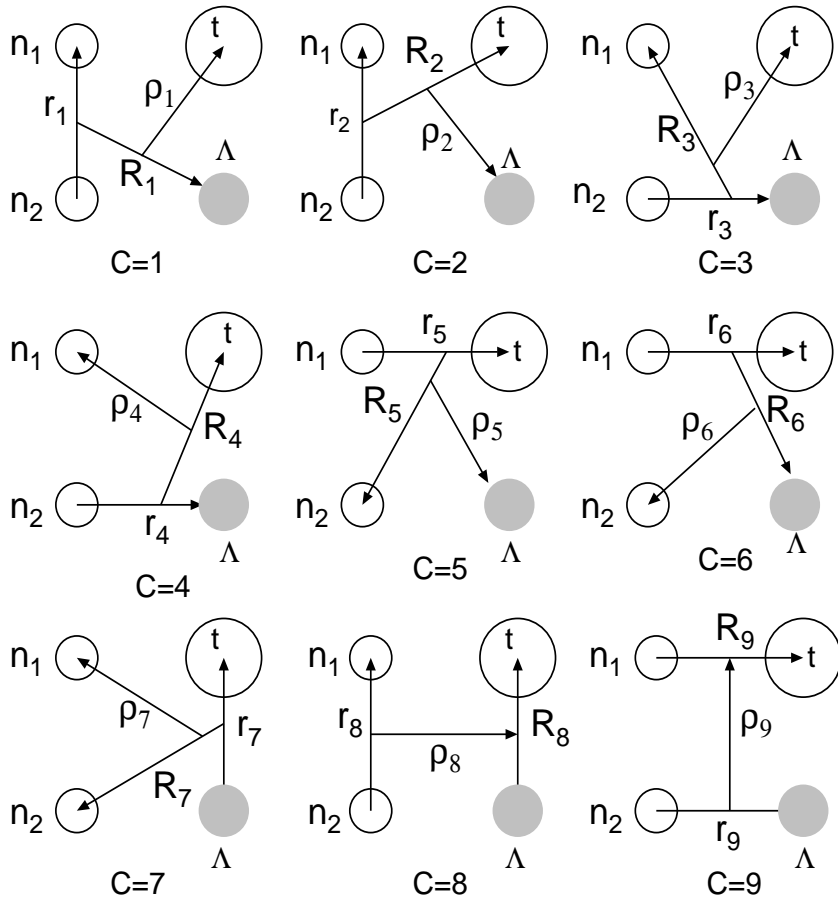
and additional $\Lambda N \leftrightarrow \Sigma N$: $\Delta V_{\Lambda N \leftrightarrow \Sigma N} = 0.15 \text{ MeV}$.

$$\text{Add to } B_{\Lambda}({}^4_{\Lambda}\text{H}) = 2.04 \pm 0.04 \text{ MeV} \Rightarrow B_{\Lambda}^{\text{SM}}({}^6_{\Lambda}\text{H}) = 4.43 \pm 0.10 \text{ MeV}.$$

Scale $\langle V_{\Lambda n} \rangle$ by $\times 0.8$ to fit halo n in ${}^6_{\Lambda}\text{He}$: $B_{\Lambda}({}^6_{\Lambda}\text{H}) \approx 3.93 \pm 0.08 \text{ MeV}$

$$\Rightarrow B_{2n}({}^6_{\Lambda}\text{H}) \approx 0.2 \pm 0.3 \text{ MeV, hardly bound w.r.t. } {}^4_{\Lambda}\text{H} + 2n.$$

