Proton rms-radius R: recent determinations from (e,e)

Ingo Sick

Disturbing

scatter of results
values between 0.84 and 0.92 fm, with error bars of typically 0.01 fm

Main problem: interpretation of data; smaller problem: difference between data

Goal of talk

go to bottom of discrepancies, understand causes for differences
analysis necessarily critical of published results
    only *one* true value of radius (.... obviously mine!)
study requires very careful look at published results
    which takes time .....

What to expect

not 'new' result of my own, but review of other results
in the end: give average of results I have no qualms with

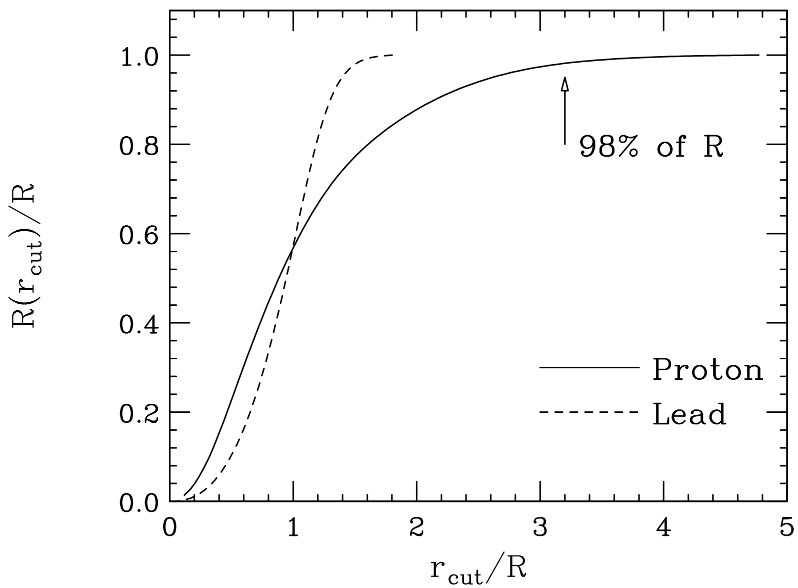
State right away: results do not fix discrepancy with μH

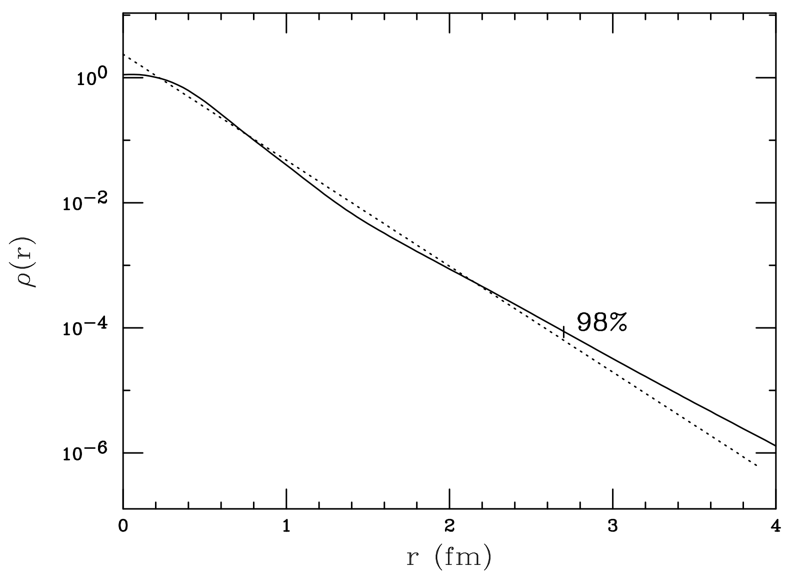
4 peculiarities of proton

consequences of fact that $G(q) \sim$ Dipole, density $\rho(r) \sim$ exponential

1. Large r and tail of $\rho(r)$ do matter more than for $A > 2$

illustration: study $[\int_0^{r_{cut}} \rho(r)\ r^4 dr / \int_0^{\infty} \rho(r)\ r^4 dr]^{1/2}$ as function of cutoff r_{cut}





to get 98% of rms-radius R must integrate out to $r_{cut} \sim 3.2 \cdot R \sim 3fm$

where $\rho(r)/\rho(0) \sim 10^{-4}$

$\Longrightarrow$ R sensitive to very large r where $\rho(r)$ poorly determined

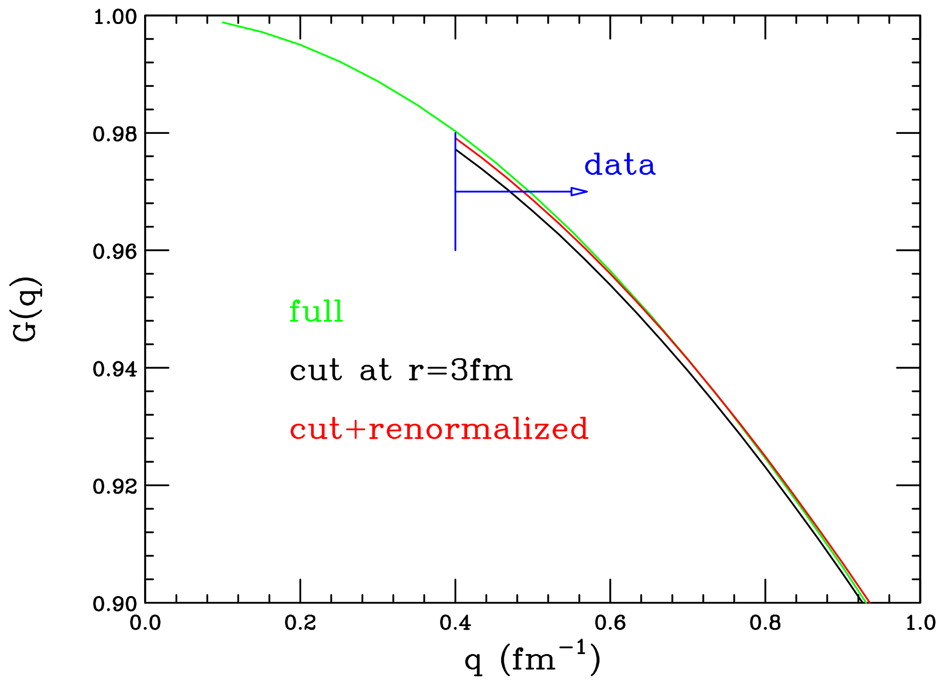
$\Longrightarrow$ large r affect $G(q)$ at very low q, below q_{min} $\Longrightarrow$ affects extrapolation to $q = 0$

$\Longrightarrow$ requires great care in extrapolation to $q = 0$

Large-r contribution not measurable in practice

model study: start from $\rho(r)$ = exponential density
determine $G(q)$ from Fourier transform

1. full $\rho(r)$
2. cut $\rho(r)$ for $r > 3fm$
3. renormalize 2. (as done in fits)



difference red-green $< 0.12\%$
not measurable

→ serious question: uncertainties of order 2% at all reachable??

.... not without doing something sensible for ρ at large r

2. Extremely large higher moments due to $\rho(r) \sim$ exponential

	$\langle r^4\rangle/\langle r^2\rangle^2$	$\langle r^6\rangle/\langle r^2\rangle^3$	
naive estimate	$\sim$1.		
exponential density	2.49	8.82	
experimental value	4.32	64.2	fit of $q \leq 5 fm^{-1}$ data Bernauer 2010

large-r contribution even worse than for exponential density

Consequence: at $q \sim 0.9 fm^{-1}$ of maximal sensitivity to $\langle r^2\rangle$:

contribution of $\langle r^4\rangle \sim$ 15% of finite-size effect, even $\langle r^6\rangle$ contributes 4%

even at q^2=0.6, where finite size effect only 0.077, $\langle r^4\rangle$ contributes 10%

$\rightarrow$ serious interference of higher moments

Wrong $\langle r^4\rangle$ or wrong $\langle r^6\rangle$ $\rightarrow$ wrong R

= short version why some determinations of R, discussed below, are wrong

Determination of higher moments: difficult

understood already in 2003, Phys. Lett. B 576 (2003) 62

For such a density (form factor) the higher moments are increasing with order, *i.e.* $\langle r^4 \rangle = 2.5\ R^4$, $\langle r^6 \rangle$ =11.6 R^6 etc, hence giving a large contribution to $G(q)$.

