Heavy flavours in high energy collisions: quenching, thermalization and correlation

Andrea Beraudo

INFN - Sezione di Torino

Heavy Quark Physics in Heavy-Ion collisions:
experiments, phenomenology and theory,
16th-20th March 2015, ECT* (Trento)

Heavy Flavor in the QGP: the conceptual setup

- Description of soft observables based on hydrodynamics, assuming to deal with a system close to local thermal equilibrium (no matter why);
- Description of jet-quenching based on energy-degradation of external probes (high-p_T partons);

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NB At high-p_T the interest in heavy flavor is no longer related to thermalization, but to the study of the mass and color charge dependence of jet-quenching (not addressed in this talk)

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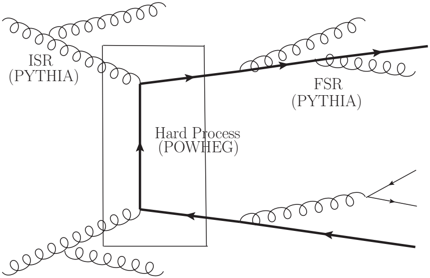
NB for realistic temperatures $g \sim 2$, so that one can wonder *whether a charm is really "heavy"*, at least in the initial stage of the evolution.

Simulating the initial hard production



- Powerful pQCD tools[1] are available to simulate the initial $Q\overline{Q}$ production, interfacing the output of a NLO event-generator (POWHEG, MC@NLO) for the hard process with a parton-shower (PYTHIA, HERWIG) describing Initial and Final State Radiation.
- This provides a *fully exclusive information on the final state*

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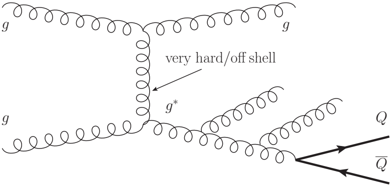
[1] For a systematic comparison (POWHEG vs MC@NLO vs FONLL): M. Cacciari *et al.*, JHEP 1210 (2012) 137.

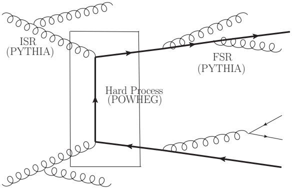
FONLL vs POWHEG+PS

FONLL



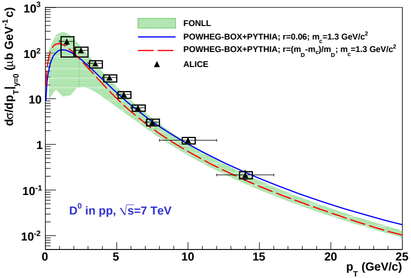
- It is a *calculation*
- It provides NLL accuracy, resumming large $\ln(p_T/M)$
- It includes processes missed by POWHEG (hard events with light partons)

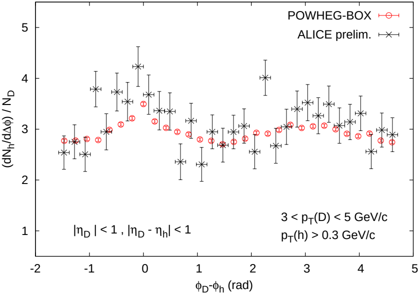
POWHEG+PS



- It is an *event generator*
- Results compatible with FONLL
- It is a more flexible tool, allowing to address more differential observables (e.g. $Q\overline{Q}$ correlations)

HF production in *pp* collisions: results





- Besides reproducing the inclusive p_T-spectra...[2]
- ...the POWHEG+PYTHIA setup allows also the comparison with $D-h$ correlation data, which start getting available.

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[2] W.M. Alberico *et al*, Eur.Phys.J. C73 (2013) 2481

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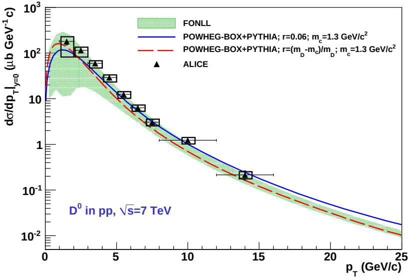

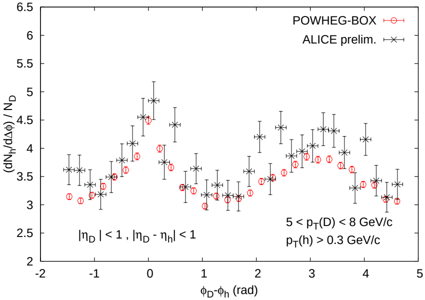

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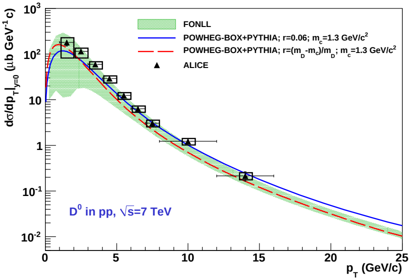

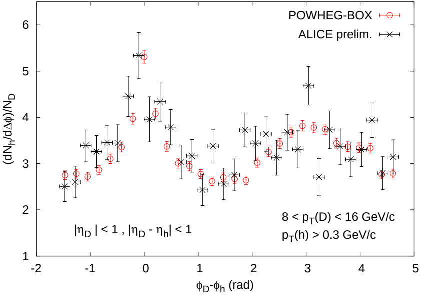

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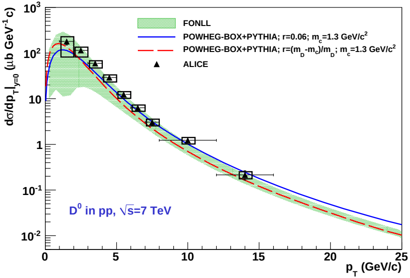