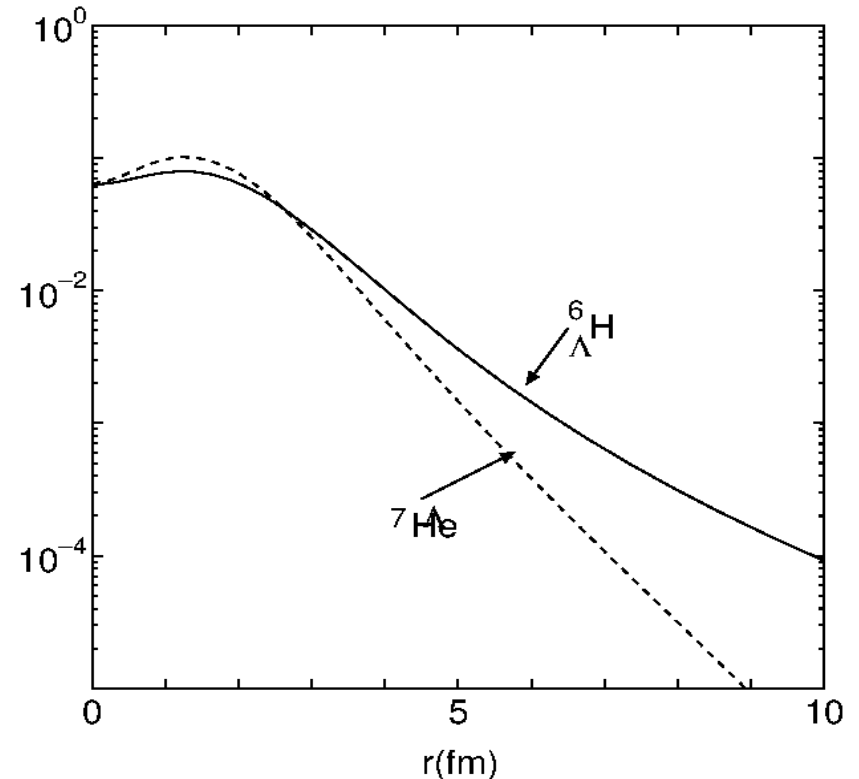
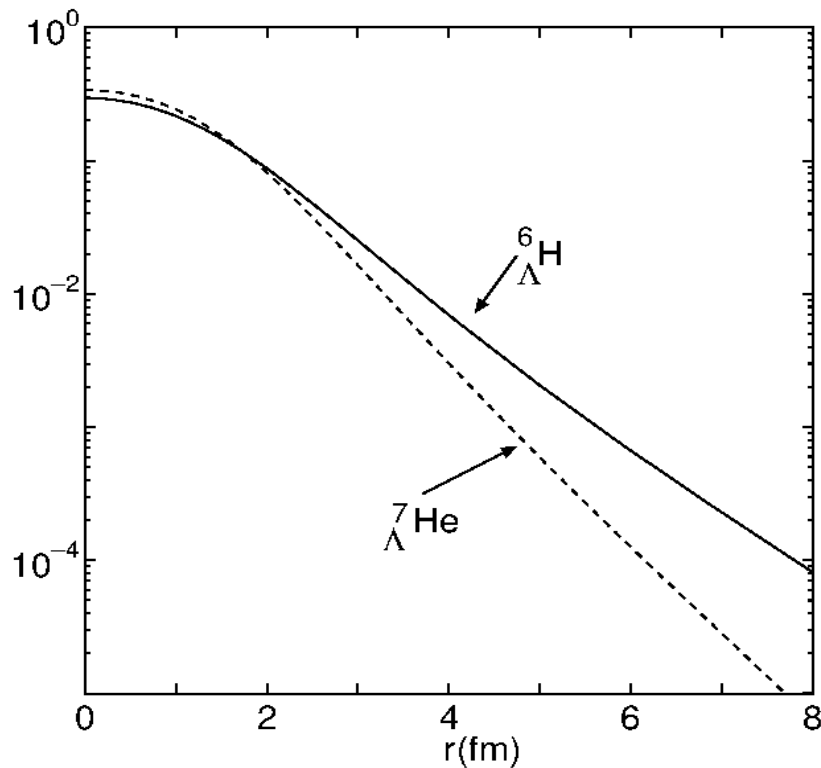
4-body calc. ${}^6_{\Lambda}\text{H}$: E. Hiyama et al. NPA 908 (2013) 29