The consequence: there is no q-region where the R^2 term dominates the finite size effect to >98% *and* the finite size effect is sufficiently big compared to experimental errors to allow a, say, 2% determination of the rms-radius . There is also no region of q where the r^4-moment can be determined accurately without getting into difficulty with the r^6-term.

etc

Consequence

in fit of low-q data uncertainty of the *highest* moments (N=4,6) large
these moments affect extracted $\langle r^2 \rangle$

Way out

fit data to highest q's with polynomial of *large* N
above uncertainty affects only the moments $\langle r^{2N} \rangle$ with the largest N
but produces significant $\langle r^2 \rangle$, $\langle r^4 \rangle$, ...

done *e.g.* by Bernauer *et al.* (2010), finds $\langle r^4 \rangle$ quoted above

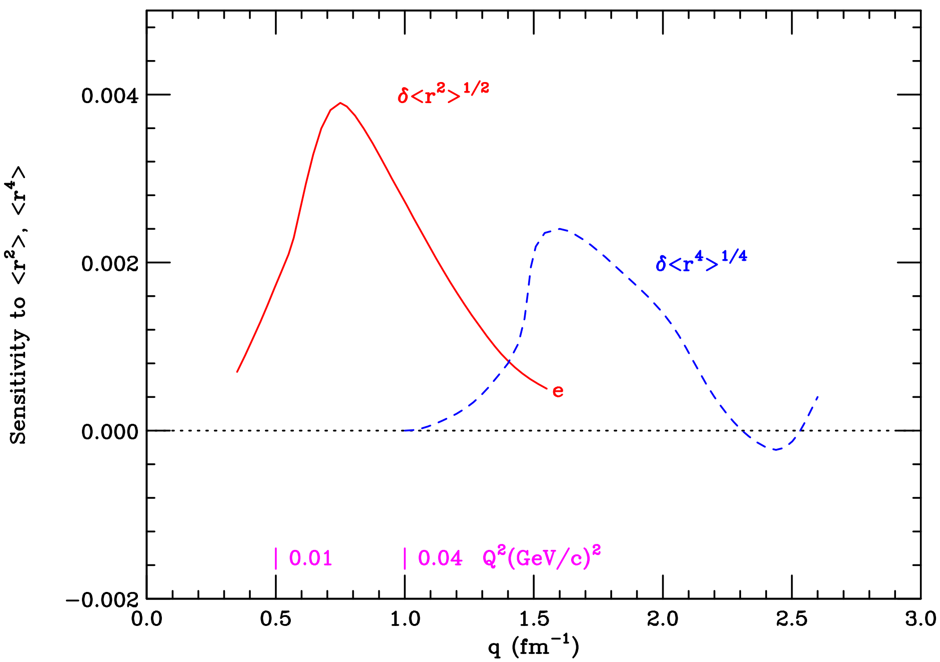
could use this $\langle r^4 \rangle$ as input in low-q fit with polynomial
.... if absolutely want to do polynomial fit

BUT: polynomial fit = bad idea, see below

3. Traditional for p and d: parameterize $G(q)$, not $\rho(r)$
creates new problems

data sensitive to R at $q > q_{min}$, want slope of $G(q = 0)$

Sensitivity to R: explored via notch-test



maximum sensitivity to R at $q \sim 0.8 fm^{-1}$ where $G(q) \sim 0.8$
note: corresponds to 0.01 - 0.04 $(GeV/c)^2$

extrapolation of $1 - G(q) \; < \; 0.2$ model-dependent

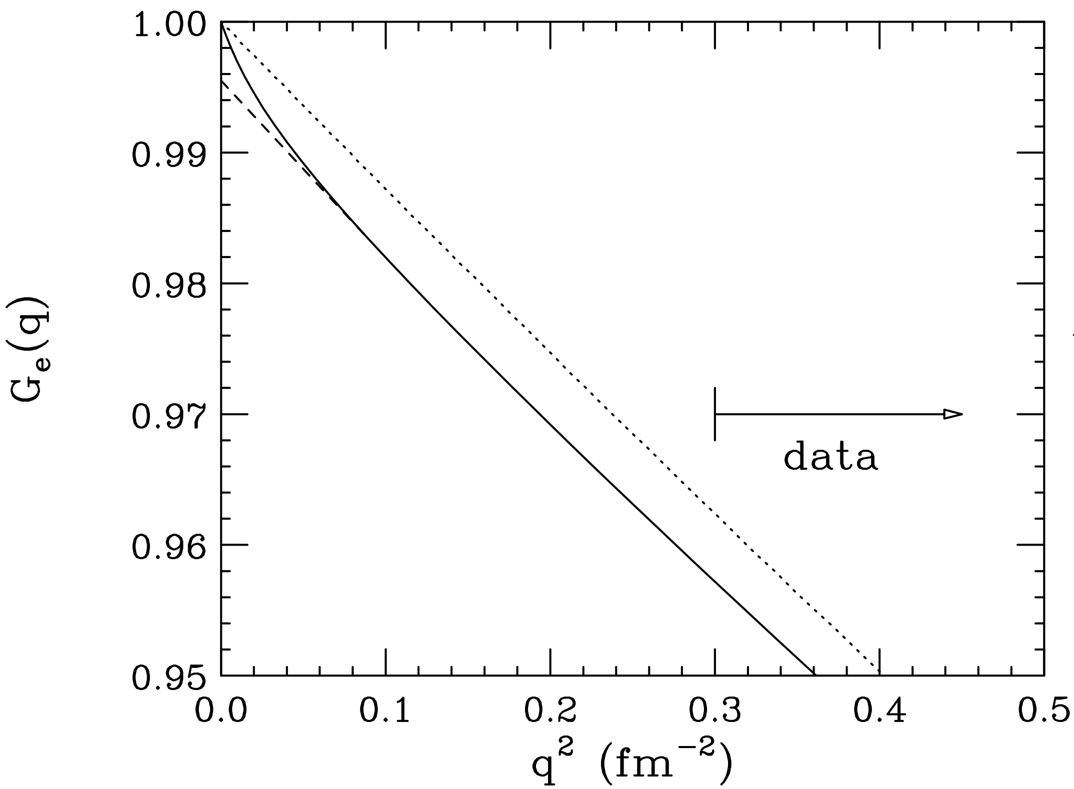
difficulty enhanced by floating normalization, *i.e.* floating "1"

4. Parameterization of $G(q)$ without considering $\rho(r)$ often generates nonsense

Pade fit of Bernauer data, $G(q) = (1 + a_1 q^2)/(1 + b_1 q^2 + b_2 q^4 + b_3 q^6)$

$q < 2fm^{-1}$, covers full region sensitive to R
excellent χ^2 (as low as Spline fit)

get $R \sim 1.48fm$ (Phys.Rev.C 89(2014)012201)



——— Pade-fit
.... Fit with 'normal' rms-radius
Note: data are floating

Why discuss $0.84fm$ vs. $0.88fm$ if $1.48fm$ fits perfectly low-q data?

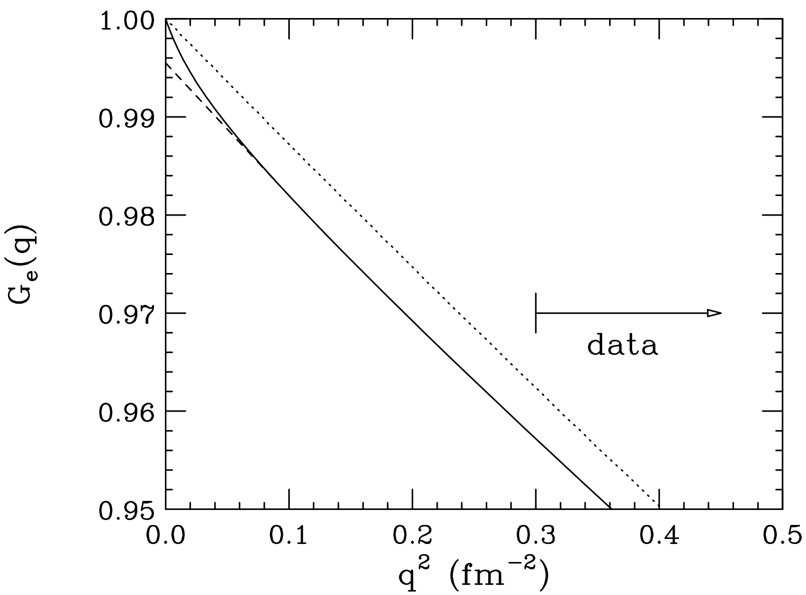
This is not a rhetorical question! Cannot be answered by ignoring it.

Understanding of $R = 1.48 fm$

split fit into two contributions $G = G_1 + G_2$:

G_1 = Pade for $q^2 > 0.06$ plus dashed line for $q^2 < 0.06$

G_2 = Pade – G_1



G_1 has 'normal' q=0 slope, norm of 0.995

$G_2 \sim e^{-q^2/(0.02 fm^2)}$
corresponds to $\rho \sim e^{-r^2/(200 fm^2)}$

G_2 leads to large rms-radius despite small norm of ~ 0.005

Choice of parameterization of $G(q)$ implies choice of $\rho(r)$
harmless-looking $G(q)$ (*e.g.* Pade) can correspond to outrageous $\rho(r)$

For sensible R *must* study behavior of $\rho(r)$ at large r
to avoid nonsense not visible in $G(q)$

Together with point 1 : use physics to constrain $\rho(r \gg) \rightarrow$ different talk
concentrate today on fits *without* large-r constraint

10 general considerations on recent fits of (e,e) data

discuss occasionally with recent examples

points 1-10 may seem trivial, but are all too often ignored

1. $G(q)$ as polynomial in q^2: $1 - q^2\langle r^2\rangle/6 + q^4\langle r^4\rangle/120 - ...$

