Can Neutron-Antineutron Oscillation Occur without CP and P Violation?

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Based on work with Kazuo Fujikawa

Baryons over antibaryons: the nuclear physics of Sakharov
ECT* Trento, 28 July 2016

- Why parity violation?

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Chang and Chang, 1980

"Note that the $n - \bar{n}$ oscillation is *maximally* parity violating. The possibility exists that the neutron oscillation is intimately related to CP violation, but we shall not discuss it further in this paper."

Recent work on CP violation on neutron-antineutron oscillation:

Berezhiani and Vainshtein, 2015
Fujikawa and Tureanu, 2015
McKeen and Nelson, 2015
Gardner and Yan, 2016

Outline

Effective $\Delta B = 2$ Lagrangian for the neutron

- Effective renormalized $\Delta B = 2$ quadratic Lagrangian:

$$\begin{aligned} \mathcal{L} &= \overline{n}(x) i\gamma^\mu \partial_\mu n(x) - m\overline{n}(x)n(x) \\ &- \frac{i}{2}\epsilon_1 [e^{i\alpha}\overline{n}^c(x)n(x) - e^{-i\alpha}\overline{n}(x)n^c(x)] \\ &- \frac{i}{2}\epsilon_5 [\overline{n}^c(x)\gamma_5 n(x) + \overline{n}(x)\gamma_5 n^c(x)], \end{aligned}$$

where n is Dirac field and C-conjugation is defined by

$$n^c(x) = C\overline{n}^T(x), \quad \text{with} \quad C = i\gamma^2\gamma^0$$

- real parameters m, ϵ_1, ϵ_5 and α

- Satisfies:

- Lorentz invariance and locality
- CPT invariance

- Majorana fermions, for example

$$\psi_\pm(x) = \frac{1}{2}[n(x) \pm n^c(x)], \quad \psi^c_\pm(x) = \pm\psi_\pm(x)$$

needed to diagonalize the Lagrangian

Parity in fermion number violating QFT

- Equivalence of parity operators under the transformation

$$P \to e^{i(aQ+bL+cB)} P$$

if electric charge (Q) and baryon (B) and lepton (L) number are conserved.

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"γ^0-parity" (conventional definition of parity for Dirac neutron)

$$n(t, \vec{x}) \to \gamma_0 n(t, -\vec{x}), \quad n^c(x) \to -\gamma^0 n^c(t, -\vec{x})$$

- $P^2 = 1$
- intrinsic parities of neutron and antineutron are ± 1
- Majorana fermions form a doublet under this transformation:

$$\psi_\pm(x) \to \gamma^0 \psi_\mp(t, -\vec{x})$$

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- ϵ_1-term odd: consequence of opposite intrinsic parities of n and n^c
- ϵ_5-term even

"$i\gamma^0$-parity" (definition of parity for Majorana particles)

$$n(x) \to i\gamma^0 n(t, -\vec{x}), \quad n^c(x) \to i\gamma^0 n^c(t, -\vec{x})$$

- $P^2 = -1$
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- ϵ_5-term odd

BCS analogy and Bogoliubov transformations

- Minimal Lagrangian which leads to oscillation:

$$\mathcal{L} = \overline{n}(x) i\gamma^\mu \partial_\mu n(x) - m\overline{n}(x)n(x) - \frac{1}{2}\epsilon_1 [\overline{n}^c(x)n(x) + \overline{n}(x)n^c(x)]$$

- diagonalized by the Majorana fermions

$$\psi_\pm(x) = \frac{1}{2}[n(x) \pm n^c(x)],$$

which satisfy Dirac equations with *different masses*:

$$[i\gamma^\mu \partial_\mu - (m \pm \epsilon)]\psi_\pm(x) = 0$$

- C-invariant
- γ_0-parity odd, but $i\gamma_0$-parity even

CP invariant

Chang and Chang, PRL 1980

- Hamiltonian in terms of Dirac field $n(x)$ mode expansion:

$$H = \int \frac{d^3p}{(2\pi)^3\omega}\Big[\omega(a_p a_p^\dagger + b_p b_p^\dagger) - \frac{m\epsilon_1}{\omega}(b_p^\dagger a_p + a_p^\dagger b_p) + \frac{\epsilon_1}{2\omega} a_p \tau_2 \vec{\tau}\cdot\vec{p} a_{-p} + h.c. + \frac{\epsilon_1}{2\omega} b_p \tau_2 \vec{\tau}\cdot\vec{p} b_{-p} + h.c.\Big]$$

