Spectral functions in QCD

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Strongly interacting matter: transport and response

aim of the meeting

common questions for diverse systems

- cold atomic gases
- hadronic and quark-gluon plasma
- neutron/nuclear matter
- ...

even though systems differ in detail, there is (hopefully!)

- common methodology
- shared insight

Outline

dynamical questions in hadronic and quark-gluon plasma

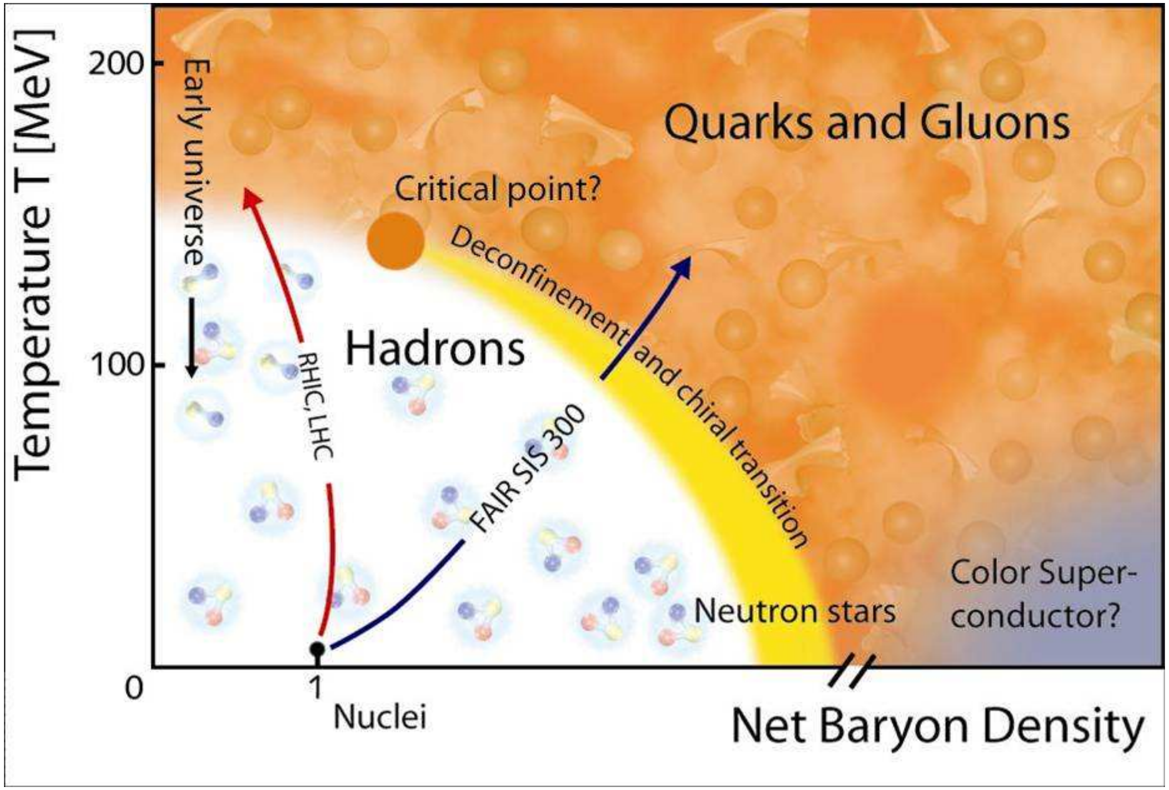
- spectral quantities
- close to equilibrium
- out of equilibrium

lattice results (FASTSUM)

- conductivity and diffusion
- baryons and baryonic spectral functions

Strongly interacting matter: QCD

hadronic and quark-gluon plasma: rich topic



spectral quantities

close to equilibrium

out of equilibrium

Strongly interacting matter: QCD

spectral quantities

- (de)confinement: light hadron spectrum ($\pi, \rho, N, ...$)
  - in-medium modification as $T \sim 0 \rightarrow T_c$
  - $T > T_c$: transition to quark degrees of freedom

Strongly interacting matter: QCD

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Strongly interacting matter: QCD

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- chiral symmetry
  - $T \to T_c$: emergent degeneracy $(\rho \leftrightarrow a_1, N \leftrightarrow N^*, ...)$
- heavy quarks/quarkonium
  - survival in QGP, channel dependent ($\bar{c}c, \bar{b}b$)
  - sequential melting, effective thermometer
- $\ldots$

Strongly interacting matter: QCD

spectral quantities

close to equilibrium

- linear response, external perturbations
  - plasma oscillations, correlation times

Strongly interacting matter: QCD

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- transport: evolution of nearly conserved quantities
  - energy-momentum: viscosities η, ξ
  - charges: conductivity σ, light/heavy quark diffusion D

Strongly interacting matter: QCD

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- transport: evolution of nearly conserved quantities
  - energy-momentum: viscosities η, ξ
  - charges: conductivity σ, light/heavy quark diffusion D
- non-strongly interacting signatures
  - thermal radiation: photon emission rate
  - dilepton production rate
- $\ldots$

Strongly interacting matter: QCD

<u>spectral quantities</u>

<u>close to equilibrium</u>

<u>out of equilibrium</u>

- “arbitrary” initial state: evolution in real time
  - heavy ion collision
  - . . .

Strongly interacting matter: QCD

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requires use of effective (field) theories

- hydrodynamics – kinetic theory, (classical) particle dynamics – classical field dynamics, . . .

Strongly interacting matter: QCD

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relation

- transport coefficients as low-energy constants
- spectral understanding, (quasi)particles

Strongly interacting matter: QCD

spectral quantities

close to equilibrium

out of equilibrium (not discussed further, except as above)

addresses seemingly very different questions:

- yet information is contained in thermal correlators

all thermal correlation functions contain *all* the information
(Euclidean, Feynman, Wightman, retarded, advanced, statistical, . . .)