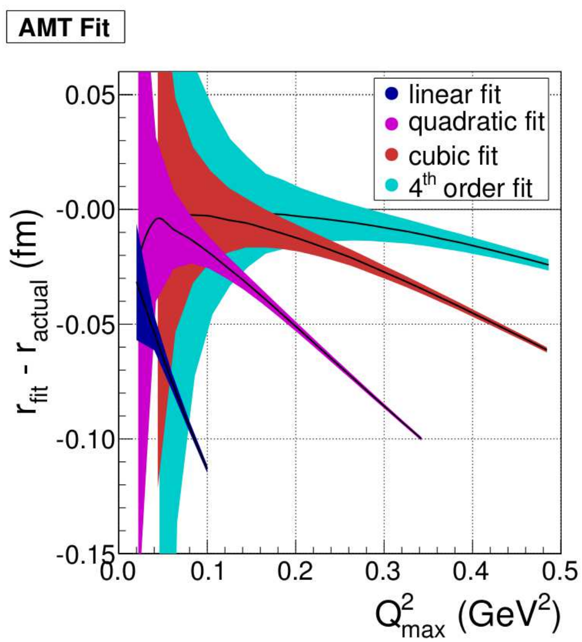
used by many authors in past

shown already in 2003 that *not* suitable (Phys. Lett. B 576 (2003) 62)

in 2014 quantitatively studied by Kraus *et al.*

parameterized $G(q) \rightarrow$ pseudo data $\pm 0.4\% \rightarrow$ power series fit $\rightarrow R_{fit}$

always gives low R_{fit}, and R_{fit} depends strongly on q_{max}



e.g. for $Q^2 = 0.03$ (typical) and linear fit

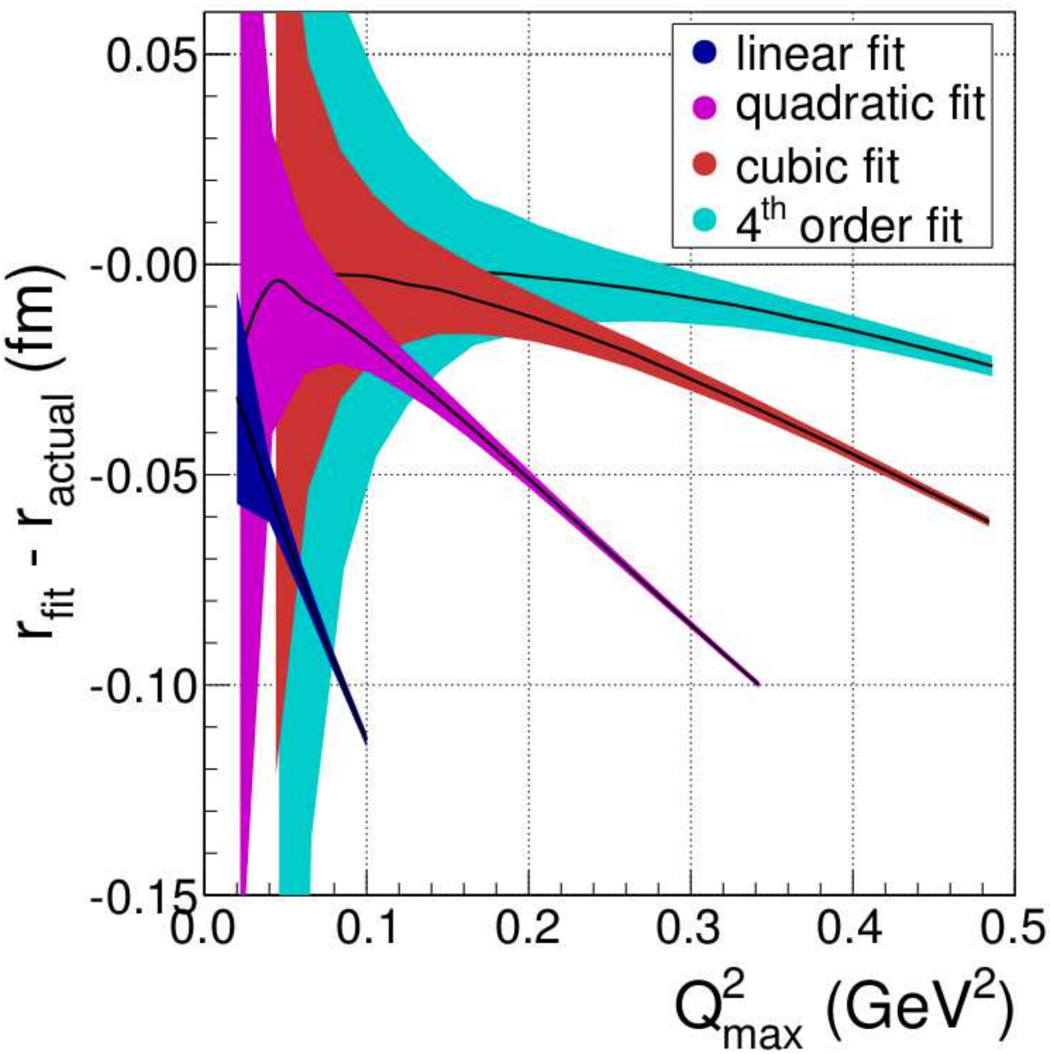
defect according to figure $= 0.04 fm$

as large as discrepancy (e,e)...μX

must be dumb to use low-order polynomial

..

AMT Fit



linear fit
quadratic fit
cubic fit
4th order fit
0.05
-0.00
-0.05
-0.10
-0.15
0.0
0.1
0.2
0.3
0.4
0.5
r_fit - r_actual (fm)
Q²_max (GeV²)

Understanding of low R's

discuss for "very low-q" linear fit in q^2

corresponds to assumption $\langle r^4 \rangle = 0$

how can produce $\langle r^2 \rangle \neq 0$ but $\langle r^4 \rangle = 0$?

What would a physicist think?

Understanding of low R's

discuss for "very low-q" linear fit in q^2

corresponds to $\langle r^4 \rangle = 0$

how can produce $\langle r^2 \rangle \neq 0$ with $\langle r^4 \rangle = 0$?

What would a physicist think?

would try to think how corresponding $\rho(r)$ would look like:
positive inside, negative at very large r
then negative tail can compensate positive part in $\langle r^4 \rangle$ to yield $\langle r^4 \rangle = 0$
given r^6 weight in $\langle r^4 \rangle$-integral

But: negative tail also impacts $\langle r^2 \rangle$
will yield too small value for R

remember: $\langle r^4 \rangle$ contributes ~15% of finite-size effect
at q of maximal sensitivity to R

= physical explanation of above results of Kraus *et al.*

= general argument why truncated polynomial (also higher order) generates problems

Illustration of problems with low-order polynomial fits

recent fit of Higinbotham *et al.*

Mainz80+Saskatoon data

$q^2_{max} = 0.8 fm^{-2}$ for R with smallest δR

$G(q) = a_0(1 + a_1 q^2)$

Find $R = 0.844 \pm 0.009 fm$

conclude that is compatible with μX result

Wrong, as a trivial back-of-the-envelope estimate shows! For q^2=0.8

$$\left.\begin{array}{ll} q^2R^2/6 & = 0.094 \\ q^4\langle r^4\rangle/120 & = 0.0138 \end{array}\right\}$$ q^4 contribution is 14.7% of q^2 contribution

→ q^2 contribution wrong by ∼14.7%

→ R^2 is wrong by ∼14.7%

... and this sort of analysis is claimed to provide insight on radius-puzzle!

Another illustration: recent fit by Griffioen, Carlson, Maddox

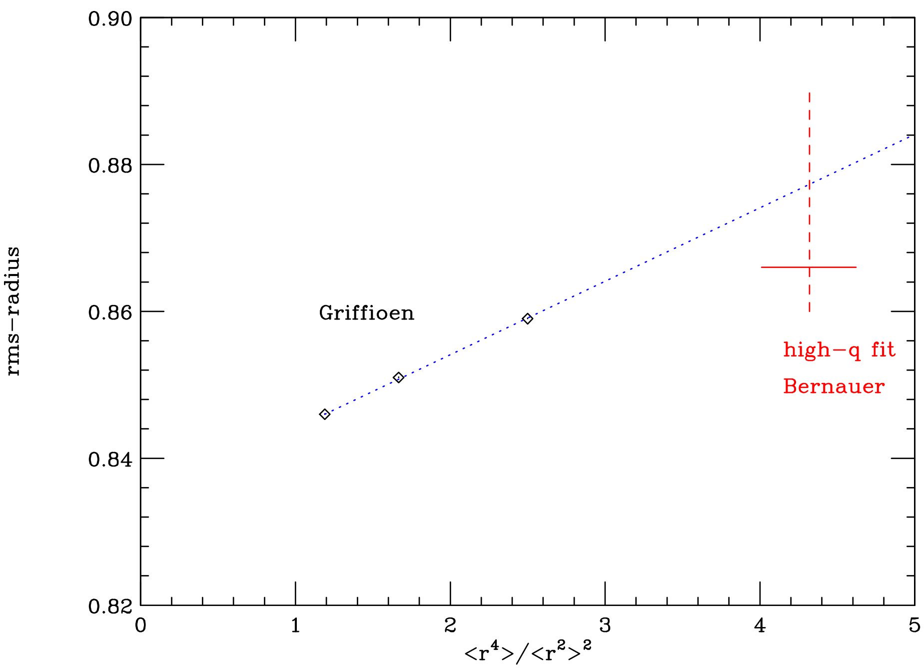
use $G(q) = 1 - q^2\langle r^2\rangle/6 + q^4\langle r^4\rangle/120$