Jacobi coordinates for $tnn\Lambda$
 missing $tnn\Lambda \leftrightarrow tnn\Sigma$

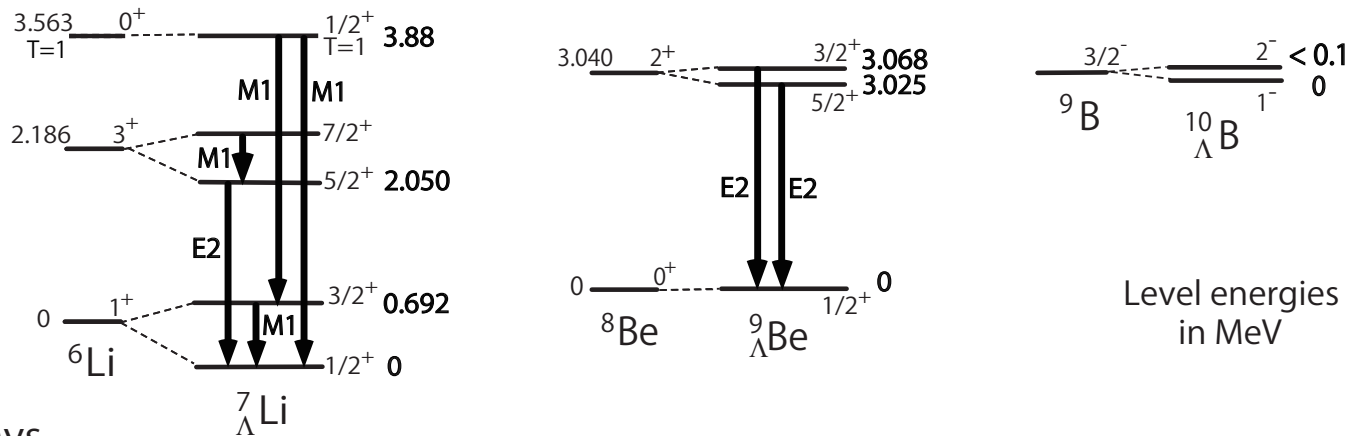
Calculated levels of ${}^5\text{H}$ & ${}^6_{\Lambda}\text{H}$
 ${}^6_{\Lambda}\text{H}$: (a) unbound (b) just bound

s -shell Λ (left) & p -shell neutrons (right) in ${}^6_{\Lambda}\text{H}$ and ${}^7_{\Lambda}\text{He}$