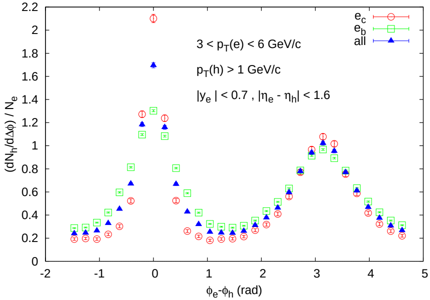

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HF in nucleus-nucleus collisions

- Transport calculations: a critical overview
- Towards a *precise determination* of the transport coefficients from QCD
- How close/far are heavy quarks go to/from thermalization? Are final (hadronic) observables able to answer this question? What could be the role of experiments at increasing $\sqrt{s_{\rm NN}}$?

Transport theory: the Boltzmann equation

Time evolution of HQ phase-space distribution $f_Q(t, \mathbf{x}, \mathbf{p})$[3]:

$$\frac{d}{dt} f_Q(t, \mathbf{x}, \mathbf{p}) = C[f_Q]$$

- Total derivative along particle trajectory

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{x}} + \mathbf{F}\frac{\partial}{\partial \mathbf{p}}$$

  Neglecting **x**-dependence and mean fields: $\partial_t f_Q(t, \mathbf{p}) = C[f_Q]$

- Collision integral:

$$C[f_Q] = \int d\mathbf{k} [\underbrace{w(\mathbf{p}+\mathbf{k}, \mathbf{k}) f_Q(\mathbf{p}+\mathbf{k})}_{\text{gain term}} - \underbrace{w(\mathbf{p}, \mathbf{k}) f_Q(\mathbf{p})}_{\text{loss term}}]$$

  $w(\mathbf{p}, \mathbf{k})$: HQ transition rate $\mathbf{p} \to \mathbf{p} - \mathbf{k}$

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[3] Approach adopted for HQs by Greco *et al*, Gossiaux *et al.* and for the whole medium in codes like BAMPS

From Boltzmann to Fokker-Planck

Expanding the collision integral for *small momentum exchange*[4] (Landau)

$$C[f_Q] \approx \int d\mathbf{k} \left[k^i \frac{\partial}{\partial p^i} + \frac{1}{2} k^i k^j \frac{\partial^2}{\partial p^i \partial p^j} \right] [w(\mathbf{p}, \mathbf{k}) f_Q(t, \mathbf{p})]$$

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The Boltzmann equation reduces to the *Fokker-Planck equation* (approx. to be quantitatively tested!)

$$\frac{\partial}{\partial t} f_Q(t,\mathbf{p}) = \frac{\partial}{\partial p^i} \left\{ A^i(\mathbf{p}) f_Q(t,\mathbf{p}) + \frac{\partial}{\partial p^j} [B^{ij}(\mathbf{p}) f_Q(t,\mathbf{p})] \right\}$$

where

$$A^i(\mathbf{p}) = \int d\mathbf{k}\, k^i w(\mathbf{p},\mathbf{k}) \longrightarrow \underbrace{A^i(\mathbf{p}) = A(p)\, p^i}_{\text{friction}}$$

$$B^{ij}(\mathbf{p}) = \frac{1}{2} \int d\mathbf{k}\, k^i k^j w(\mathbf{p},\mathbf{k}) \longrightarrow \underbrace{B^{ij}(\mathbf{p}) = \hat{p}^i \hat{p}^j B_0(p) + (\delta^{ij} - \hat{p}^i \hat{p}^j) B_1(p)}_{\text{momentum broadening}}$$

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Problem reduced to the *evaluation of three transport coefficients*

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The relativistic Langevin equation

The Fokker-Planck equation can be recast into a form suitable to follow the dynamics of each individual quark: the Langevin equation

$$\frac{\Delta p^i}{\Delta t} = -\underbrace{\eta_D(p)p^i}_{\text{determ.}} + \underbrace{\xi^i(t)}_{\text{stochastic}} ,$$

with the properties of the noise encoded in

$$\langle \xi^i(\mathbf{p}_t)\xi^j(\mathbf{p}_{t'})\rangle = b^{ij}(\mathbf{p}_t)\frac{\delta_{tt'}}{\Delta t} \qquad b^{ij}(\mathbf{p}) \equiv \kappa_\parallel(p)\hat{p}^i\hat{p}^j + \kappa_\perp(p)(\delta^{ij} - \hat{p}^i\hat{p}^j)$$

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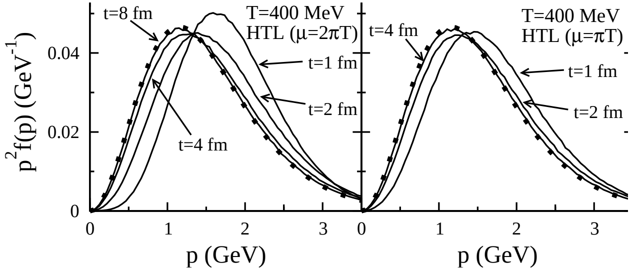
Transport coefficients (to *derive from theory*):