$Q^2 \leq 0.02 GeV^2$, "Bernauer" data, $R = 0.850 \pm 0.019 fm$ compatible with μX?

find contribution q^4-term	0.0018	} $\Delta \sim 15\%$ error in $\langle r^2\rangle$. R=0.850 simply wrong!
experimental $\langle r^4\rangle$ yields	0.011	

Another demonstration of effect of $\langle r^4 \rangle$

$\langle r^2 \rangle$ *vs* $\langle r^4 \rangle$ for different densities
$\delta(r-c)$, exponential, gaussian
Griffioen *et al.*



R linear function of $\langle r^4 \rangle / \langle r^2 \rangle^2$

extrapolate to true $\langle r^4 \rangle$ (Bernauer high-q fit)

get $R \sim 0.88 fm$

2. Poles of parameterized $G(q)$ cause problems

some parameterization have poles at $q > q_{max}$

power series

inverse polynomials

some Pade

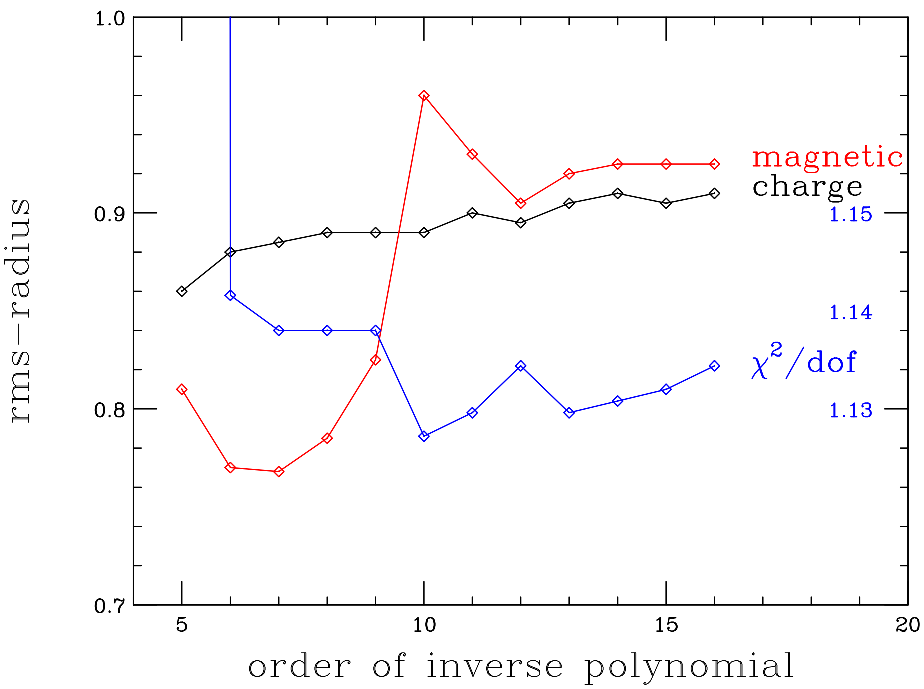
.....

poles → oscillations in $\rho(r)$ out to extremely large r

consequences

large-r contributions can have adverse effects on R

cannot judge if large-r behavior of $\rho(r)$ sensible



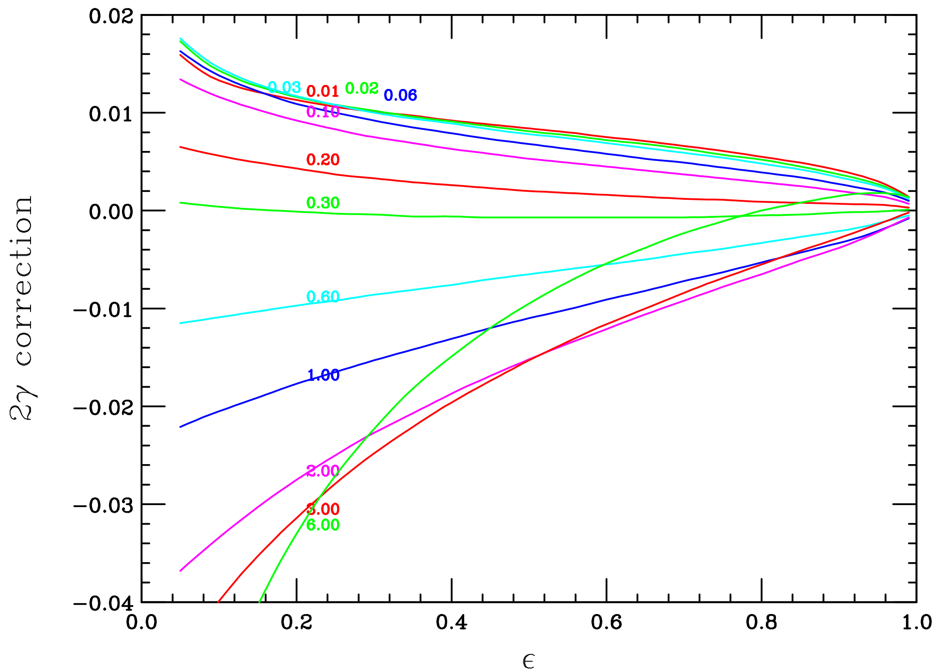
example: IP fit Bernauer

jump at N=10 due to close pole

note: choice of N → arbitrariness of R

3. Two-photon effects

PWIA relation $\sigma \leftrightarrow G(q)$ complicated by 2-γ exchange TPE
at low q mainly Coulomb distortion, well under control



At q of maximum sensitivity to R $1 - G(q) \sim 0.2$, so TPE$\sim$0.01 do matter!

Inappropriate TPE:

no corrections
corrections for point nucleus (McKinley-Feshbach)
phenomenological corrections (not determined at low q)

For valid result on R must use valid TPE

4. Good χ^2

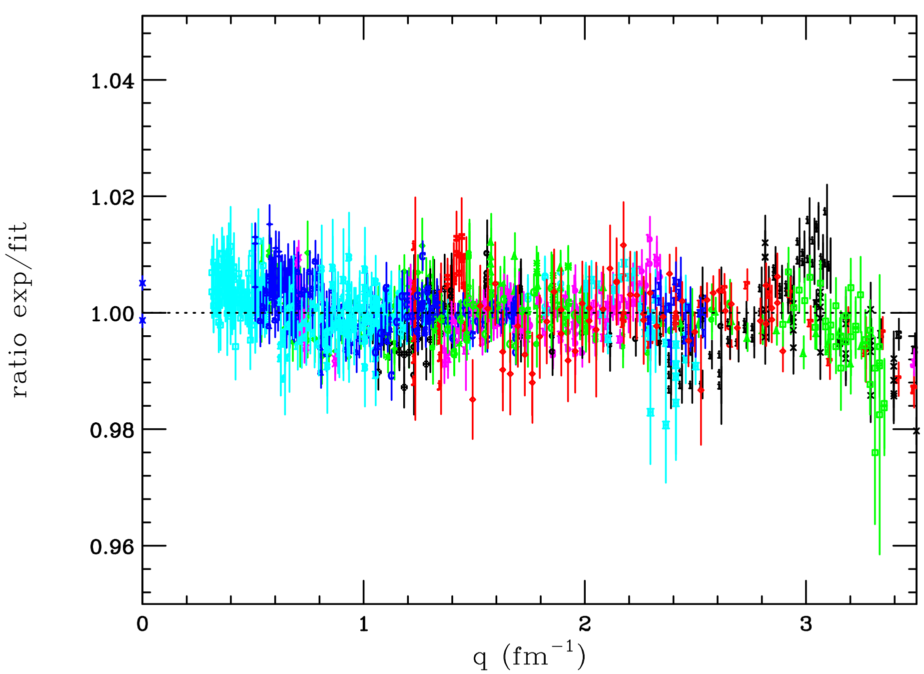
R valid only if data is fit with good χ^2

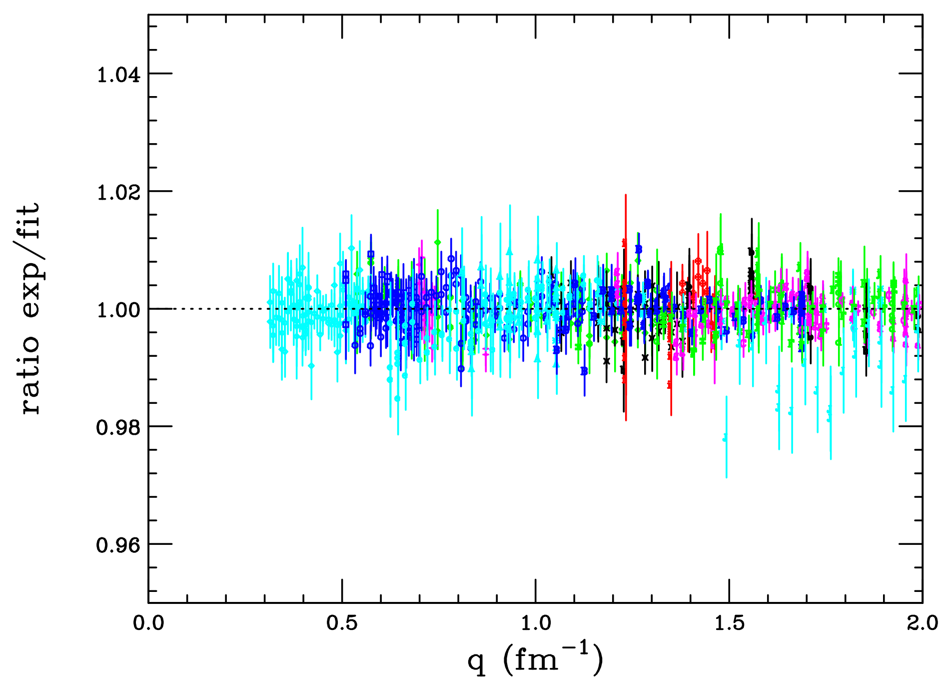
Remember: one standard deviation σ corresponds to $\Delta\chi^2 = 1$

many analyses take cavalier-attitude about χ^2
accept χ^2 of (say) 1.5 *per degree of freedom*
while good fits give 1.1

