Two-colour QCD at $T > 0$ in the presence of a strong magnetic field

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with collaborators

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QCD-TNT III Workshop, ECT*, Trento, September 2 - 6, 2013

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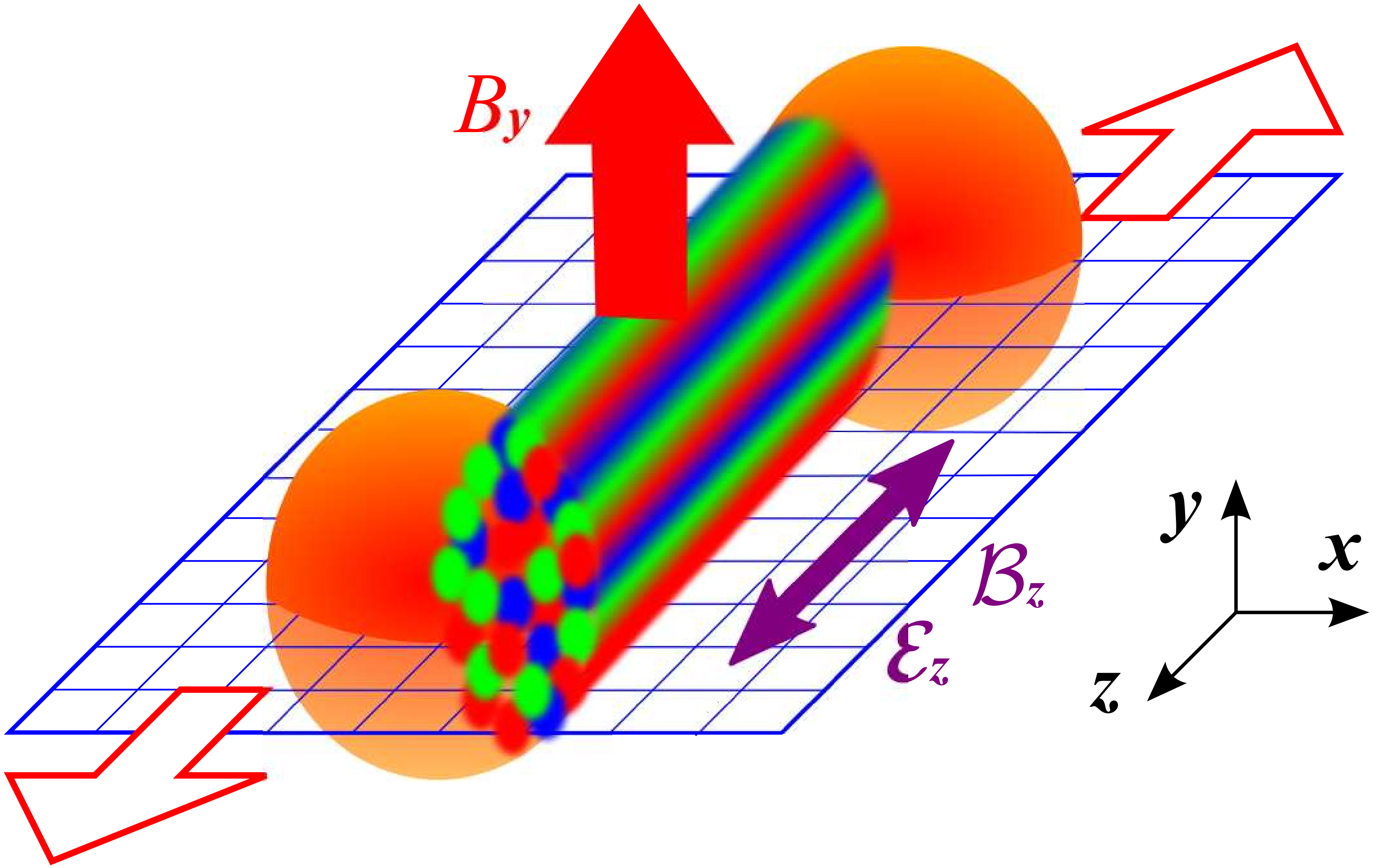
1. Introduction