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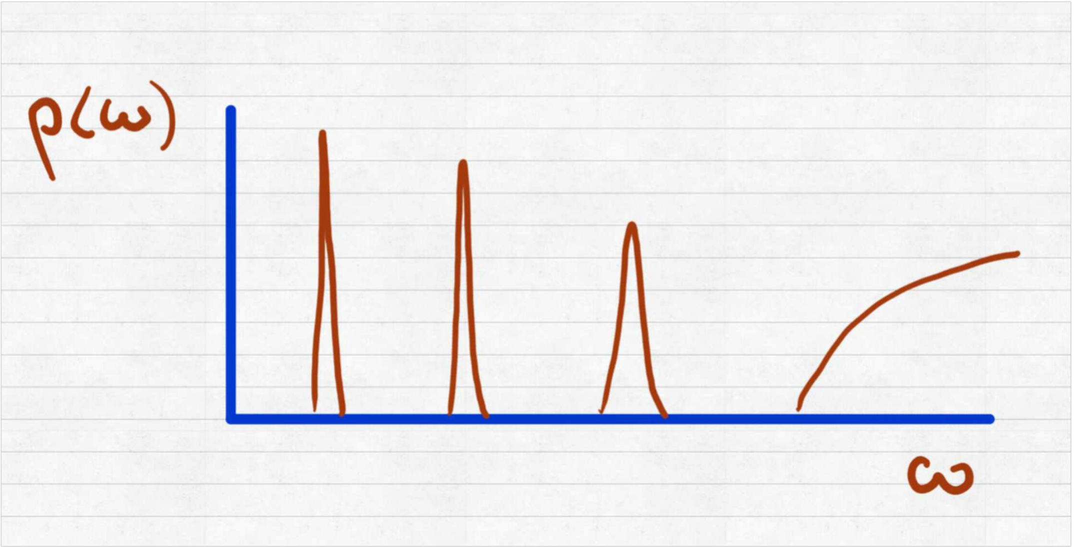
- but info might be more accessible in specific form
- or easier defined in certain representations

$\Rightarrow$ spectral function $\rho(\omega, \mathbf{p})$

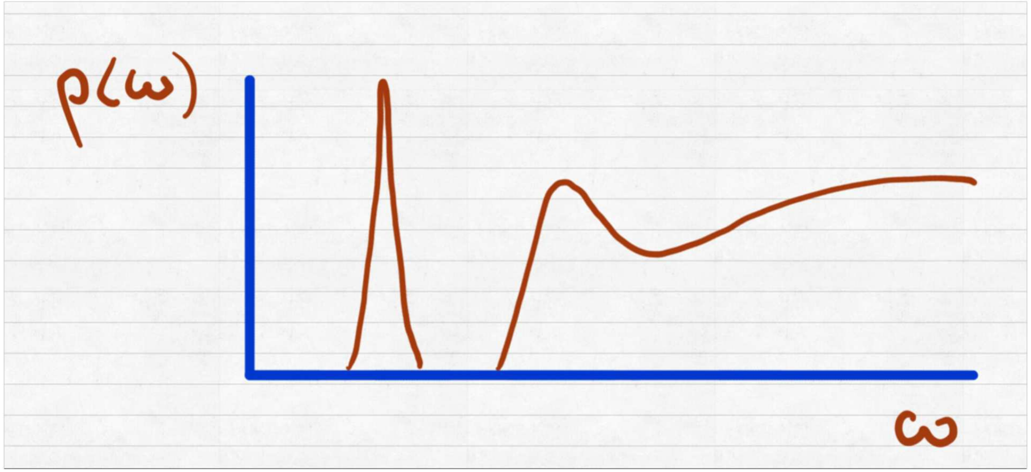
Spectral functions

- spectral quantities at low temperatures



Spectral functions

- spectral quantities at low temperatures
- spectral quantities at higher temperatures



thermal broadening, dissociation, quarkonium

Spectral functions

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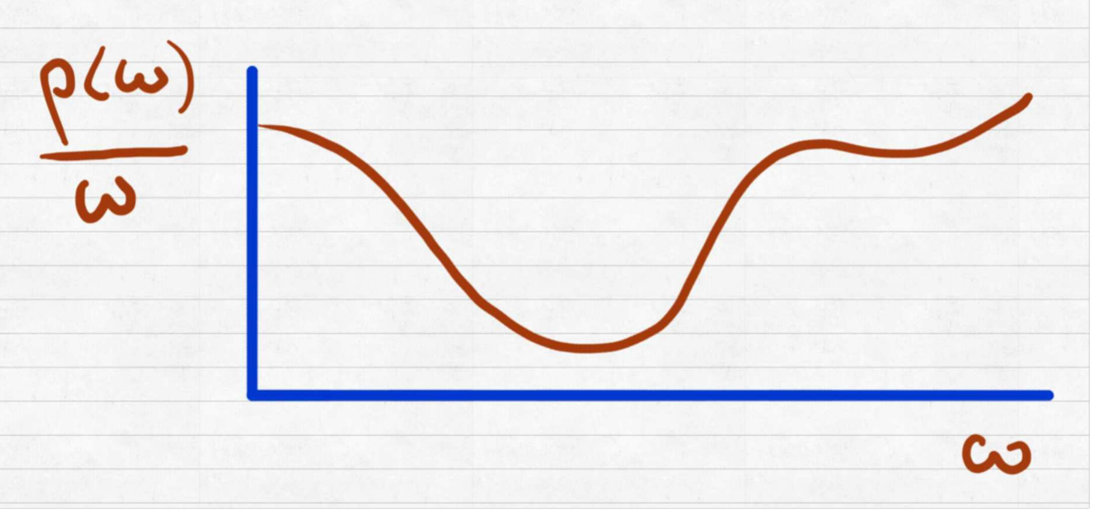
- spectral quantities at higher temperatures
- transport at small frequencies



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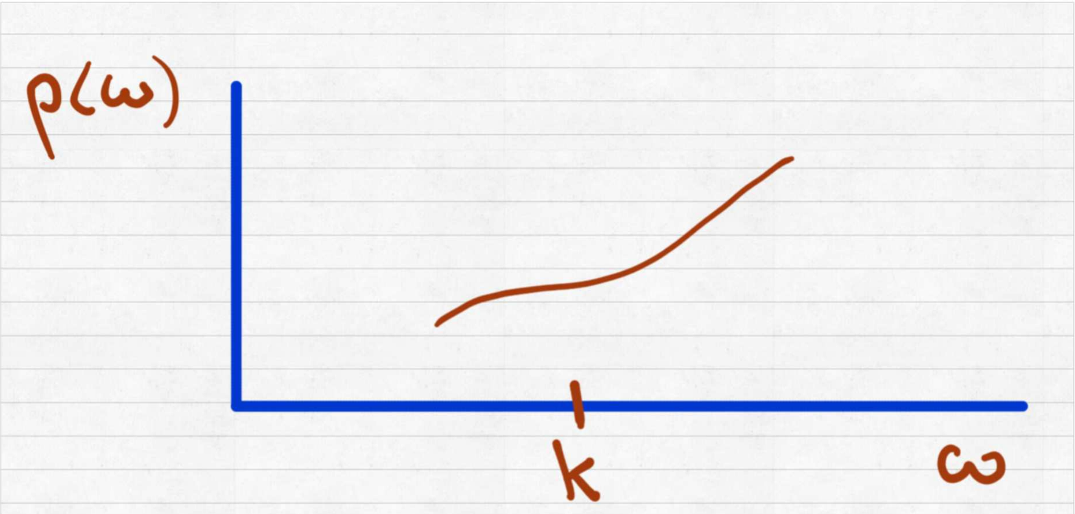
- photon/dilepton production in kinematic range