- *Momentum diffusion* $\kappa_\perp \equiv \frac{1}{2}\frac{\langle \Delta p_\perp^2 \rangle}{\Delta t}$ and $\kappa_\| \equiv \frac{\langle \Delta p_\|^2 \rangle}{\Delta t}$;
- *Friction* term (dependent on the discretization scheme!)

$$\eta_D{}^{\text{Ito}}(p) = \frac{\kappa_\|(p)}{2TE_p} - \frac{1}{E_p^2}\left[(1-v^2)\frac{\partial \kappa_\|(p)}{\partial v^2} + \frac{d-1}{2}\frac{\kappa_\|(p) - \kappa_\perp(p)}{v^2}\right]$$

fixed in order to assure approach to equilibrium (Einstein relation):

A first check: thermalization in a static medium



For $t \gg 1/\eta_D$ one approaches a relativistic Maxwell-Jüttner distribution[5]

$$f_{\rm MJ}(p) \equiv \frac{e^{-E_p/T}}{4\pi M^2 T\, K_2(M/T)}, \qquad \text{with} \int d^3p\, f_{\rm MJ}(p) = 1$$

(Test with a sample of c quarks with $p_0 = 2$ GeV/c and weak-coupling HTL transport coefficients)

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[5] A.B., A. De Pace, W.M. Alberico and A. Molinari, NPA 831, 59 (2009)

The realistic case: expanding fireball

Within our POWLANG setup (POWHEG+LANGevin) the HQ evolution in heavy-ion collisions is simulated as follows

- $Q\overline{Q}$ pairs initially produced with the POWHEG-BOX package (with nPDFs) and distributed in the transverse plane according to $n_{\rm coll}(\mathbf{x}_\perp)$ from (optical) Glauber model;

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[6] P.F. Kolb, J. Sollfrank and U. Heinz, Phys. Rev. C **62** (2000) 054909
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- update of the HQ momentum and position to be done at each step *in the local fluid rest-frame*
  - $u^\mu(x)$ used to perform the boost to the fluid rest-frame;
  - $T(x)$ used to set the value of the transport coefficients

  with $u^\mu(x)$ and $T(x)$ fields taken from the output of hydro codes[6];
- Procedure iterated until hadronization

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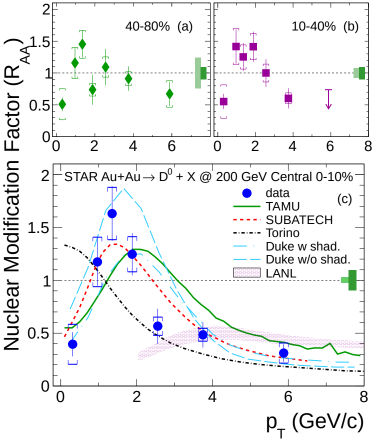
Results vs experimental data

D-mesons at low-p_T: STAR data compared to various model predictions (see the various talks).

Sharp peak ≈ 1.5 GeV in central $(0 - 10\%)$ collisions:

- from charm radial flow?
- from coalescence with light quarks (included in some of the models)?

More in the following...



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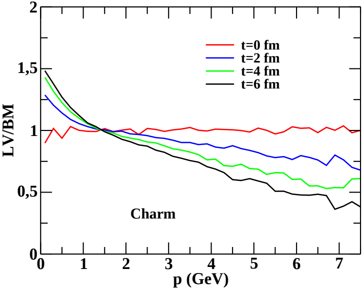

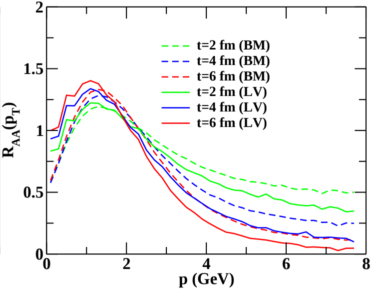

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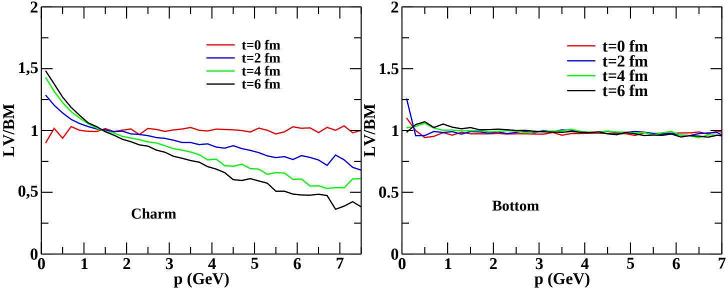

For beauty on the other hand Langevin≡Boltzmann!

The Langevin/FP approach: a critical perspective

At the same time the Langevin/FP approach, although formally derived as a soft-scattering limit of the Boltzmann equation, can be considered *more general than the latter*, requiring simply the knowledge of a few transport coefficients (friction and diffusion) *meaningful even in a non-perturbative framework* and *not relying on quasi-particle picture* of the medium.

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Notice that, for the light quarks/gluons of the medium one has

- Thermal de Broglie wavelength: $\lambda_{\rm th} \sim 1/T$
- Mean free path: $\lambda_{\rm mfp} \sim 1/g^2 T$

In the weak-coupling regime one has $\lambda_{\rm th} \ll \lambda_{\rm mfp}$, so that between the relatively rare scatterings one has the propagation of *localized on-shell particles*. However as the coupling gets large $\lambda_{\rm th} \sim \lambda_{\rm mfp}$, the two scales are no longer well separated and a *picture based on on-shell distribution function* may be *no longer valid*

HF transport coefficients

- Weak-coupling calculation (pQCD+HTL)
- Non-perturbative calculation (lattice-QCD)

Transport coefficients: perturbative evaluation

It's the stage where the various models differ!