Very strong magnetic fields in quark-gluon matter may exist (or have existed)

- **during the electroweak phase transition ($\sqrt{eB} \sim 1 - 2$ GeV),**
- **in the interior of dense neutron stars (magnetons) ($\sqrt{eB} \sim 1$ MeV),**
- **in noncentral heavy ion collisions at RHIC ($\sqrt{eB} \sim 100$ MeV) and LHC ($\sqrt{eB} \sim 500$ MeV), because antiparallel currents of the spectators create a strong magnetic field.**

Non-central heavy ion collision **Kharzeev, McLerran, Warringa, '08**



Such strong magnetic fields may lead to

- **a strengthening of the chiral symmetry breaking (increase of the chiral condensate, increase of F_π, decrease of M_π),**
- **a change of the finite temperature chiral transition both in temperature and in strength,**
- **the chiral magnetic effect (CME), leading to an event by event charge asymmetry in peripheral heavy ion collisions.**

Chiral model at $T = 0$ (Shushpanov, Smilga, '97)

$$< \bar{\psi}\psi >_B = < \bar{\psi}\psi >_0 \left(1 + \frac{1}{F_\pi^2} \frac{(eB)^2}{96\pi^2 M_\pi^2} + \mathcal{O}\left(\frac{(eB)^4}{F_\pi^4 M_\pi^4}\right)\right)$$

In the chiral limit, $M_\pi \ll \sqrt{eB} \ll 2\pi F_\pi \sim \Lambda_{hadr}$**:**

from J. Schwinger's ('51) solution

$$< \bar{\psi}\psi >_B = < \bar{\psi}\psi >_0 \left(1 + \frac{1}{F_\pi^2} \frac{(eB)\log 2}{16\pi^2} + \mathcal{O}\left(\frac{(eB)^2}{F_\pi^4}, \frac{(eB)^2}{\Lambda_{hadr}^4}\right)\right)$$

$$M_{\pi^0}(B) = M_{\pi^0}(0)\left(1 - \frac{1}{F_\pi^2} \frac{(eB)\log 2}{16\pi^2} + \dots\right)$$

$$F_\pi(B) = F_\pi(0)\left(1 + \frac{1}{F_\pi^2} \frac{(eB)\log 2}{8\pi^2} + \dots\right)$$

$$M_{\pi^+}(B) = M_{\pi^-}(B) \propto \sqrt{eB}$$

Strong fields $\sqrt{eB} \gg F_\pi, M_\pi, \Lambda_{hadr}$ **or deconfined phase** $(T > T_c)$

$< \bar{\psi}\psi >_B \sim |eB|^{3/2}$ $\Longrightarrow$ **e**B **the only scale**

Dyson-Schwinger equations suggest a selfconsistent quark mass:

(Shushpanov, Smilga, '97)

$$m_q(B) \sim \sqrt{|eB|} \exp\left[-\sqrt{\pi/(\alpha_s c_F)}\right]$$

$$< \bar{\psi}\psi >_B \sim |eB|^{3/2} \exp\left[-\frac{\pi}{2}\sqrt{\pi/(2\alpha_s c_F)}\right]$$

where $\alpha_s \equiv \alpha_s(|eB|)$**.**

Talk by P. Watson: argued for $(eB)^2$ **behavior in the chiral limit.**

2. The lattice model

Pioneering calculations (quenched SU(2)): Buividovich, Lushchevskaya, Polikarpov†,...

Full QCD:

Bonati, D'Elia, Negro, ... Bali, Bruckmann, Endrodi, Fodor, Kovacs, Schäfer, ...

see also review by M. D'Elia, arXiv:1209-0375.