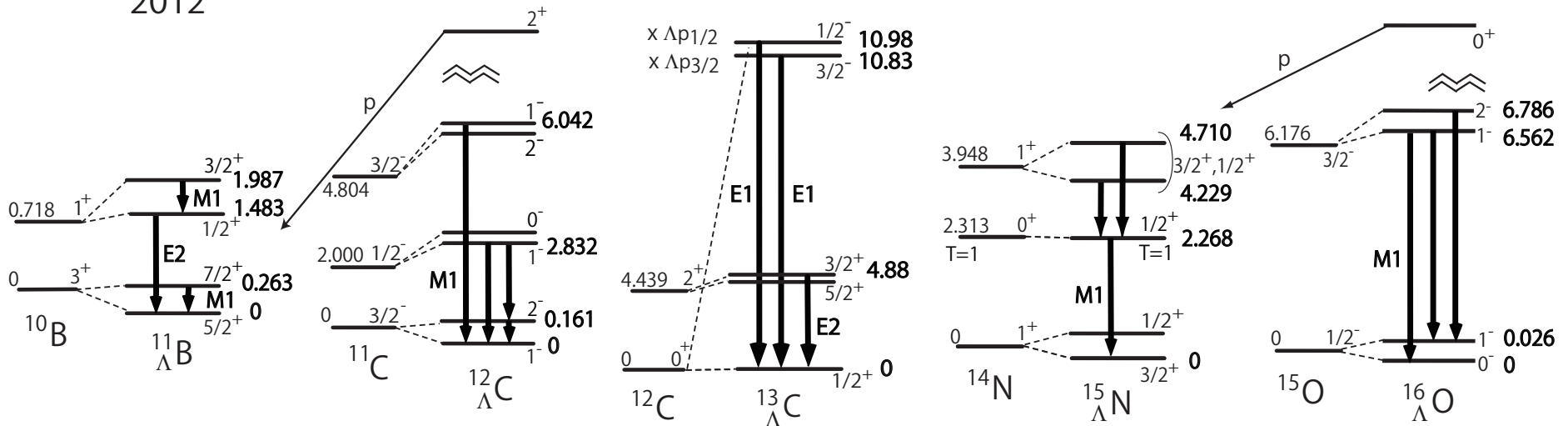


- Wavefunctions from Hiyama's 4-body calculations suggest that $\langle V_{\Lambda n} \rangle$ in ${}^6_{\Lambda}\text{H}$ is smaller than in ${}^7_{\Lambda}\text{He}$, consistent with $\approx 20\%$ decrease from ${}^6_{\Lambda}\text{He}$ phenomenology ($B_n = 0.17$ MeV)
- Gal-Millener's final estimate: $B_{2n}({}^6_{\Lambda}\text{H}) = 0.2 \pm 0.3$ MeV

Other neutron-rich hypernuclei



Hypernuclear γ rays
2012



Level schemes of Λ hypernuclei from γ -ray measurements

H. Tamura et al., Nucl. Phys. A 835 (2010) 3 [HYP09], updated at HYP12

Λ spin-orbit splitting: 150 keV in $^{13}_{\Lambda}\text{C}$ & related 43 keV in $^9_{\Lambda}\text{Be}$

p-shell Λ hypernuclei

$$V_{\Lambda N} = V_0(r) + V_\sigma(r) s_N \cdot s_\Lambda + V_{LS}(r) l_{N\Lambda} \cdot (s_\Lambda + s_N) + V_{ALS}(r) l_{N\Lambda} \cdot (s_\Lambda - s_N) + V_T(r) S_{12}$$

For $p_N s_Y$: $V_{\Lambda N} = \bar{V} + \Delta s_N \cdot s_\Lambda + S_\Lambda l_N \cdot s_\Lambda + S_N l_N \cdot s_N + T S_{12}$

R.H Dalitz, A. Gal, Ann. Phys. 116 (1978) 167

D.J. Millener, A. Gal, C.B. Dover, R.H. Dalitz, PRC 31 (1985) 499

$N\Lambda-N\Lambda$	\bar{V}	Δ	S_Λ	S_N	T	(MeV)
$A = 7 - 9$	(-1.32)	0.430	-0.015	-0.390	0.030	
$A = 11 - 16$	(-1.32)	0.330	-0.015	-0.350	0.024	
$N\Lambda-N\Sigma$	1.45	3.04	-0.085	-0.085	0.157	

D.J. Millener, Nucl. Phys. A 804 (2008) 84

Doublet spacings in p -shell hypernuclei (in keV)

D.J. Millener, NPA 881 (2012) 298

	J_u^π	J_l^π	$\Lambda\Sigma$	Δ	S_Λ	S_N	T	ΔE^{th}	ΔE^{exp}
${}^7_\Lambda\text{Li}$	$3/2^+$	$1/2^+$	72	628	-1	-4	-9	693	692
${}^7_\Lambda\text{Li}$	$7/2^+$	$5/2^+$	74	557	-32	-8	-71	494	471
${}^8_\Lambda\text{Li}$	2^-	1^-	151	396	-14	-16	-24	450	(442)
${}^9_\Lambda\text{Be}$	$3/2^+$	$5/2^+$	-8	-14	37	0	28	44	43
${}^{11}_\Lambda\text{B}$	$7/2^+$	$5/2^+$	56	339	-37	-10	-80	267	264
${}^{11}_\Lambda\text{B}$	$3/2^+$	$1/2^+$	61	424	-3	-44	-10	475	505
${}^{12}_\Lambda\text{C}$	2^-	1^-	61	175	-22	-13	-42	153	161
${}^{15}_\Lambda\text{N}$	$3/2_2^+$	$1/2_2^+$	65	451	-2	-16	-10	507	481
${}^{16}_\Lambda\text{O}$	1^-	0^-	-33	-123	-20	1	188	23	26
${}^{16}_\Lambda\text{O}$	2^-	1_2^-	92	207	-21	1	-41	248	224