We account for the effect of 2 → 2 collisions in the medium

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We account for the effect of 2 → 2 collisions in the medium

Intermediate cutoff $|t|^* \sim m_D^2$[7] *separating the contributions of*

- hard collisions ($|t| > |t|^*$): kinetic pQCD calculation
- soft collisions ($|t| < |t|^*$): Hard Thermal Loop approximation (*resummation of medium effects*)

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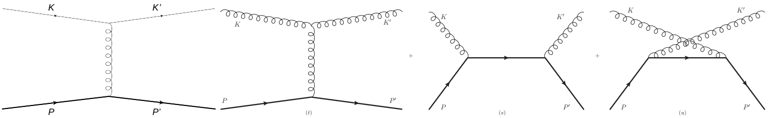
Transport coefficients $\kappa_{T/L}(p)$: hard contribution

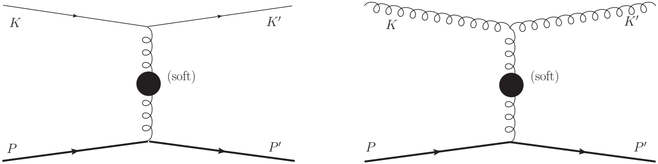


$$\kappa_T^{g/q(\mathrm{hard})} = \frac{1}{2}\frac{1}{2E}\int_k \frac{n_{B/F}(k)}{2k}\int_{k'}\frac{1 \pm n_{B/F}(k')}{2k'}\int_{p'}\frac{1}{2E'}\theta(|t|-|t|^*)\times$$
$$\times\,(2\pi)^4\delta^{(4)}(P+K-P'-K')\left|\overline{\mathcal{M}}_{g/q}(s,t)\right|^2 q_T^2$$

$$\kappa_L^{g/q(\mathrm{hard})} = \frac{1}{2E}\int_k \frac{n_{B/F}(k)}{2k}\int_{k'}\frac{1 \pm n_{B/F}(k')}{2k'}\int_{p'}\frac{1}{2E'}\theta(|t|-|t|^*)\times$$
$$\times\,(2\pi)^4\delta^{(4)}(P+K-P'-K')\left|\overline{\mathcal{M}}_{g/q}(s,t)\right|^2 q_L^2$$

where: $(|t| \equiv q^2 - \omega^2)$

Transport coefficients $\kappa_{T/L}(p)$: soft contribution



When the exchanged 4-momentum is **soft** the t-channel gluon feels the presence of the medium and requires **resummation**.

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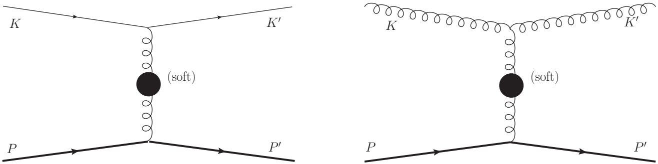


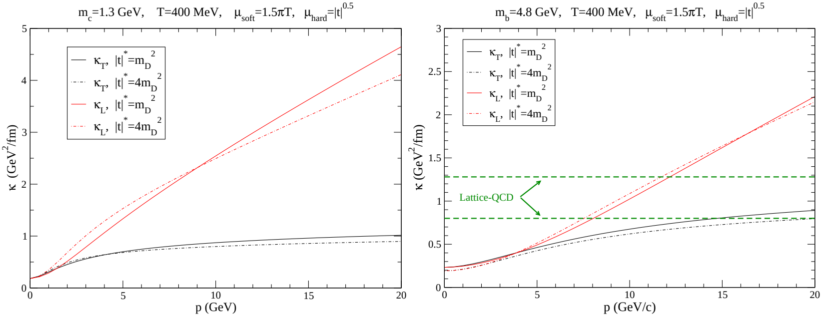
When the exchanged 4-momentum is **soft** the t-channel gluon feels the presence of the medium and requires **resummation**.
The *blob* represents the *dressed gluon propagator*, which has longitudinal and transverse components:

$$\Delta_L(z,q) = \frac{-1}{q^2 + \Pi_L(z,q)}, \quad \Delta_T(z,q) = \frac{-1}{z^2 - q^2 - \Pi_T(z,q)},$$

where *medium effects* are embedded in the HTL gluon self-energy.

Transport coefficients: numerical results

Combining together the hard and soft contributions...



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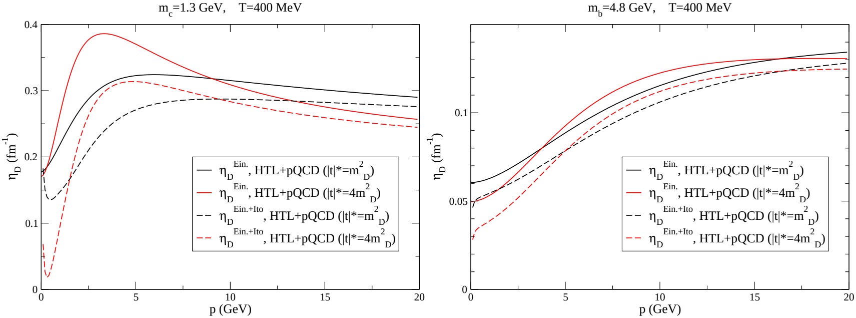

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Non perturbative information on HF transport coefficients can be obtained from lattice-QCD simulations, so far treating the HQ's as static ($M = \infty$) color sources placed in a thermal bath.