With (typically) 500 data points

difference 1.1 $\leftrightarrow$ 1.5 corresponds to $\Delta\chi^2$ of 200
this difference corresponds to 14σ!
which is not acceptable when discussing a 5σ difference (e,e)$\leftrightarrow$ μH





Important distinction: absolute value of χ^2 is not the main issue

depends on optimism of experimentalist assigning $\delta\sigma$
depends on eventual rescaling of $\delta\sigma$

Really relevant: comparison of fits to *same* $\sigma \pm \delta\sigma$

if fit "A" gives significantly larger χ^2 than fit "B"
then fit "A" has *systematic* differences to data
then fit "A" must be discarded

Many published fits have $\chi^2 \gg \chi^2_{min}$, hence are irrelevant

Illustration

recent fit of Higinbotham *et al.*
use dipole form factor
fit Mainz80+Saskatoon+Stanford+JLab data

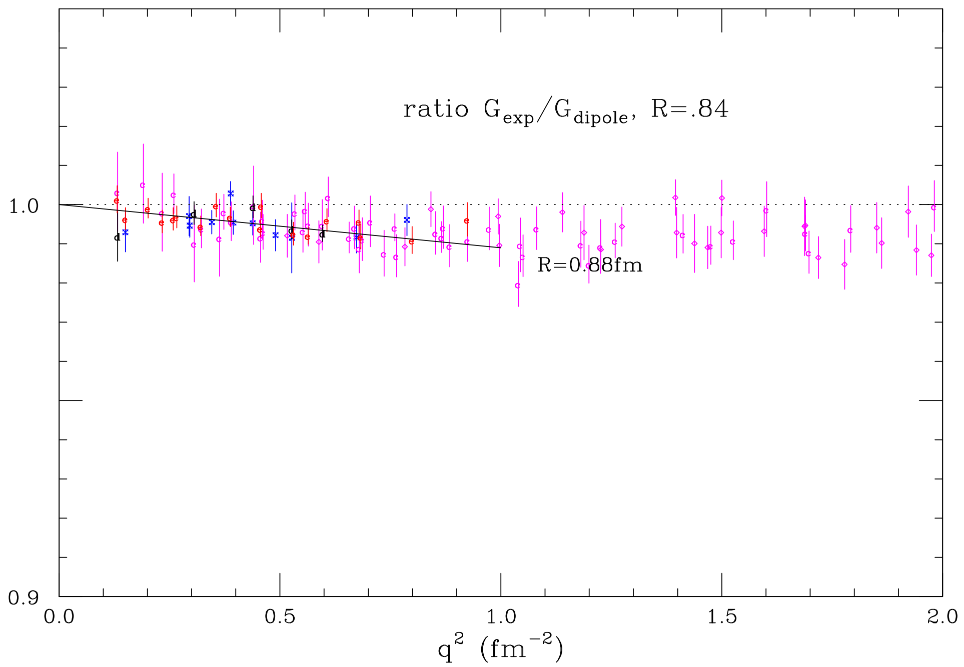
fit has reduced χ^2 of 1.25
find $R = 0.849 \pm 0.006 fm$

conclude that R is compatible with μX

But χ^2 is much too large

take one of my fits of *world* data (603 data points)
find reduced χ^2 of 0.96, *not* 1.25

Consequence: systematic deviation of Higibotham dipole fit from cross section data

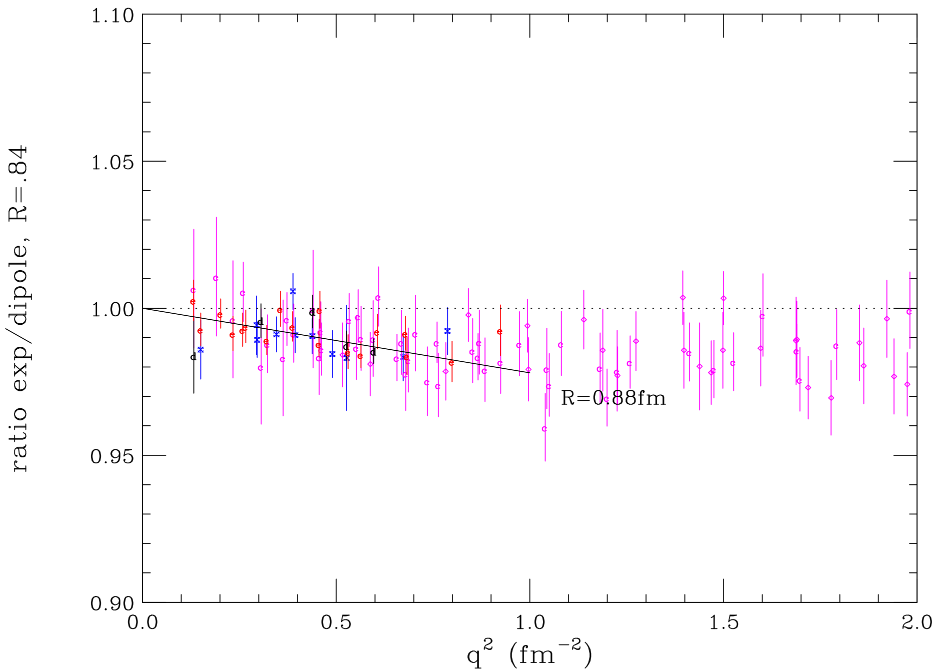


solid line shows change of low-q slope to $R = 0.88 fm$
fits data!

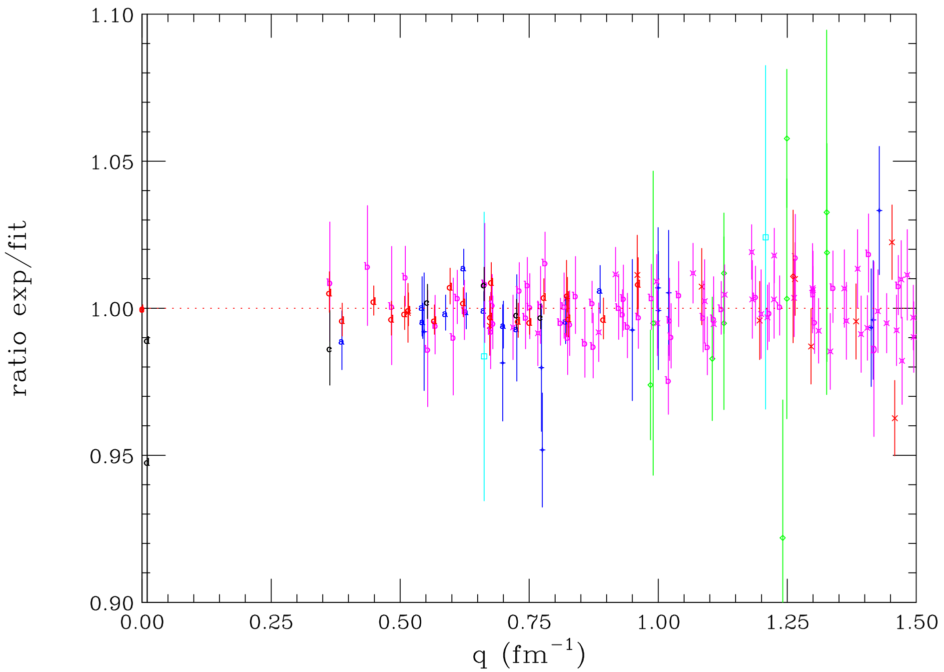
the dipole "fit" (dotted line) is simply wrong

A direct comparison of cross section ratios of "fit" and fit

Higiginbotham dipole



MD



Another illustration: CF fit of Griffioen *et al.*

fit Bernauer data
$q_{max} = 5 fm^{-1}$

get reduced χ^2 of 1.61 (+pole ...)
find $R = 0.8389 \pm 0.0004 fm$

conclude that agrees with μX

But Bernauer data can be fit with reduced χ^2 of 1.14
shown years ago

For 1400 data points difference 1.61 .. 1.14 is $\Delta\chi^2 = 660$!

Who in his right mind would call that a "fit"?