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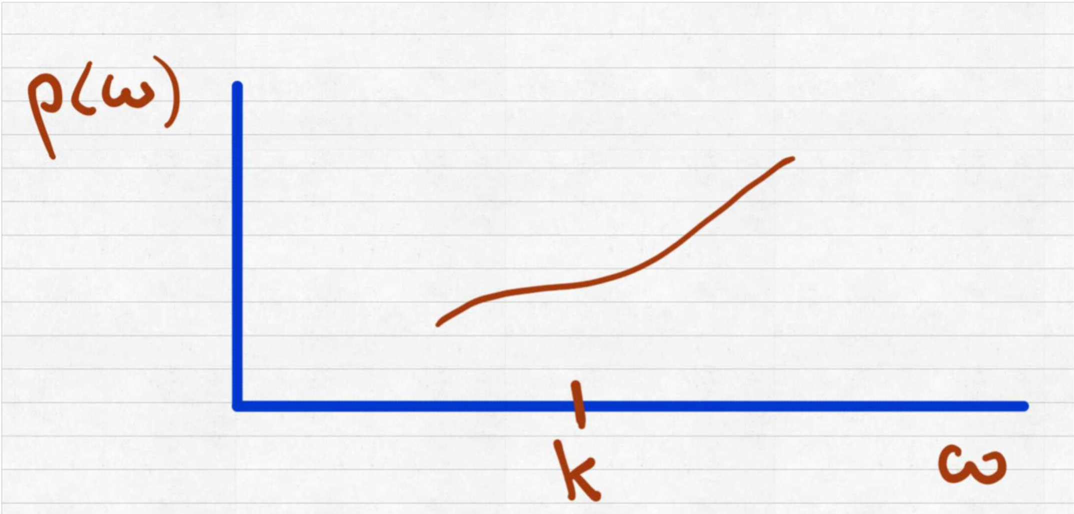

- lots of diverse information in various regimes

Example: Vector channel

electromagnetic current $j_\mu(t,\mathbf{x}) = \bar{\psi}(t,\mathbf{x})\gamma_\mu\psi(t,\mathbf{x})$

spectral function $\rho_{\mu\nu}(t-t',\mathbf{x}-\mathbf{x}') = \langle [j_\mu(t,\mathbf{x}), j_\nu(t',\mathbf{x}')] \rangle$

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- photon production, rate $\sim n_{\rm B}(k)/k\, \rho_{\mu\mu}(k,\mathbf{k})$
- dilepton production, rate $\sim n_{\rm B}(\omega)/M^2\, \rho_{\mu\mu}(\omega,\mathbf{k})$ with $M^2 = \omega^2 - \mathbf{k}^2$ and $m_\ell \sim 0$

Some FASTSUM results

- conductivity and diffusion

- baryons

FASTSUM

- anisotropic $N_f = 2 + 1$ Wilson-clover ensembles
- many temperatures below and above T_c

FASTSUM collaboration

GA (Swansea)
Chris Allton (Swansea)
Seyong Kim (Sejong University)
Maria-Paola Lombardo (Frascati)
Sinead Ryan (Trinity College Dublin)
Jonivar Skullerud (Maynooth)
Simon Hands (Swansea)
Don Sinclair (Argonne)

Ale Amato (Swansea -> Helsinki)
Wynne Evans (Swansea -> Bern)
Pietro Giudice (Swansea->Münster)
Tim Harris (TCD->Mainz)
Benjamin Jäger (Swansea)
Aoife Kelly (Maynooth)
Bugra Oktay (Utah)
Kristi Praki (Swansea)
Davide de Boni (Swansea)

On the lattice

FASTSUM ensembles

- $N_f = 2 + 1$ dynamical quark flavours, Wilson-clover
- many temperatures, below and above T_c
- anisotropic lattice, $a_s/a_\tau = 3.5$, many time slices
- strange quark: physical value
- two light flavours: somewhat heavy $m_\pi = 384(4)$ MeV

N_s	24	32	24	24	32/24	32/24	32/24	24	32/24
N_τ	128	48	40	36	32	28	24	20	16
T/T_c	0.24	0.63	0.76	0.84	0.95	1.09	1.27	1.52	1.90
$N_{\rm cfg}$	400	600	500	500	500	500	500	1000	1000

- tuning and $N_\tau = 128$ data from HadSpec collaboration

diffusion and conductivity

GA, Chris Allton, Alessandro Amato, Pietro Giudice, Simon Hands and Jonivar Skullerud

PRL 111 (2013) 172001 [arXiv:1307.6763 [hep-lat]]

JHEP 02 (2015) 186 [arXiv:1412.6411 [hep-lat]]

Conductivity/diffusion

linear response: Kubo relation

$$\sigma = \lim_{\omega\to 0} \frac{1}{6\omega}\rho_{ii}(\omega,\mathbf{0})$$

where $$\rho_{\mu\nu}(t,\mathbf{x}) = \langle [j_\mu(t,\mathbf{x}), j_\nu(0,\mathbf{0})]\rangle$$

is current-current spectral function, j_μ is EM current

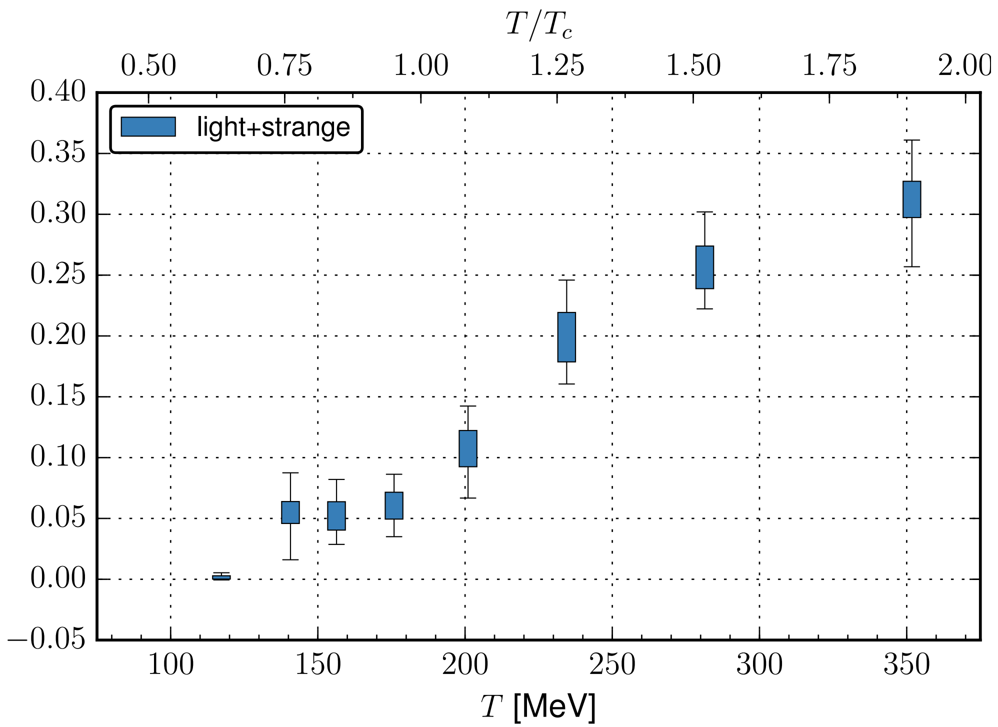
- real-time correlator in equilibrium
- on the lattice: euclidean correlator
- related to spectral function