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Non perturbative information on HF transport coefficients can be obtained from lattice-QCD simulations, so far treating the HQ's as static ($M\!=\!\infty$) color sources placed in a thermal bath.
One consider the non-relativistic limit of the Langevin equation:

$$\frac{dp^i}{dt} = -\eta_D p^i + \xi^i(t), \quad \text{with} \quad \langle \xi^i(t)\xi^j(t')\rangle = \delta^{ij}\delta(t-t')\kappa$$

Hence, in the $p\to 0$ limit:

$$\kappa = \frac{1}{3}\int_{-\infty}^{+\infty} dt \langle \xi^i(t)\xi^i(0)\rangle_{\rm HQ} \approx \frac{1}{3}\int_{-\infty}^{+\infty} dt\, \underbrace{\langle F^i(t)F^i(0)\rangle_{\rm HQ}}_{\equiv D^>(t)}$$

Lattice-QCD transport coefficients: setup

Non perturbative information on HF transport coefficients can be obtained from lattice-QCD simulations, so far treating the HQ's as static ($M=\infty$) color sources placed in a thermal bath.
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κ is then given by the $\omega\to 0$ limit of the *spectral density* $\sigma(\omega)$ of the above E-field correlator

$$\kappa \equiv \lim_{\omega\to 0}\frac{D^>(\omega)}{3} \equiv \lim_{\omega\to 0}\frac{1}{3}\frac{\sigma(\omega)}{1-e^{-\beta\omega}} \underset{\omega\to 0}{\sim} \frac{1}{3}\frac{T}{\omega}\sigma(\omega)$$

Lattice-QCD transport coefficients: results

The spectral function $\sigma(\omega)$ has to be reconstructed starting from the *euclidean electric-field correlator*

$$D_E(\tau) = -\frac{\langle \mathrm{Re}\,\mathrm{Tr}[U(\beta,\tau)gE^i(\tau,\mathbf{0})U(\tau,0)gE^i(0,\mathbf{0})]\rangle}{\langle \mathrm{Re}\,\mathrm{Tr}[U(\beta,0)]\rangle}$$

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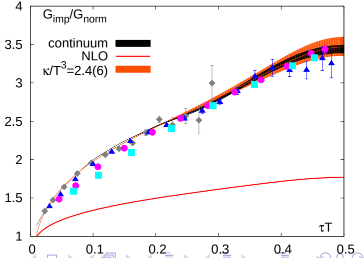
∼3-5 times larger then the perturbative result (W.M. Alberico *et al*, EPJC 73 (2013) 2481). Challenge: approaching the continuum limit in full QCD (see Kaczmarek talk)!



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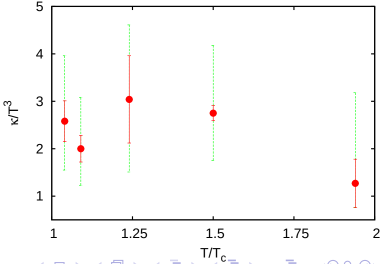

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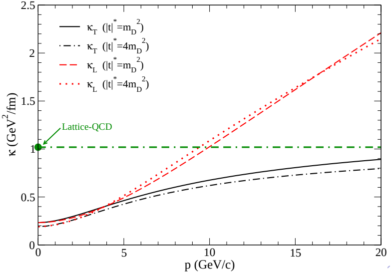

First message: look at beauty!

Measurements of beauty in *AA* collisions (with future *detector upgrades*) in the next years will allow one to establish a link between first-principle theoretical predictions (*continuum-extrapolated* lattice-QCD calculations) and experimental observables:

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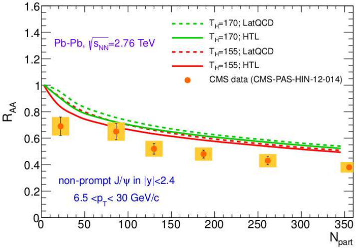

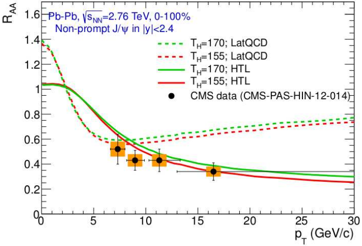

Measurements so far limited to non-prompt J/ψ's at quite high p_T

Heavy Quark thermalization?

Heavy quark thermalization?

Wondering whether heavy quarks thermalize entails a number of related questions...