- second term indicates fermion-antifermion transitions
- last terms indicate pairings of fermion and antifermions (BCS analogy)
- diagonalization in terms of the Majorana fields modes:

$$i\sqrt{\frac{2\omega}{\Omega_1}}A_\pm = (a_p \mp b_p) - \frac{\epsilon_1}{2\omega^2}\vec{\tau}\cdot\vec{p}\tau_2(a^\dagger_{-p} \mp b^\dagger_{-p}),$$

$$\Omega_\pm = \sqrt{\vec{p}^2 + (m \pm \epsilon)^2}$$

- Physical (BCS) vacuum ($A_{\pm}|vac\rangle = 0$) satisfies:

$$2\omega^2 a_p|vac\rangle = \epsilon_1\tau_2\vec{\tau}\cdot\vec{p}a^{\dagger}_{-p}|vac\rangle, \quad 2\omega^2 b_p|vac\rangle = \epsilon_1\tau_2\vec{\tau}\cdot\vec{p}b^{\dagger}_{-p}|vac\rangle$$

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- Physical vacuum is a condensed state of pairs of neutrons and antineutrons
- Neutron created at $t = 0$ by $\bar{n}(\vec{x}, 0)|vac\rangle$ contains a small admixture of antineutron:

$$|n\rangle = [a_p^{\dagger} - (\epsilon_1 p^2/2m\omega^2) b_p^{\dagger}]|vac\rangle$$

- Transition probability

$$P_{n\bar{n}} = [\sin(\epsilon_1 mt/\omega)]^2 + (\epsilon_1^2 p^4/m^2\omega^4)\cos^2(\epsilon_1 mt/\omega)$$

(second term is due to the fermion pairing, but strongly suppressed and likely undetectable)

- Lagrangian with $\Delta B = 2$ which does not lead to (conventional) oscillation:

$$\begin{aligned}\mathcal{L} &= \overline{n}(x) i\gamma^\mu \partial_\mu n(x) - m\overline{n}(x)n(x) \\ &- \frac{i}{2}\epsilon_5[\overline{n^c}(x)\gamma_5 n(x) + \overline{n}(x)\gamma_5 n^c(x)],\end{aligned}$$

- diagonalized by *degenerate* ($M = \sqrt{m^2 + \epsilon_5^2}$) Majorana fields:

$$\psi_\pm = \frac{1}{2}e^{\pm i\phi\gamma_5}(n(x) - n^c(x)), \quad \sin 2\phi = \epsilon_5/\sqrt{m^2 + \epsilon_5^2}$$

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- Define new Dirac field with mass M by

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- related to original n, n^c by *Bogoliubov transformations*:

$$\begin{pmatrix} n(x) \\ n^{c}(x) \end{pmatrix} = \begin{pmatrix} \cos\phi N_{+}(x) - i\gamma_5 \sin\phi N^{c}_{+}(x) \\ \cos\phi N^{c}_{+}(x) - i\gamma_5 \sin\phi N_{+}(x) \end{pmatrix}$$

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consequently the symmetry properties of the Lagrangian
-$n(x)$ and $n^c(x)$ are not orthogonal

$$\langle T n^c(x) \bar{n}(y) \rangle = \int \frac{d^4 p}{(2\pi)^4} \frac{\gamma_5 \sin 2\phi M}{p^2 - M^2 + i\epsilon} e^{-ip(x-y)}, \qquad (1)$$

therefore decay of neutron to $\bar{p}$ or annihilation with ordinary matter may occur.

Review of parity properties of the $\Delta B = 2$ Lagrangians

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- $\epsilon_1, \epsilon_5 \neq 0$: γ_0-parity, $i\gamma_0$-parity broken, CP violated, oscillation occurs

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Conclusion

γ_0-parity breaking provides a criterion for neutron oscillation

Is there *intrinsic CP violation* in $\Delta B = 2$ Lagrangian and how can it be observed?

CP violation in $\Delta B = 2$ Lagrangian

$$\begin{aligned}\mathcal{L} &= \overline{n}(x) i\gamma^\mu \partial_\mu n(x) - m\overline{n}(x)n(x) \\ &- \frac{1}{2}\epsilon_1[\overline{n}^c(x)n(x) + \overline{n}(x)n^c(x)] \\ &- \frac{i}{2}\epsilon_5[\overline{n}^c(x)\gamma_5 n(x) + \overline{n}(x)\gamma_5 n^c(x)],\end{aligned}$$

- P violated, C invariant, CP violated
- Mass eigenfields

$$\frac{1}{2}\begin{pmatrix} n(x) + n^c(x) \\ n(x) - n^c(x) \end{pmatrix} = e^{-i\Theta\gamma_5} e^{-i\tau_3\gamma_5\bar{\theta}} \begin{pmatrix} \tilde{\psi}_+(x) \\ \tilde{\psi}_-(x) \end{pmatrix},$$

with $M_\pm = \sqrt{(m \pm \epsilon_1)^2 + \epsilon_5^2}$, $M_\pm e^{2i\theta_\pm\gamma_5} = m \pm \epsilon_1 \pm i\epsilon_5\gamma_5$
and $\Theta \equiv \frac{1}{2}(\theta_+ + \theta_-)$ and $\bar{\theta} \equiv \frac{1}{2}(\theta_+ - \theta_-)$.