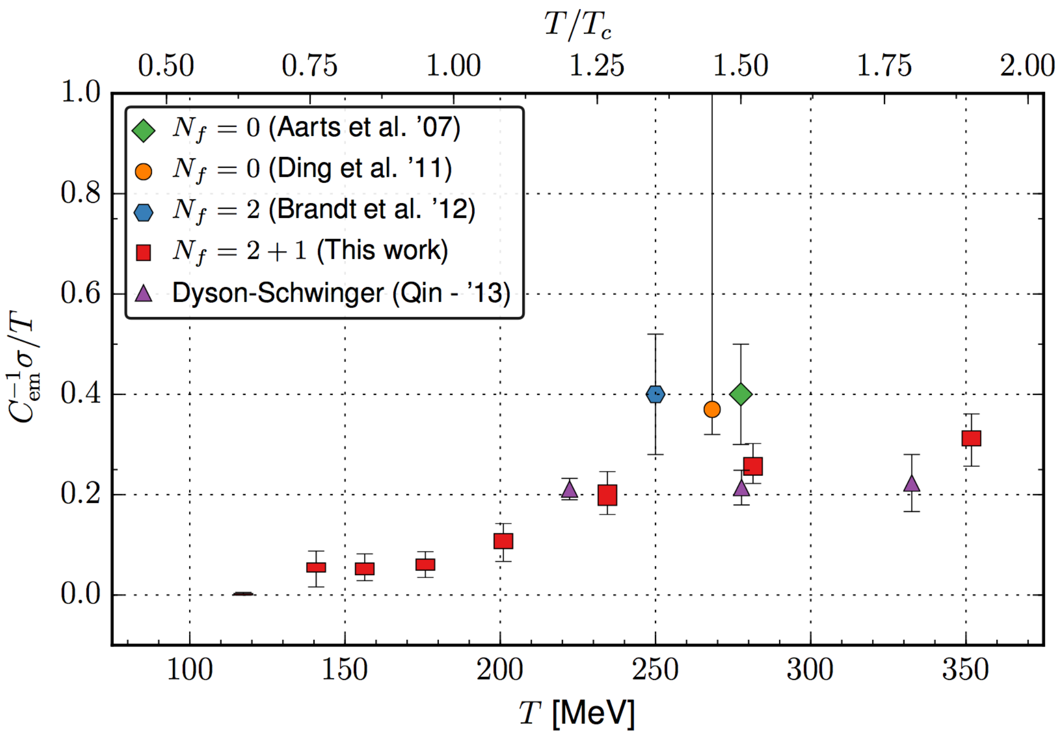
$$G(\tau) = \int d\omega\, K(\tau,\omega)\rho(\omega) \qquad K(\tau,\omega) = \frac{\cosh[\omega(\tau-1/2T)]}{\sinh(\omega/2T)}$$

- inversion/analytical continuation

Conductivity

- conductivity $C_{\rm em}^{-1}\sigma/T$ $\qquad C_{\rm em} = e^2 \sum_f q_f^2$





- temperature dependent
- agreement with previous results above T_c

Conductivity/diffusion

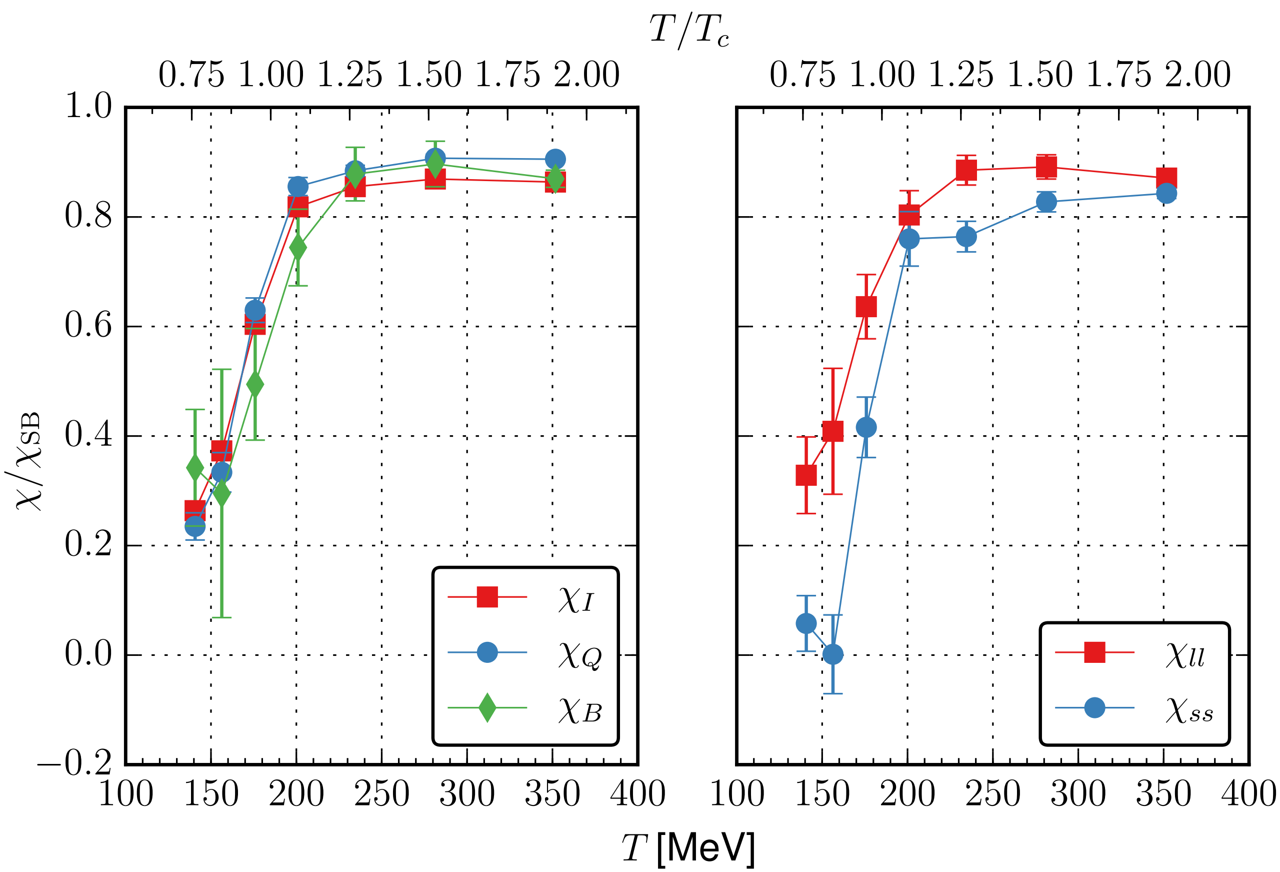
- electrical conductivity σ
- charge susceptibility χ
- both σ and χ proportional to EM factor