- Are theoretical tools able to describe their approach to *thermal* equilibrium in a evolving medium?
- What are the indications coming from experiment? Are final hadronic/leptonic observables able to provide an unambiguous answer on *what happens in the partonic stage*?
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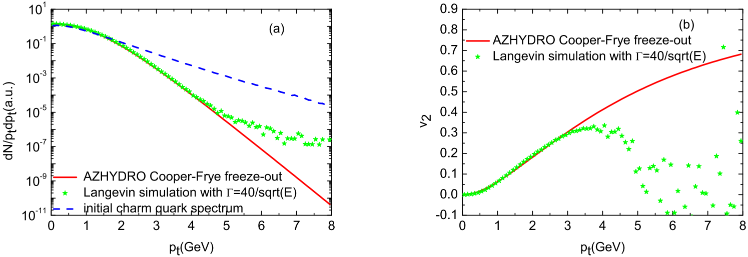
NB thermal equilibrium of HQ's at the end of the QGP phase is assumed in the description of hidden and open charm production within the Statistical Hadronization Model: answering this question may support or rule out such an hypothesis

Validation of the theoretical tools

In the limit of large transport coefficients heavy quarks should reach local thermal equilibrium and decouple from the medium as the other light particles, according to the Cooper-Frye formula:

$$E(dN/d^3p) = \int_{\Sigma_{\rm fo}} \frac{p^\mu \cdot d\Sigma_\mu}{(2\pi)^3} \exp[-p\cdot u/T_{\rm fo}]$$



This was verified to be actually the case (M. He, R.J. Fries and R. Rapp, PRC 86, 014903).

Experimental indications

It is possible to compare the *experimental* D-meson R_{AA} with the *theoretical expectation* in the case of *kinetic equilibrium*

- Spectrum in *pp* given by POWHEG+PYTHIA setup
- Final spectrum in *AA* given by hydro + Cooper-Frye

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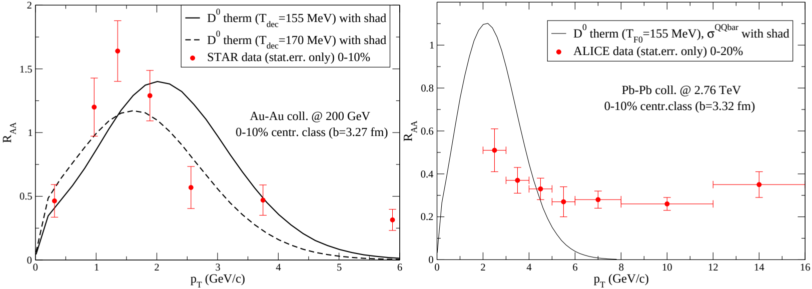

Evidence of a bump from radial flow at RHIC, while more data at low-p_T (waiting for ALICE ITS upgrade) necessary at LHC; in any case charm *partially out of* ***kinetic*** *equilibrium*, at least at LHC

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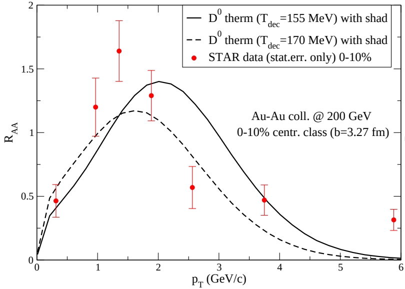

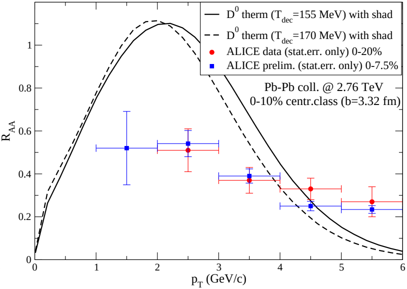

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HF in the POWLANG setup: recent developments

(arXiv:1410.6082)

The major novelty concerns the simulation of heavy-quark hadronization, which now can be performed via

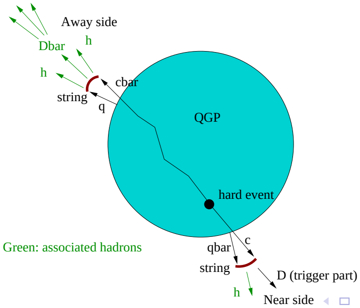
- standard vacuum Fragmentation Functions
- recombination with thermal light partons

From quarks to hadrons

In-medium hadronization may affect the R_{AA} and v_2 of final D-mesons due to the *collective flow* of light quarks. We tried to estimate the effect through this model interfaced to our POWLANG transport code:

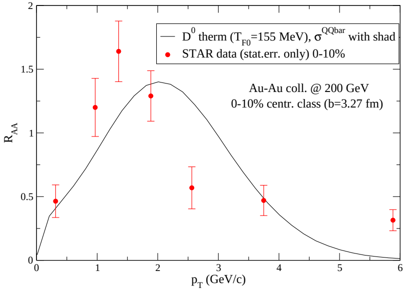
- At $T_{\rm dec}$ c-quarks coupled to light $\overline{q}$'s from a local *thermal distribution*, eventually *boosted* ($u^\mu_{\rm fluid} \neq 0$) to the lab frame;
- Strings are formed and given to PYTHIA 6.4 to simulate their fragmentation and produce the final hadrons ($D + \pi + \dots$)

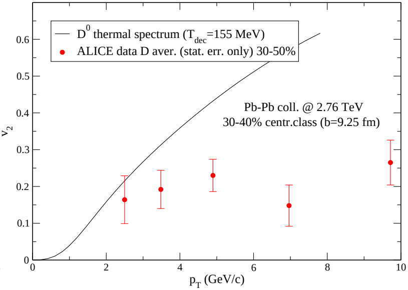
One can address the study of $D - h$ and $e - h$ correlations in AA collisions



From quarks to hadrons: effect on R_{AA} and v_2

Experimental data display a peak in the R_{AA} and a sizable v_2 one would like to interpret as a signal of *charm radial flow and thermalization*