$\Lambda - \Sigma$ coupling contributions normally are below 100 keV

YN interaction contributions to g.s. binding energies

	${}^7_{\Lambda}\text{Li}$	${}^8_{\Lambda}\text{Li}$	${}^9_{\Lambda}\text{Li}$	${}^{10}_{\Lambda}\text{B}$	${}^{11}_{\Lambda}\text{B}$	${}^{11}_{\Lambda}\text{Be}$	${}^{12}_{\Lambda}\text{B}$	${}^{13}_{\Lambda}\text{C}$	${}^{15}_{\Lambda}\text{N}$	${}^{16}_{\Lambda}\text{N}$
keV	$1/2^+$	1^-	$3/2^+$	1^-	$5/2^+$	$1/2^+$	1^-	$1/2^+$	$3/2^+$	1^-
Λ - Σ	78	160	183	35	66	99	103	28	59	62
Δ	419	288	350	125	203	2	108	-4	40	94
S_{Λ}	0	-6	-10	-13	-20	0	-14	0	12	6
S_N	94	192	434	386	652	540	704	841	630	349
T	-2	-9	-6	-15	-43	0	-29	-1	-69	-45
sum	589	625	952	518	858	641	869	864	726	412
Exp	5.58	6.80	8.50	8.89	10.24		11.37	11.69		13.76
MeV		6.84	8.29	9.11						
\bar{V}	-0.94	-1.02	-1.06	-1.05	-1.04		-1.05	-0.96		-0.93

$$B_{\Lambda}^{\text{exp}}(\text{g.s.}) = [B_{\Lambda}^{\text{exp}}({}^5_{\Lambda}\text{He}) = 3.12 \pm 0.02 \text{ MeV}] - (A - 5)\bar{V} + \text{'sum'}$$

Improve fit by adding a spin-independent ΛNN term,

see Millener-Gal-Dover-Dalitz, PRC **31** (1985) 499

Beyond-mean-field $\Delta B_{\Lambda}^{\text{g.s.}}$ shell-model contributions (in keV) to normal-parity g.s. of neutron-rich hypernuclei

Millener, NPA 881 (2012) 298; Gal & Millener, PLB 725 (2013) 445

target	n -rich	$\Lambda\Sigma$	$\Lambda\Sigma$	$\Delta B_{\Lambda}^{\text{g.s.}}$
AZ	${}_{\Lambda}^A(Z-2)$	diag.	total	total
${}^9\text{Be}$	${}_{\Lambda}^9\text{He}(\frac{1}{2}^+)$	210	253	879
${}^{10}\text{B}$	${}_{\Lambda}^{10}\text{Li}(1^-)$	202	275	1022
${}^{12}\text{C}$	${}_{\Lambda}^{12}\text{Be}(0^-)$	184	158	748
${}^{14}\text{N}$	${}_{\Lambda}^{14}\text{B}(1^-)$	189	255	785

- Production by ${}^AZ(K^-, \pi^+)_{\Lambda}^A(Z-2)$ or by ${}^AZ(\pi^-, K^+)_{\Lambda}^A(Z-2)$
- **Modest $\Lambda\Sigma$ coupling effects from neutron excess.**
- **Beyond-mean-field $\Delta B_{\Lambda}^{\text{g.s.}}$ is dominated by ΛN spin dependence, mostly by induced $s_N \cdot \ell_N$.**