- Note that $\Theta = 0 \leftrightarrow \epsilon_1\epsilon_5 = 0$

- Due to C-symmetry, $\tilde{\psi}_+$ are purely Majorana fields:

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- Apply usual CP transformations to n, n^c and $\tilde{\psi}_\pm$:

$$\begin{pmatrix} n(x) \\ n^c(x) \end{pmatrix} \to (-i\tau_2)\gamma^0 \begin{pmatrix} n(t, -\vec{x}) \\ n^c(t, -\vec{x}) \end{pmatrix}$$

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which cannot be compensated by any field redefinition $n \to e^{i\delta} n$

CP violation is intrinsic and cannot be eliminated

- Transfer CP violating chiral $U(1)$ phase to parity violating mass term:

$$im' \bar{n} \gamma_5 n$$

by $n(x) \to e^{-i\Theta\gamma_5} n(x)$ to the order $O(\Theta) = O(\epsilon_1 \epsilon_5 / 2m^2)$ which is very small

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- Analogy with topological θ-term in QCD
- Intrinsic CP violation may be observed as contribution to EDM of neutron
- CP violating phase *unobservable* in neutron oscillation

On observability of CP violation in neutron oscillation

- Observable CP violation $\equiv$ different transition probabilities:

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- Observable CP violation $\equiv$ different transition probabilities:

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- The two probabilities are related by C, CP and T transformations (if CPT is conserved)

Consequently, in spite of the Lagrangian being intrinsically CP-violating, this violation may not be observed in neutron oscillation due to other good symmetries, for example C

What happens if C and CP are both violated?

$$
\begin{aligned}
\mathcal{L} &= \overline{n}(x) i\gamma^\mu \partial_\mu n(x) - m\overline{n}(x)n(x) \\
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- Lagrangian violates P, C, CP
- perform *canonical* Bogoliubov transformation

$$\begin{pmatrix} n(x) \\ n^c(x) \end{pmatrix} = \begin{pmatrix} \cos\phi N_+(x) - i\gamma_5 \sin\phi N_-(x) \\ \cos\phi N_-(x) - i\gamma_5 \sin\phi N_+(x) \end{pmatrix}.$$

to diagonalize the Lagrangian in terms of N_+, except ϵ_1 term

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- Lagrangian violates P, C, CP
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$$\begin{pmatrix} n(x) \\ n^c(x) \end{pmatrix} = \begin{pmatrix} \cos\phi N_+(x) - i\gamma_5 \sin\phi N_-(x) \\ \cos\phi N_-(x) - i\gamma_5 \sin\phi N_+(x) \end{pmatrix}.$$

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- perform $N_+ \to e^{-i(\pi/4+\alpha/2)}N_+ \to$ manifest C invariance!

Lagrangian is transformed to

$$\begin{aligned}\mathcal{L} &= \overline{N}_+(x) i\gamma^\mu \partial_\mu N_+(x) - M\overline{N}_+(x)N_+(x) \\ &- im'\overline{N}_+(x)\gamma_5 N_+(x) \\ &- \frac{1}{2}\epsilon_1(1 + O(\epsilon_5/m)^2)\left(\overline{N}^c_+(x)N_+(x) + h.c.\right)\end{aligned}$$

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- CP violation contributes to EDM of neutron, but not observable in oscillation

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- *parity violating mass term* originating from other sources
- if transformed by chiral rotation to a ϵ_5-type of term, then $\epsilon_5 \sim \epsilon_1$
- if transformed back to parity violating mass, then $m'' \sim \epsilon_1^2$
- thus, a considerable effect (m') may be transformed into a negligible one ($m'' \sim \epsilon_1^2$)

- A trial to eliminate CP violation altogether in

McKeen and Nelson, 2015

$$\begin{aligned}\mathcal{L} &= \overline{n}(x) i\gamma^\mu \partial_\mu n(x) - \overline{n}(x)(m_n P_L + m_n^* P_R) n(x) \\ &- \frac{1}{2}[\overline{n}^c(x)(\delta_1 P_L + \delta_2^* P_R) n(x) + \overline{n}(x)(\delta_2 P_L + \delta_1^* P_R) n^c(x)]\end{aligned}$$

which is CP invariant if m_n, δ_1 and δ_2 are all real
- use chiral transformation, phase transformation and finally

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Conclusions

- Neutron-antineutron oscillation may occur without CP violation

see also Fujikawa and Tureanu, 2015
McKeen and Nelson, 2015

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- BCS analogy and Bogoliubov quasiparticles appear naturally in $\Delta B = 2$ Lagrangian
- canonical Bogoliubov transformation is essential for a reliable analysis of CP and other discrete symmetries
- even if the Lagrangian is intrinsically CP violating, the effect is not seen in oscillation, but could contribute to neutron EDM