$$C_{\rm em} = e^2 \sum_f q_f^2 \qquad\qquad q_f = \frac{2}{3}, -\frac{1}{3}$$

- diffusion coefficient $D = \sigma/\chi$
- $C_{\rm em}$ cancels
- in SU(N_c) theories, factors of N_c cancel
- finite large N_c limit
- weak coupling: $D \sim 1/g^4 T$
- strong coupling: $D = 1/2\pi T$ (holography)

Susceptibilies

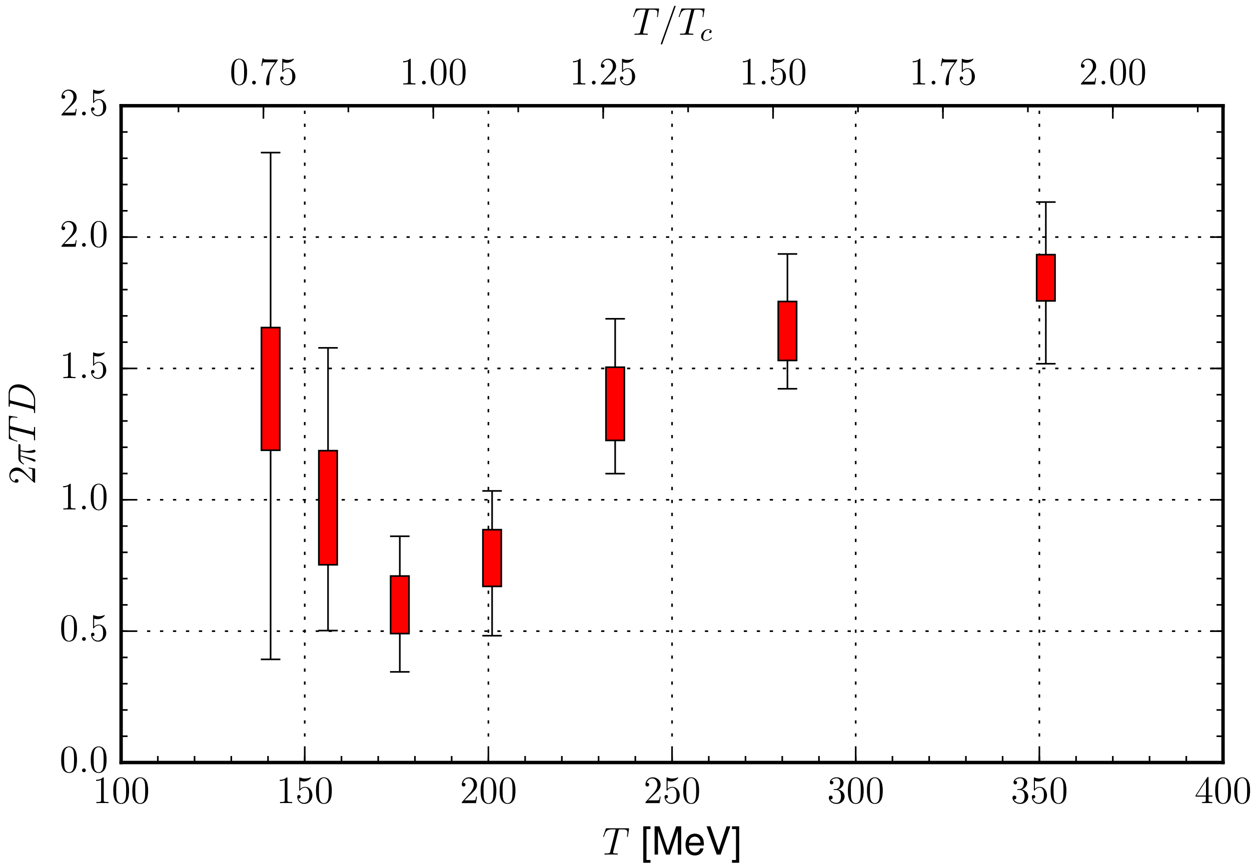
- fluctuations of isospin, electrical charge, baryon number, flavour



- agreement with previous (mostly staggered) results
- some flavour dependence

Diffusion coefficient

- combination of results: $D = \sigma/\chi_Q$



- consistent with strongly coupled plasma, $2\pi TD \sim 1$
- minimum around transition, c.f. η/s

baryons across the deconfinement transition

GA, Chris Allton, Simon Hands, Benjamin Jäger, Kristi Praki and Jonivar Skullerud

PRD 92 (2015) 014503 [arXiv:1502.03603 [hep-lat]]

+ Davide de Boni

in preparation

Baryons in a medium

- mesons in a medium very well studied

lattice studies of baryons at finite temperature limited

- screening masses De Tar and Kogut 1987
- with a small chemical potential QCD-TARO: Pushkina, de Forcrand, Kim, Nakamura, Stamatescu et al 2005
- temporal correlators Datta, Gupta, Mathur et al 2013

not much more . . . but what about:

- in-medium modification?
- chiral symmetry?
- spectral functions?
- ...