$B_{\Lambda}^{\text{g.s.}}$ predictions (in MeV) for n-rich hypernuclei

A. Gal & D.J. Millener, PLB 725 (2013) 445

n -rich	normal	normal	normal	n -rich	n -rich
${}^A_{\Lambda}Z$	${}^A_{\Lambda}Z'$	$B_{\Lambda}^{\text{g.s.}}$	$\Delta B_{\Lambda}^{\text{g.s.}}$	$\Delta B_{\Lambda}^{\text{g.s.}}$	$B_{\Lambda}^{\text{g.s.}}$
${}^9_{\Lambda}\text{He}(\frac{1}{2}^+)$	${}^9_{\Lambda}\text{Li}/{}^9_{\Lambda}\text{B}$	8.44 ± 0.10	0.952	0.879	8.37 ± 0.10
${}^{10}_{\Lambda}\text{Li}(1^-)$	${}^{10}_{\Lambda}\text{Be}/{}^{10}_{\Lambda}\text{B}$	8.94 ± 0.11	0.518	1.022	9.44 ± 0.11
${}^{12}_{\Lambda}\text{Be}(0^-)$	${}^{12}_{\Lambda}\text{B}$	11.37 ± 0.06	0.869	0.748	11.25 ± 0.06
${}^{14}_{\Lambda}\text{B}(1^-)$	${}^{14}_{\Lambda}\text{C}$	12.17 ± 0.33	0.904	0.785	12.05 ± 0.33

- Small $B_{\Lambda}^{\text{g.s.}}$ modifications induced by $\Lambda\Sigma$ coupling.
- Small $\Lambda\Sigma$ coupling effects persist also in particle-stable neutron-rich Λ hypernuclei.

$\Lambda\Sigma$ matrix elements & contributions to $B_{\Lambda}^{\text{g.s.}}$ (in MeV) across the periodic table

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$N-Z$	${}^A_{\Lambda}Z$	$\bar{V}_{\Lambda\Sigma}$	$\Lambda\Sigma(\bar{V})$	$\Delta_{\Lambda\Sigma}$	$\Lambda\Sigma(\Delta)$	$\Delta B_{\Lambda}^{\text{g.s.}}(\Lambda\Sigma)$
4	${}^9_{\Lambda}\text{He}$	1.194	0.143	4.070	0.104	0.246
8	${}^{49}_{\Lambda}\text{Ca}$	0.175	0.010	0.946	0.014	0.024
22	${}^{209}_{\Lambda}\text{Pb}$	0.0788	0.052	0.132	0.001	0.053

- $\Lambda\Sigma$ from Halderson, following NSC97f.
- $\bar{V}_{\Lambda\Sigma}$ & $\Delta_{\Lambda\Sigma}$ decrease drastically as overlap between $0s$ hyperon and high- ℓ excess neutrons becomes poorer with A ($0f_{7/2}$ in ${}^{49}_{\Lambda}\text{Ca}$, $0h_{9/2}$ & $0i_{13/2}$ in ${}^{209}_{\Lambda}\text{Pb}$).
- **Conclusion:** $\Lambda\Sigma$ contributes less than 100 keV to binding of medium & heavy n-rich hypernuclei.

Conclusion

- Akaishi's substantial $\Lambda\Sigma$ coupling effects are not supported by FINUDA's observation of ${}^6_{\Lambda}\text{H}$ and by realistic shell-model calculations rooted in Λ hypernuclear phenomenology.
- Why didn't J-PARC Experiment E10, ${}^6\text{Li}(\pi^-, K^+){}^6_{\Lambda}\text{H}$, see any strength near threshold? **Do ${}^9\text{Be}(\pi^-, K^+){}^9_{\Lambda}\text{He}$.**
- Realistic shell-model calculations done for binding energies of neutron-rich Λ hypernuclei that could be formed in (π^-, K^+) or in (K^-, π^+) reactions on stable p -shell nuclear targets do not support the large effects conjectured by Akaishi to arise from $\Lambda\Sigma$ coupling. **$\Lambda\Sigma$ contributions to binding energies of heavier neutron-rich hypernuclei are less than 100 keV.**