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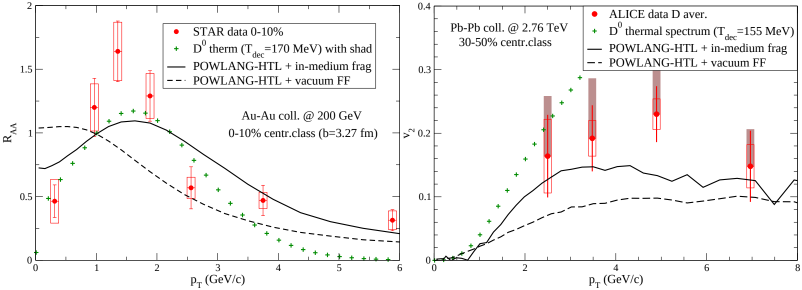

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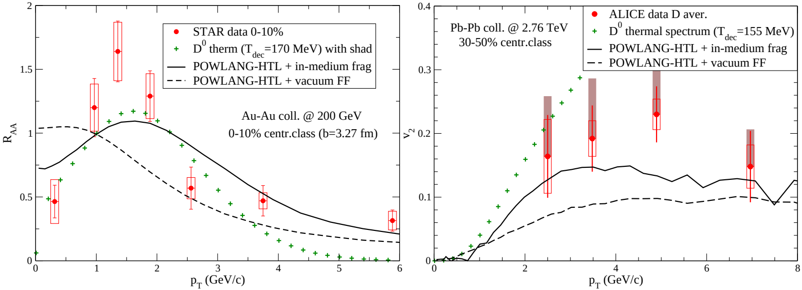

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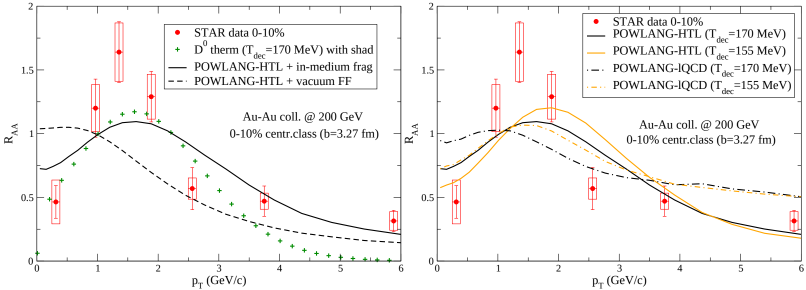
Rescattering in the hadronic phase and its effect on v_2 should be also investigated (in progress)!

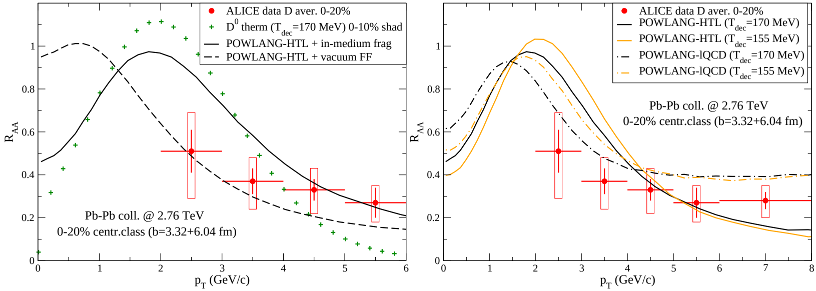
D-meson R_{AA} at RHIC



It is possible to perform a systematic study of different choices of

- Hadronization scheme (left panel)
- Transport coefficients (weak-coupling pQCD+HTL vs non-perturbative l-QCD) and decoupling temperature (right panel)

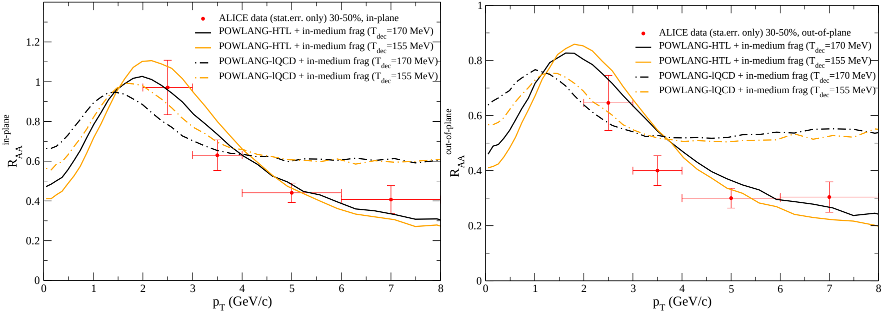
D-meson R_{AA} at LHC



Experimental data for central (0−20%) Pb-Pb collisions at LHC display a strong quenching, but – at least with the present bins and p_T range – don't show strong signatures of the bump from radial flow predicted by "thermal" and "transport + $Q\overline{q}_{\rm therm}$-string fragmentation" curves.