Spectral functions

- mesons/bosonic operators

$$\tilde{\tau} = \tau - 1/2T$$

$$G(\tau, \mathbf{p}) = \int \frac{d\omega}{2\pi} K(\tau, \omega) \rho(\omega, \mathbf{p})$$

$$K(\tau, \omega) = \frac{\cosh(\omega\tilde{\tau})}{\sinh(\omega/2T)}$$

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- baryons are fermionic: matrix in Dirac space

$$G(x) = \langle O(x)\overline{O}(0)\rangle = \sum_\mu \gamma_\mu G_\mu(x) + \mathbb{1} G_m(x)$$

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- baryons are fermionic: matrix in Dirac space

$$G(x) = \langle O(x)\overline{O}(0)\rangle = \sum_\mu \gamma_\mu G_\mu(x) + 1\!\!1 G_m(x)$$

- spectral relations (dropping momentum dependence)

$$G_4(\tau) = \int \frac{d\omega}{2\pi} K_{\rm e}(\tau, \omega) \rho_4(\omega), \qquad K_{\rm e}(\tau, \omega) = \frac{\cosh(\omega\tilde{\tau})}{\cosh(\omega/2T)}$$

$$G_{i,m}(\tau) = \int \frac{d\omega}{2\pi} K_{\rm o}(\tau, \omega) \rho_{i,m}(\omega), \qquad K_{\rm o}(\tau, \omega) = -\frac{\sinh(\omega\tilde{\tau})}{\cosh(\omega/2T)}$$

Kernels

- mesons/bosonic operators $\qquad \tilde{\tau} = \tau - 1/2T$

$$K(\tau,\omega) = \frac{\cosh(\omega\tilde{\tau})}{\sinh(\omega/2T)} = [1+n_B(\omega)]\,e^{-\omega\tau} + n_B(\omega)e^{\omega\tau}$$

- baryons/fermionic operators

$$K_{\rm e}(\tau,\omega) = \frac{\cosh(\omega\tilde{\tau})}{\cosh(\omega/2T)} = [1-n_F(\omega)]\,e^{-\omega\tau} + n_F(\omega)e^{\omega\tau}$$

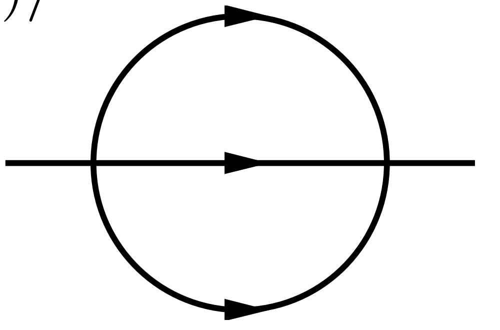
$$K_{\rm o}(\tau,\omega) = -\frac{\sinh(\omega\tilde{\tau})}{\cosh(\omega/2T)} = [1-n_F(\omega)]\,e^{-\omega\tau} - n_F(\omega)e^{\omega\tau}$$

- at $T=0$: all kernels $e^{-\omega\tau}$

pos/neg parity combinations: $\qquad \rho_\pm(\omega) = \frac{1}{2}\left[\rho_m(\omega) \pm \rho_4(\omega)\right]$

Free spectral functions

lowest order contribution $\quad G(x) = \langle O(x)\overline{O}(0)\rangle$



$$O(x) = \epsilon_{abc} u_a(x)\left[u_b^T(x)\mathcal{C}\gamma_5 d_c(x)\right]$$

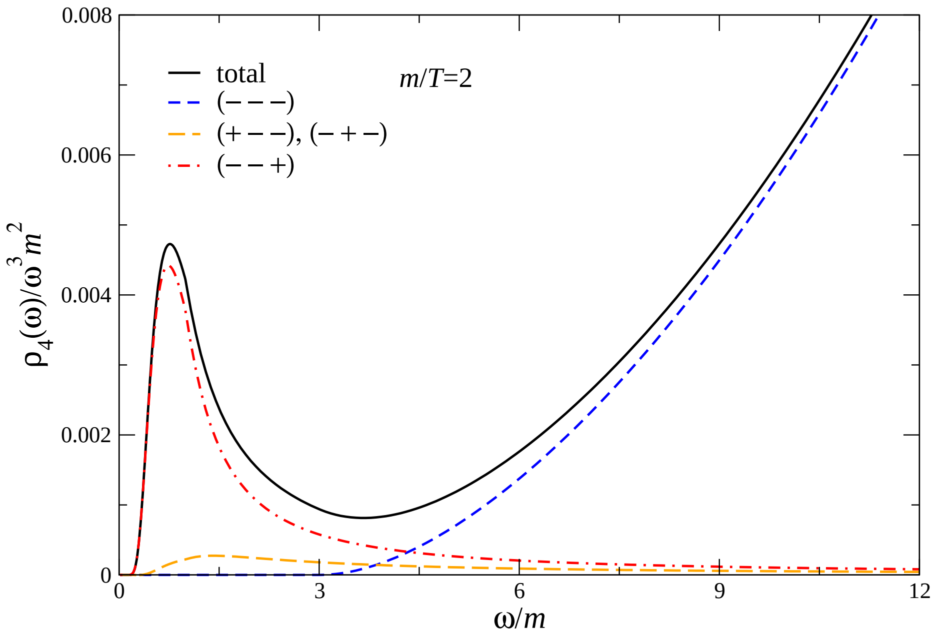
two-loop diagram $\qquad (c = 4, i, m)$

$$\rho_c(\omega) = 3\int_{\mathbf{k}_{1,2,3}} d\Phi_{123} \sum_{s_j=\pm} 2\pi\delta\left(\omega + \sum\nolimits_j s_j\omega_{\mathbf{k}_j}\right)[\text{stat.}] f_c(\omega, s_i, \mathbf{k}_i)$$

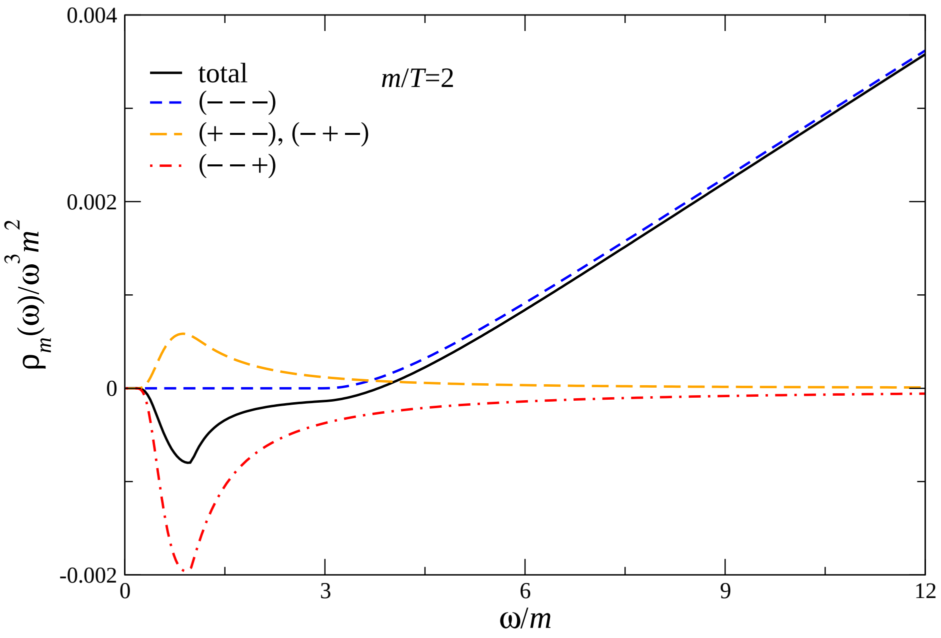
with
$$d\Phi_{123} = \prod_{j=1}^{3} \frac{d^3k_j}{(2\pi)^3 2\omega_{\mathbf{k}_j}}(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)$$

$$[\text{stat.}] = n_F(s_1\omega_{\mathbf{k}_1})n_F(s_2\omega_{\mathbf{k}_3})n_F(s_3\omega_{\mathbf{k}_3}) + n_F(-s_1\omega_{\mathbf{k}_1})n_F(-s_2\omega_{\mathbf{k}_3})n_F(-s_3\omega_{\mathbf{k}_3})$$