5. Correct treatment of data

some data sets are floating
i.e. Bernauer data have 31 norm factors for 34 data sets
this property must be respected in the fits
even if this *is* a nuisance
ignoring this (out of laziness) invalidates the results

6. **$G_e(q)$ and $G_m(q)$ are correlated**

    the (e,e) cross sections depend on G_e *and* G_m
    both have to be determined from the data
    use of convenient recipe for G_m: G_e not reliable, must be discarded

7. **Use of relevant data**

    R is determined mainly by cross sections at low q
    omission of σ-data yields R *not* coming from (e,e)
    then R determined by other input
        cannot be claimed to come from (e,e)

    statement seems trivial, but relevant in practice (see VDM-discussion)

8. **Choice of q_{max}**

    fit of (e,e) involves choice of q_{max} of data employed
        choice a priori arbitrary
        must explore consequences
    radii depend strongly on q_{max} (see below for more detail)
        trivial example already shown in figure by Kraus *et al.*

    R without exploration of q_{max}-dependence not valid

9. Model-dependence due to choice of fit-function

some authors use 1- parameter fits at low q
power of q^2, linear in z, single dipole, ...
then $\langle r^4\rangle/\langle r^2\rangle$ is fixed by fit-function, not data
$\langle r^4\rangle \neq$ true value known from fit of data over whole q-range

then $\langle r^2\rangle$ must compensate for wrong $\langle r^4\rangle \rightarrow$ wrong value of R
also gives larger χ^2

Illustration: low-q fits of Horbatsch+Hessels

fit Bernauer data, $q_{max} \sim 1.6 fm^{-1}$
use 1-parameter dipole resp. 1-parameter linear function in z
find $R = 0.842(2)$ resp. $0.888(1) fm$

fit-function fixes $\langle r^4\rangle$ to $1.244 fm^4$ resp. $2.15 fm^4$. True value $2.58 fm^4$ (fit to all data)

explains both low and discrepant radii

is reason why $\chi^2/dof = 1.11$ instead of 1.03
for 761 points $\Delta\chi^2 = 61!$ ($\sim 8\sigma's$)

'Fits' 8σ's from minimum are irrelevant when discussing 5σ difference (e,e)$\leftrightarrow \mu X$!

10. Use of polarization data

G_e and G_m are strongly coupled
best separation needed for accurate G_e
must include polarization data measuring G_e/G_m

in same vein:
use *world* data, not only biased selection
world without Bernauer: 600 data points
for $q < 1 fm^{-1}$ ~90 points
Bernauer: 1400 points
use of *all* data improves in particular L/T separation

Summary of recent fits to (e,e) data: see table next page

lists the various shortcomings
the most serious ones:
power series in q^2
poor χ^2
$\langle r^4 \rangle / \langle r^2 \rangle$ fixed by parameterization
q_{max} dependence ignored

Summary of recent fits

	poles/singularities	polynomial in q^2	incorrect/phen./no 2γ	too large χ^2	data mistreated	$G_M(q)$ assumed	no relevant (e,e) data	q_{max}-dependence ignored	no PT data	$\langle r^4\rangle/\langle r^2\rangle$ fix, or $\langle r^6\rangle$=0	rms-radius	q_{max} (fm^{-1})	$\rho(r)$
Adamuscin (VDM) [?]							X			(x)	0.848	8.	✓
Bernauer [?]	(x)	(x)	X					X	X		0.879	5.	(–)
Borisyuk (CM) [?]									X		0.912	5.	✓
Graczyk (BI) [?]			(x)	?							0.899	15.	k
Griffioen (q4) [?]		X	X		X	X		X		X	0.850	0.7	–
Griffioen (CF) [?]	X			X		X		X	X		0.840	5.	–
Higinbotham(dip) [?]			X	X					X	X	0.849	10.	k
Higinbotham(q2) [?]	X	X	X							X	0.844	0.9	–
Horbatsch (dip)[?]			X						X	X	0.840	1.5	k
Horbatsch (CM, z1) [?]	X	X	X							X	0.890	1.5	–
Lee (CM) [?]		(x)									0.904	5.	–
Lorenz (VDM) [?]			(x)	X					X		0.840	5.	✓
Sick (+TC) [?]											0.886	12.	✓

CM=conformal mapping, BI=Bayesian Inference, TC=Tail Constraint, k=kink, –=no $\rho(r)$

Three important observations

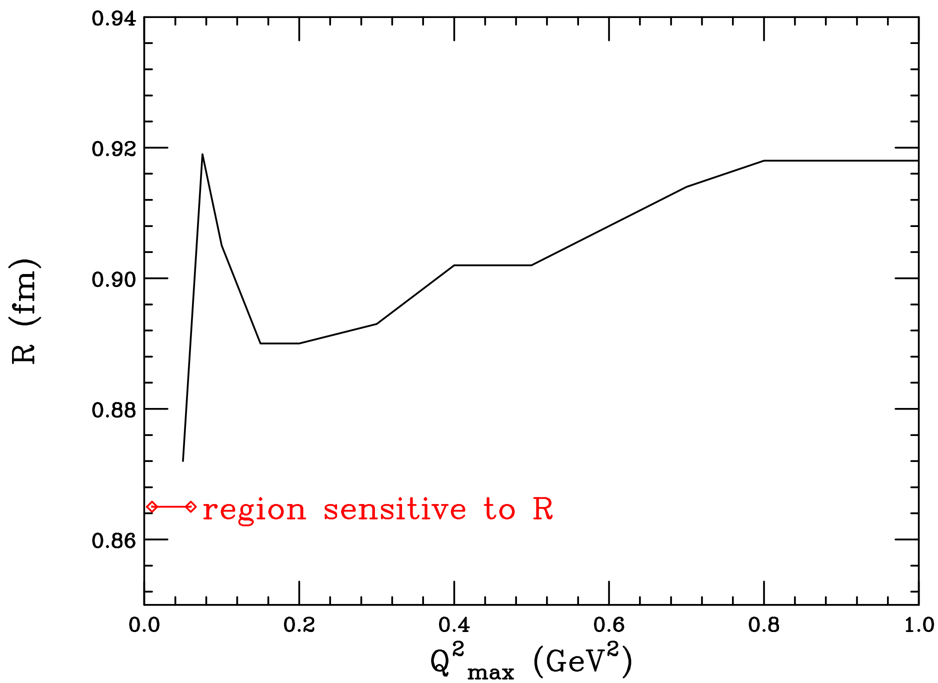
1. Dependence of extracted R on q_{max}

dependence exhibited for polynomial series at very low q
trivial, discussed already above

fits with *large* q_{max}, $5fm^{-1} - 10fm^{-1}$, tend to give $R > 0.88fm$

q_{max} dependence found by various authors, show result of Lee *et al.*

How can these large q's affect R?
rms-radius is 'measured' at $q = 0$!



This behavior calls for explanation!

Understanding: effect of large-r tail of $\rho(r)$

remember: rms-radius sensitive to r due to r^4 weight

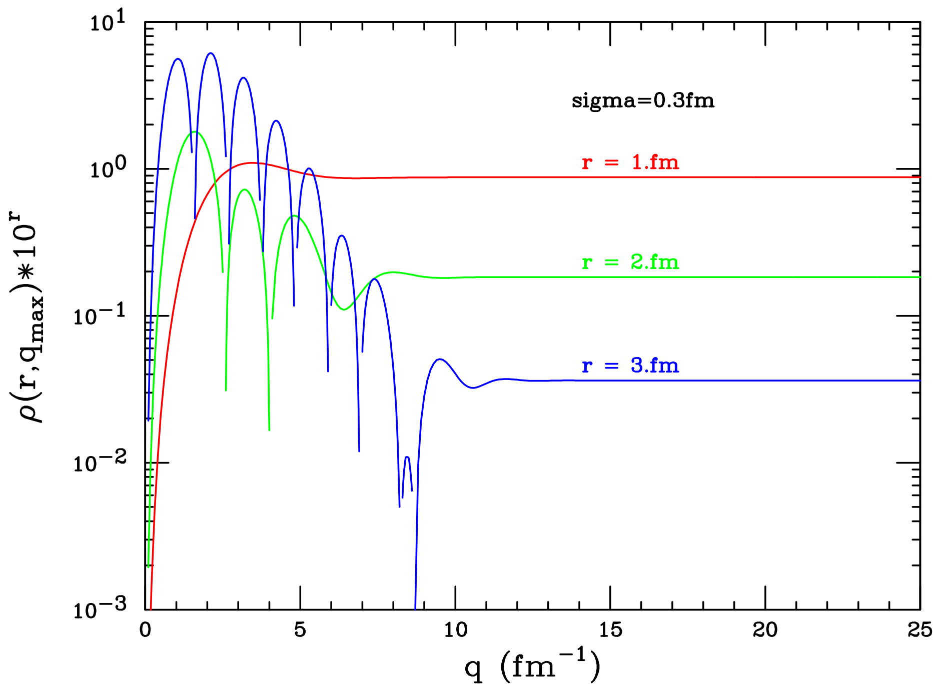
large r affect low q and curvature of $G(q)$ below q_{min}

Data up to large q fix *shape* of $\rho(r)$ *including* large-r tail

this reduces arbitrariness of shape of fitted $G(q)$ at low q

this leads to more reliable extrapolation from q_{min} to $q = 0$

For demonstration study $\rho(r, Q_{max}) = ... \frac{1}{r} \int_0^{Q_{max}} G(q)\ sin(qr)\ q\ dq$



converged $\rho(r)$ for larger r

needs higher q_{max}

Important observation: to fix $\rho(r)$ at the larger r must include $G(q)$ at the higher q's