D meson R_{AA}: in-plane vs out-of-plane

One can study di R_{AA} in- and out-of-plane in non-central $(30-50\%)$ Pb-Pb collisions at LHC:



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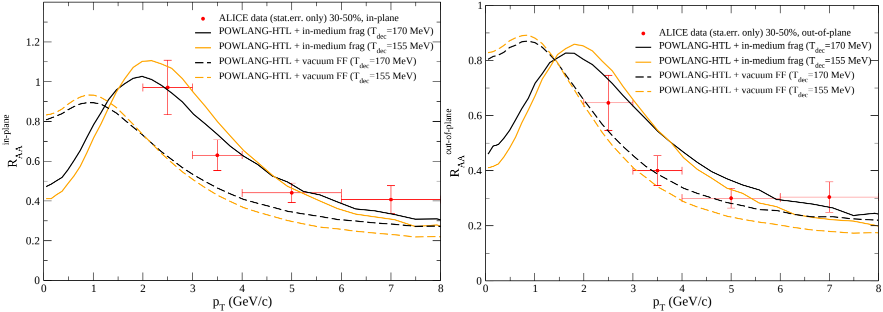

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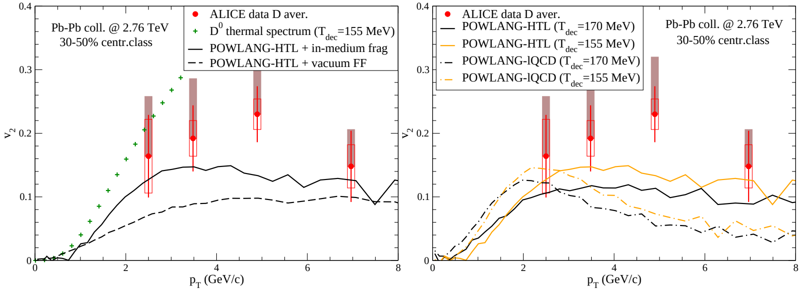
- $Q\overline{q}_{\rm therm}$-string fragmentation describes data slightly better than in-vacuum independent Fragmentation Functions.

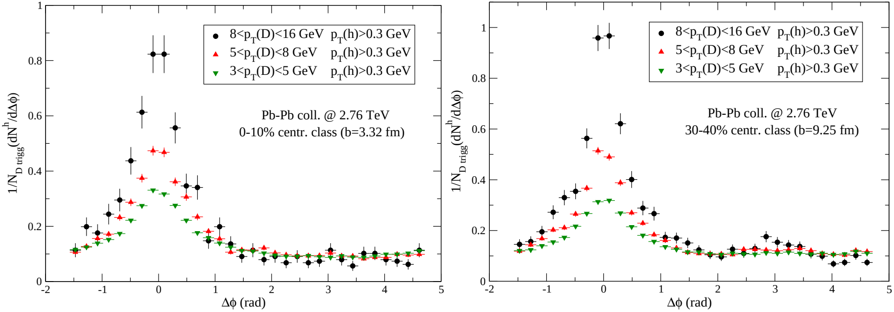
D-meson v_2 at LHC



Concerning D-meson v_2 in non-central (30−50%) Pb-Pb collisions:

- $Q\overline{q}_{\rm therm}$-string fragmentation routine significantly improves our transport model predictions compared to the data;
- HTL curves with a *lower decoupling temperature* display the best agreement with ALICE data

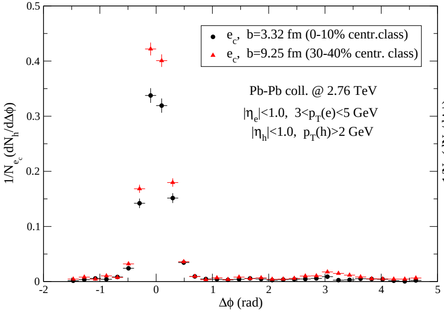
Azimuthal correlations: $D - h$

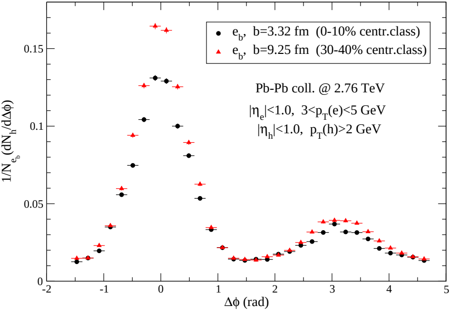


Away-side peak strongly suppressed
both in central and semi-central collisions

Azimuthal correlations: $e-h$

We plot the separate e_c (left) and e_b (right) contributions from charm and beauty decays





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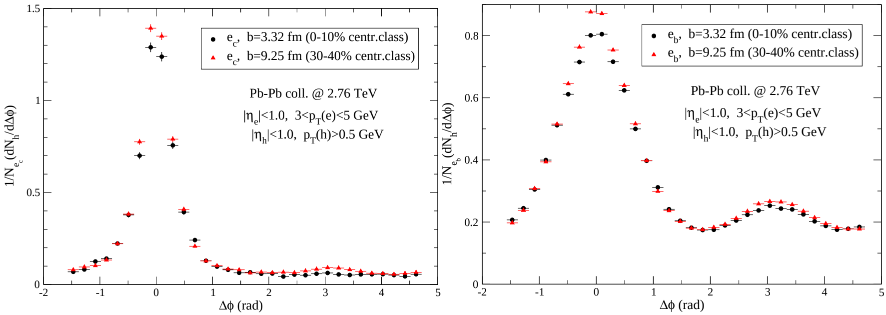

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Summary and perspectives

- We have tried to answer the question of kinetic equilibration of charm in heavy-ion collisions. We have shown how predictions of two different scenarios (hydro and transport+recombination) can be hardly distinguished within the current kinematic range covered by the experiment and they can both describe some qualitative features of the data at low-moderate p_T

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- Work in progress:
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