Free spectral functions



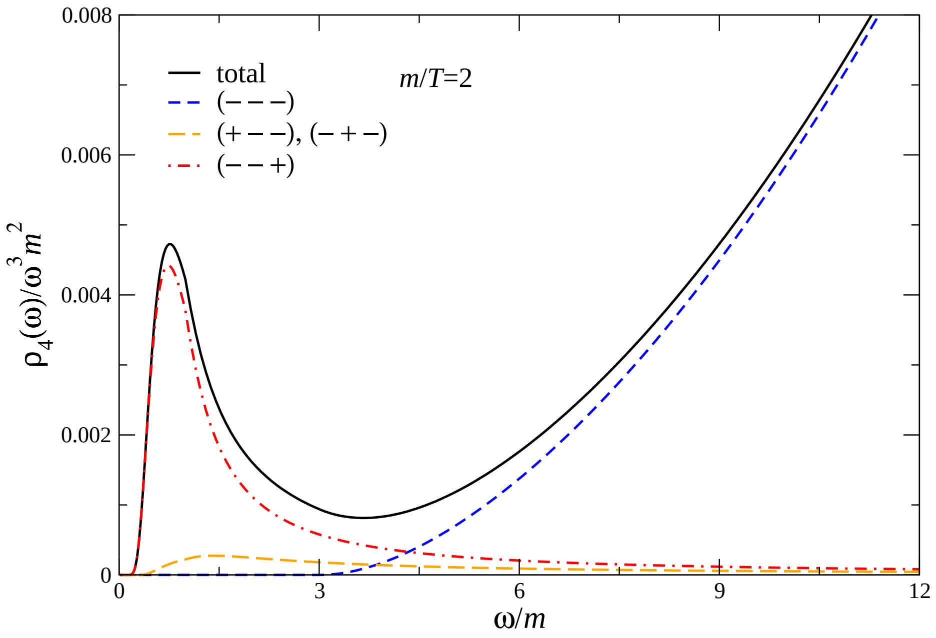
$\rho_4(\omega)$



$\rho_m(\omega)$

- decay: $\omega > 3m$ with m quark mass
- at $T > 0$ scattering contributions for all ω
- large ω: thermal contributions suppressed
- $\rho_m(\omega)$ not positive definite

Free spectral functions



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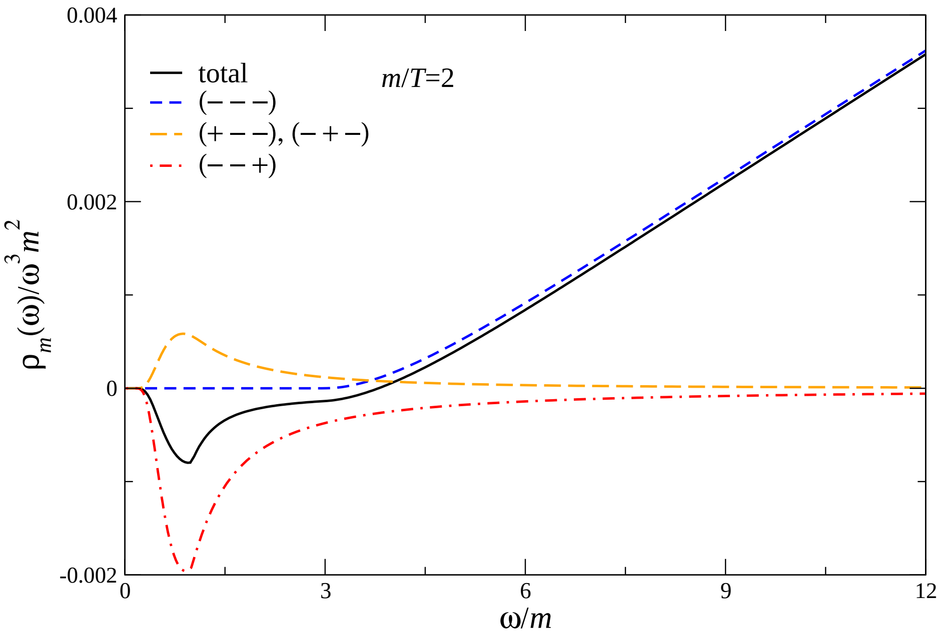

$\rho_m(\omega)$

$$\omega \gg T \gg m$$

$$\rho_4(\omega) = \frac{5\omega^5}{2048\pi^3}\left(1 + \frac{112\pi^4}{3}\frac{T^4}{\omega^4} + \ldots\right)$$

$$\rho_m(\omega) = \frac{7m\omega^4}{512\pi^3}\left(1 - 4\pi^2\frac{T^2}{\omega^2} + \ldots\right)$$