Fits with maximal q_{max} yield the most reliable extrapolation to $q = 0$

Importance of high q data for $dG/dq^2(q=0)$ of fit is not a contradiction

low-q data important to fix $G(q)$ in region where data sensitive to R

high-q data fix *shape* of $\rho(r)$ *i.e.* shape of $G(q)$ needed for extrapolation to $q = 0$

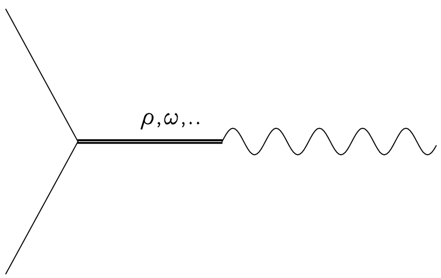
Or more simply said:

rms-radius depends strongly on density at large r: $R^2 = ... \int \rho(r) r^4 dr$
to fix (implicitly) this density need $G(q)$ up to large q

2. Radii from VDM fits always low, typically in $0.84 fm$ region

always much too large χ^2
needs understanding

Basic assumption of VDM



leads a priori to form factor $G(q) = \sum_i a_i/(1+q^2\gamma_i)$, $\rho(r) = \sum a_i e^{-\gamma_i r}/r$
γ_i^{-1} = masses squared of vector mesons

The promise: VDM could fix problem with large-r behavior

tail $\sim e^{-\gamma r}/r$ is given by *physics*

Complication

'pole' closest to physical region (responsible for low q) is *not* a pole

it is a cut starting at $4m_\pi^2$

accounts for interaction with pion tail of N (triangle diagram)

Strength distribution in cut: difficult to come by

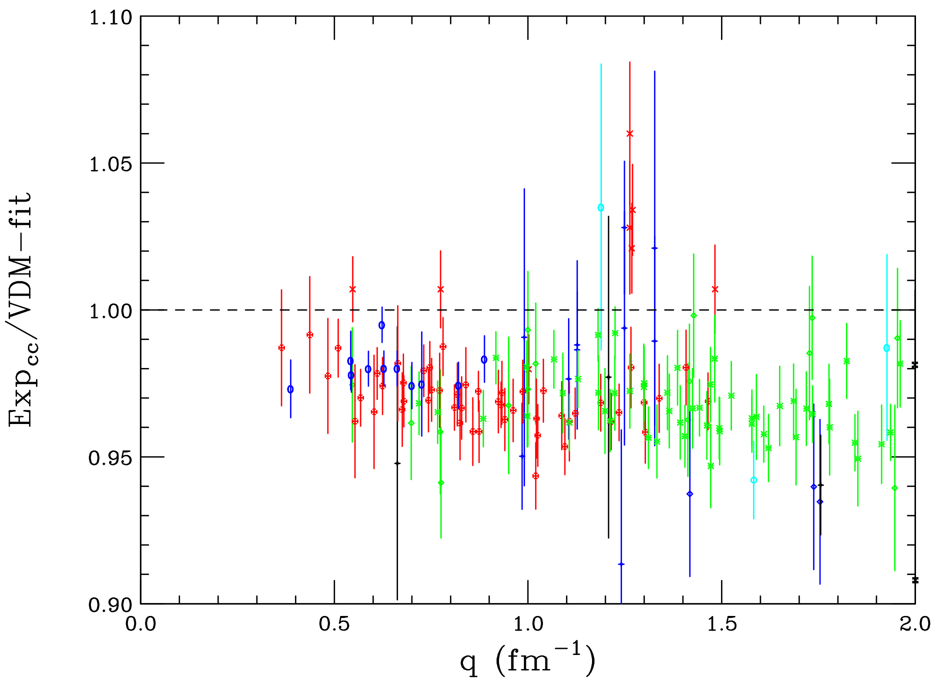
determined by Hoehler *et al.* in 1976 using dispersion relations

only partly updated

Generic problem of VDM analyses

χ^2 is too large

systematic differences to data at low q: fit Mergel *et al.*



difference to true R

can be trivially read off figure

±same for all VDM fits

since Hoehler's time

Reason for too large χ^2

VDM spectral function is too strong a constraint

has not enough flexibility to allow good fit of (e,e)

Demonstration of constraining role of spectral function

VDM analysis of Adamuscin *et al.*

want to fit (e,e) data over largest q^2-range ($< 0, > 0$)

Difficulty: TPE in (e,e) cross sections (G_e from L/T and PT disagree)

'solution': omit σ, fit only polarization transfer PT data

result: $R = 0.848 \pm 0.007 fm$

amazingly small error bar!

BUT...

PT data measure *only* ratio G_e/G_m

contain no information whatsoever on charge radius

Conclusion

spectral function all by itself fixes R to amazing precision

adding *relevant* (e,e) data, σ, only leads to bad χ^2!

explains the problem of VDM analyses since Hoehler's time

find always poor χ^2 and $0.84 fm$ + systematic deviation from data

Further complication: large-q behavior of $G(q) = \sum$ monopoles

falls like q^{-2}
leads to $1/r$ singularity of $\rho(r) \sim e^{-\kappa r}/r$
also contradicts QCD which wants fall-off faster than q^{-4}
and does not allow to even qualitatively fit data

Solution in standard VDM analyses

different 'fixes' employed
effect in the end:
multiply $\sum$monopoles with dipole form factor
with mass Γ^{-1} substantially greater than the γ_i's

In practice: dipole contributes $0.05 - 0.2 fm$ of rms-radius

this contribution is purely phenomenological (choice of dipole, choice of Γ)
[theory+phenomenology] no better than [phenomenology alone]
removes hope to get from VDM better fix on ρ in tail

What remains: too large χ^2 of VDM fits

systematic deviations from (e,e)
cannot claim to "fit" (e,e) data
can only say to "use" (e,e) data

Disgression

VDM adherents claim that analytic structure of $G(q)$ important
could be, *e.g.* poles at $q > q_{max}$ certainly are detrimental

Test if analytic structure would *really* change results

use VDM-type form factor:
sum of monopoles times dipole = M·D-parameterization

$$G(q) = \sum_i a_i/(1+q^2\gamma_i) \;\; 1/(1+q^2\Gamma)^2$$

with free $a_i,\ \gamma_i$, with VDM-constraint $\gamma_i^{-1} > 4m_\pi^2$

Result of fit of *world* data up to $12 fm^{-1}$

variation of handpicked γ_i's not even needed
fit of parameters a_i enough
χ^2 as low as other best fits (SOG, Laguerre) of same data
within a $\Delta\chi^2$ of <1! (χ^2=587 for 603 data points)

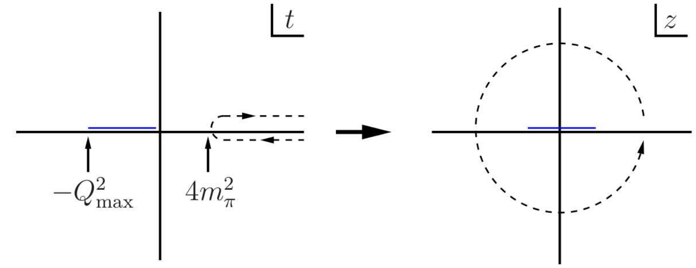
R differs by $0.002 fm$

→ G_{SOG}, $G_{Laguerre}$ not different from G_{MD}
SOG, Laguerre analytic structure equivalent to VDM

M·D parameterization optimal for (partial) control of large-r density

3. Conformal Mapping approaches could be helpful, but...

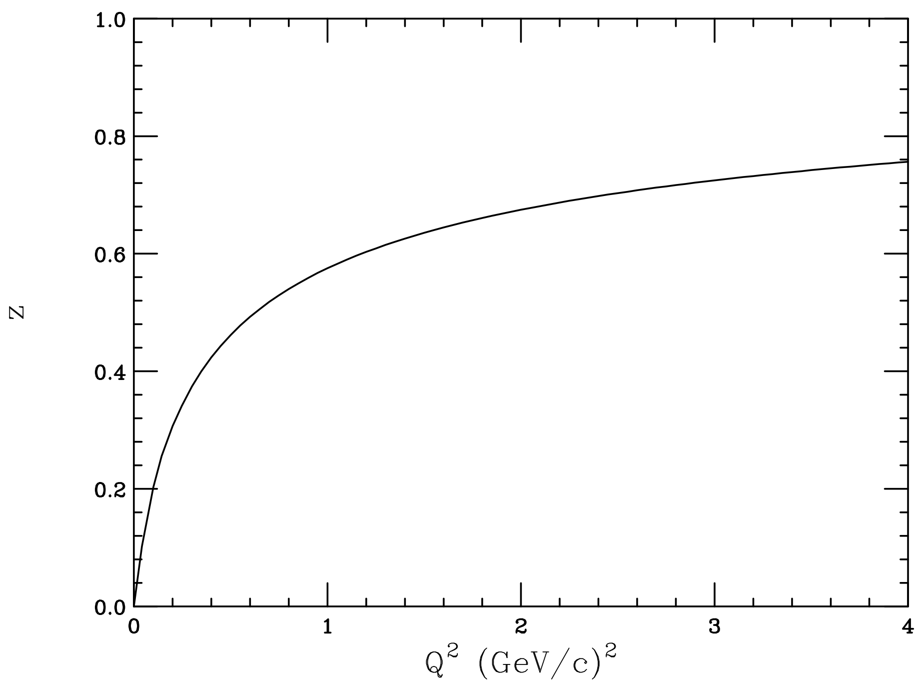
flexible expansions of $G(q)$ in terms of q not optimal
expansion parameter can become large, >1
variable transformation could help

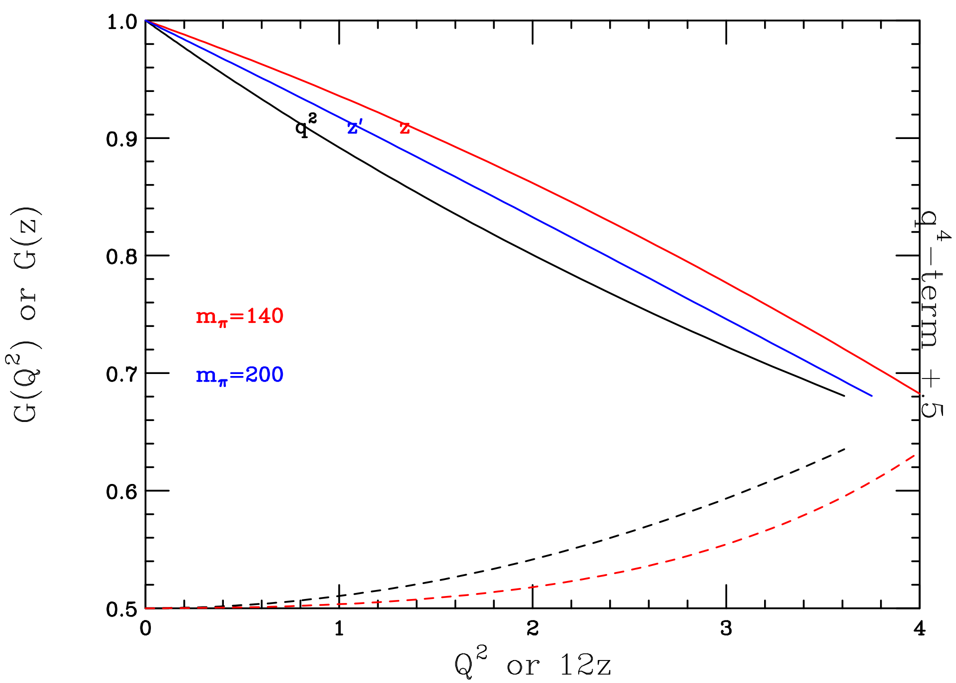


Standard choice $z = \frac{\sqrt{t_c - t} - \sqrt{t_c - t_0}}{\sqrt{t_c - t} + \sqrt{t_c + t_0}}$, $\quad t = q^2$

most often with $t_0 = 0$ and $t_c = 4m_\pi^2$
yields expansion parameter $z < 1$, see figure

Claimed to 'linearize' extrapolation: not really true



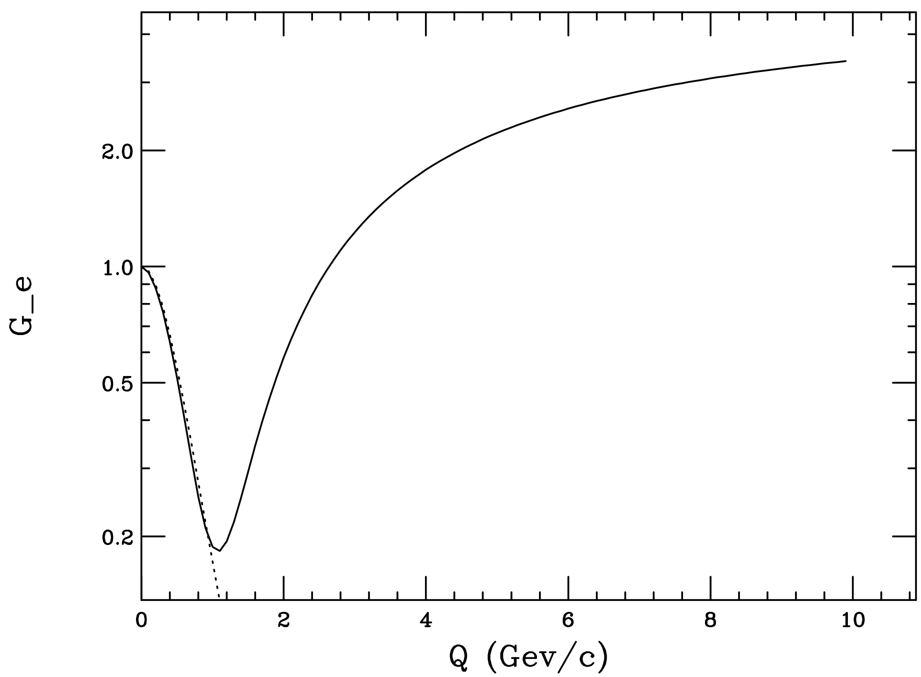


CM does ease multi-parameter expansion of $G(z)$

q^{-4} fall-off of $G(q)$ justifies bound on size of coefficients of $G(z)$ (Hill, Paz)
avoids divergent results due to over-fitting of data involving large coefficients

but some care is still needed
see *e.g.* $G(q)$ of Lee *et al.* , contradicts justification for bound

Even in terms of z, use of power-series is highly unwise



Better choice: Borisyuk

$G(q)$ = polynomial in z times dipole in q
then q^{-4} fall-off guaranteed, polynomial remains of order 1

Typical parameterizations used in above table

VDM, powers in q^2 many authors

already discussed, gives wrong R

Inverse Polynomials, Continued Fraction, some Pade many authors

have often singularities above q_{max}

have often no $\rho(r)$, cannot judge whether large-r tail sensible

[N][N+2]Pade with $b_i > 0$ Arrington, Sick

OK, but $\rho(r)$ requires effort

Conformal Mapping + power-series in z Lee

$G(q \rightarrow \infty) \rightarrow$ constant value, no $\rho(r)$

Conformal Mapping + power series in z times dipole Borisyuk

OK, $\rho(r)$ requires effort

Sum sigmoid functions times dipole Graczyk

OK, $\rho(r)$ requires effort

Dipole times spline modification at larger q's Horbatsch

fixes $\langle r^4 \rangle / \langle r^2 \rangle$, wrong

Sum-of-Gaussians, Laguerre polynomial, M·D Sick

OK

Average of R-values

use analyses without *serious* deficiencies, large q_{max}
take brute-force average without considering quoted δR

Adjustments applied

Borisyuk

does not include polarization transfer data
when I omit PT data, R increases by $.017 fm$, apply this correction

Graczyk

uses phenomenological TPE, not justified at low q
correct R for difference to standard TPE

1. author	adjustment	rms-radius
Bernauer		$0.879 \pm 0.008 fm$
Borisyuk	$-.017 fm$	$0.895 \pm 0.012 fm$
Graczyk	$0.009 fm$	$0.890 \pm 0.003 fm$
Lee		$0.904 \pm 0.015 fm$
Sick (+tail constraint)		$0.886 \pm 0.008 fm$
Average (unweighted)		$0.891 \pm 0.009 fm$

estimated standard deviation from the average R: $0.009 fm$

Outliers: 0.879 (Bernauer data), 0.904 (power series?)

Fits

Adamuscin: VDM
Bernauer: power series, inverse polynomials, spline, B, 1.14
Borisyuk: power series in z times dipole, W, 0.75
Graczyk: sum of dipoles·sigmoid functions, W, χ^2 not known
Griffioen: power series q^4, ”B”, 1.0
Griffioen: continued fraction (pole), B, 1.6
Higinbotham: power series q^2, S+M, 0.7
Higinbotham: dipole, S+M, 1.25
Horbatsch: dipole, B, 1.11
Horbatsch: 1-parameter fit in z, B, 1.11
Lee: power series in z, W $\delta\sigma$ rescaled, 0.64
Lee: power series in z, B $\delta\sigma$ rescaled, 0.82
Lorenz: VDM, B, 2.2
Lorenz, continued fraction (pole), B, <1.6
Sick: SOG, Laguerre polynomials, M·D, W, 0.98
Sick: SOG, Laguerre polynomials, M·D, W, 1.35 (norm fixed)

Overall: even results without obvious flaws differ somewhat

remaining model dependence of extrapolation, q_{max}, choice of data

For more reliable R respect 3 insights

1. Fit data to largest q possible

this determines $\rho(r)$ *including* large-r
this fixes *shape* of $G(q)$ below q_{min} needed for extrapolation to $q = 0$

2. Use parameterization accessible in *both* q- and r-space

SOG, Laguerre, M·D (as in VDM fits)
then can check implied behavior of $\rho(r)$ at large r
can check whether physically sensible
can fit data + eventual large-r constraint simultaneously

3. Constrain large-r behavior via physics knowledge

$\rho(r \gg)$ is given by wave function of least-bound Fock state: $n + \pi^+$
shape (not absolute norm) given by known removal energy
can be imposed at asymptotic r where $\rho(r) < 1\%$ of ρ(center)

this fixes most of present problems with R

See PRC 89 (14) 012201