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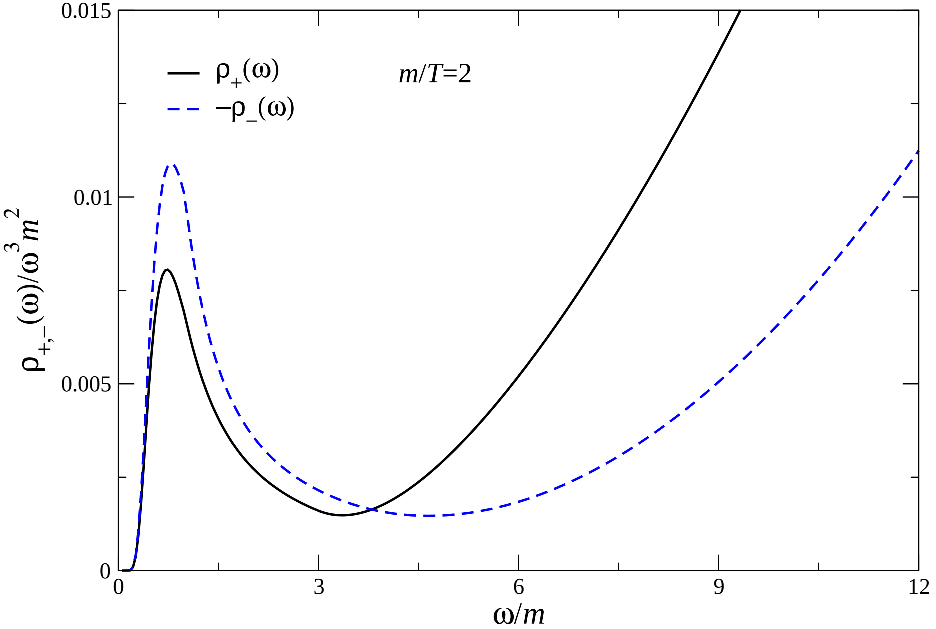
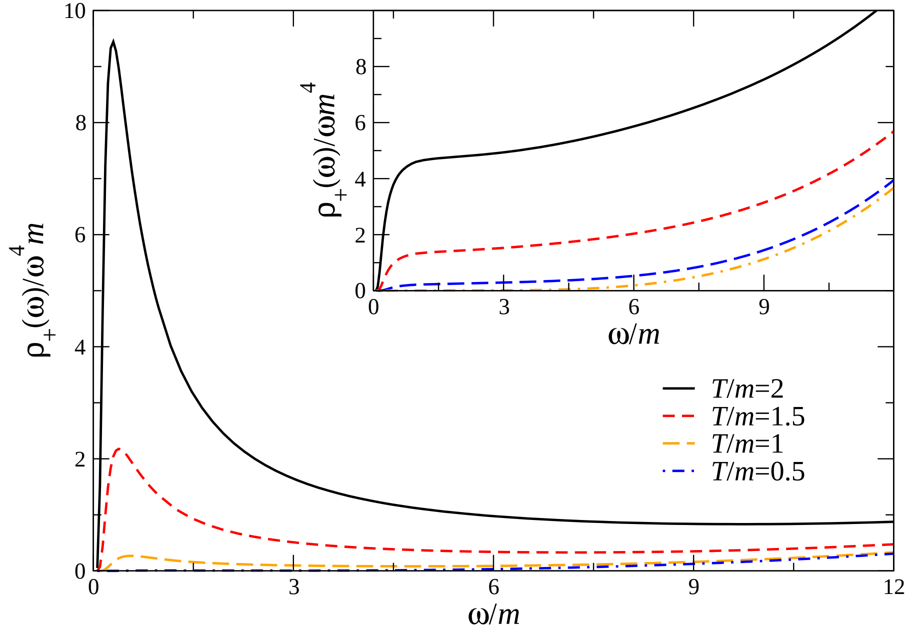




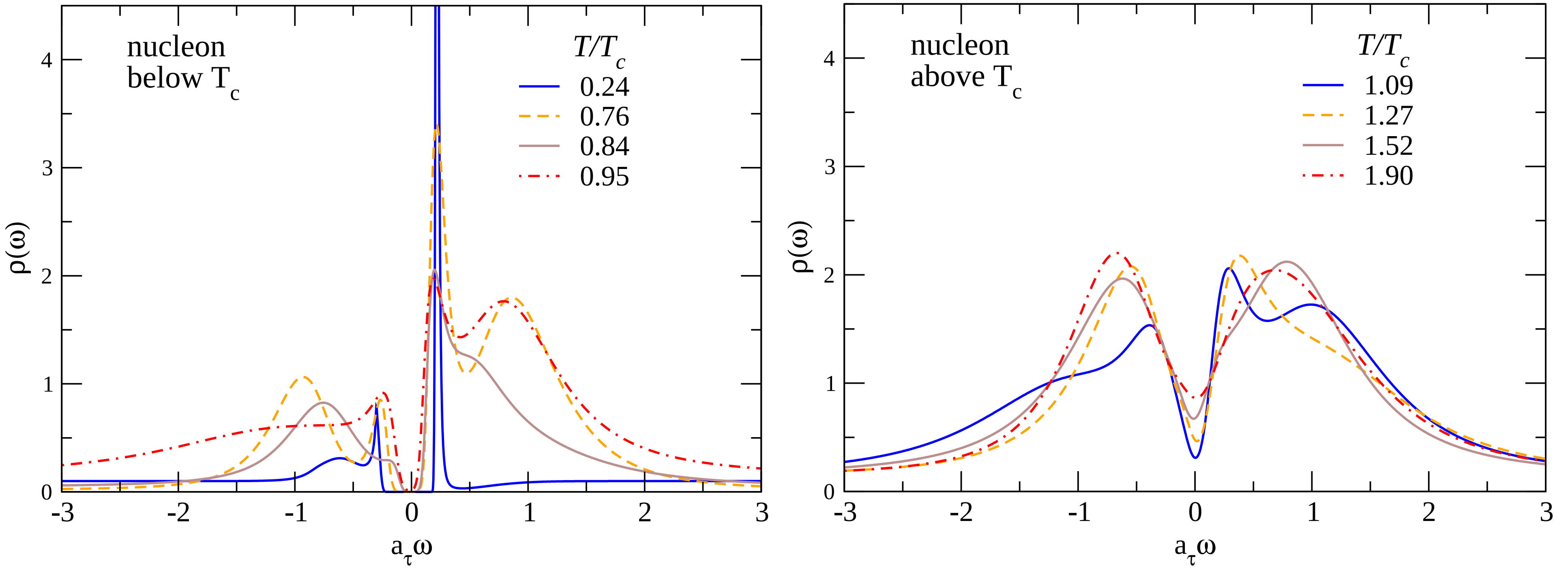
$$\rho_{\pm}(\omega) = \frac{1}{2}\left[\rho_m(\omega) \pm \rho_4(\omega)\right] \qquad \rho_+(\omega)$$

- thermal enhancement at $\omega \sim T \sim m$
- apparent peak depends on presentation/normalisation
- exponentially suppressed as $\omega \to 0$

$$\pm\rho_{\pm}(\omega) \geq 0 \qquad \rho_-(\omega) = -\rho_+(-\omega)$$

Nucleon spectral functions

- interacting case



- $\omega > 0 : \rho_+(\omega) \qquad \omega < 0 : \rho_-(\omega) = -\rho_+(-\omega)$
- groundstates at $T < T_c$
- above T_c: degeneracy in spectral functions emerges

Summary and outlook

- dynamics in QCD plasmas rich topic
- many – diverse – observables expressed in terms of spectral functions
- detailed studies of mesonic/vector current correlators

baryons at nonzero temperature

- underexplored area
- theoretically interesting
- phenomenologically relevant
- see Chris Allton's talk for more details