Our simplified quark-gluon matter:

- colour $SU(2)$,
- staggered fermions without rooting of fermionic determinant, i.e $N_f = 4$ flavours,
- unique e.-m. charge.

Why this model?

- Very similar chiral behavior as in $SU(3)$ colour.
- Can be extended to finite baryon chemical potential without sign problem.
- Topology (important also for the CME) can be studied in a simpler case.
- Faster to simulate. Use a farm of PC's with GPU's.

Lattice gauge action: standard Wilson plaquette action

$$U_{n,\mu\nu} \equiv U_{n,\mu}\, U_{n+\hat{\mu},\nu}\, U^{\dagger}_{n+\hat{\nu},\mu}\, U^{\dagger}_{n,\nu}\,, \qquad U_{n,\mu} \in SU(N_c), \quad N_c = 2$$

$$\begin{aligned} S_G^W &= \beta \sum_{n,\mu<\nu} \left(1 - \frac{1}{N_c}\,\mathrm{Re\ tr}\ U_{n,\mu\nu}\right), \qquad \beta = \frac{2N_c}{g_0^2} \\ &= \frac{1}{2}\sum_n a^4\ \mathrm{tr}\ G^{\mu\nu}G_{\mu\nu} + O(a^2). \end{aligned}$$

Staggered fermion action: (Kogut, Susskind, '75)

- **Use naive discretization and diagonalize action w.r.t. to spinor degrees of freedom.**
- **Neglect three of four degenerate Dirac components.**
- **Attribute the 16 fermionic degrees of freedom localized around one elementary hypercube to four *tastes* with four Dirac indices each.**

Chirally symmetric Lagrangian $\Longleftrightarrow$ flavor symmetry broken.

Naturally describes the mass-degenerated four-flavor case.

Path integral quantization for Euclidean time $\Longrightarrow$ 'statistical averages'.

Partition function:

$$Z = \int [dU][d\psi]\,[d\bar{\psi}]\; e^{-S^G(U) \;+\; \bar{\psi}M(U)\psi}$$

$$= \int [dU]\; e^{-S^{eff}(U)}, \qquad S^{eff}(U) = S^G(U) - \log(\mathrm{Det} M(U))$$

with $M(U) \equiv D_{\mathrm{Latt}}(U) + m$**.**

Simulation on a finite lattice $N_t \times N_s^3$,

with (anti-) periodic boundary conditions for gluons (quarks).

Rooting prescription:

for $N_f = 2 + 1\ (+1)$ 4th-root of the fermionic determinant is taken.

$\Longrightarrow$ **Locality violated (??)**

$\Longrightarrow$ **We prefer not to apply rooting (Listening to M. Creutz)**

Study temperature dependence: $N_t \ll N_s$, $T \equiv 1/L_t = 1/(N_t\ a(\beta))$

Order parameters:

Polyakov loop: $L(\vec{x}) \equiv \frac{1}{N_c}\mathrm{tr} \prod_{x_4=1}^{N_t} U_4(\vec{x}, x_4)$, $\langle L(\vec{x})\rangle = \exp(-\beta F_Q)$,

F_Q = **free energy of an isolated infinitely heavy quark.**

$\Longrightarrow$ $F_Q \to \infty$, **i.e.** $\langle L(\vec{x})\rangle \to 0$ **within the confinement phase** $(T < T_c)$**.**

$\Longrightarrow$ $\langle L(\vec{x})\rangle$ **order parameter for the deconfinement transition** $(T = T_c)$**.**

Chiral condensate: $\langle\bar{\psi}\psi\rangle$

in the chiral limit order parameter for chiral symmetry breaking $(T < T_c)$

and restoration $(T > T_c)$

Find T_c **(or** β_c**) from maxima of susceptibilities of** $L(\vec{x})$ **and/or** $\bar{\psi}\psi$**.**

External constant magnetic field eB

$\bar{B} = (0, 0, B)$ $\quad\quad$ $\bar{A}(\bar{r}) = \frac{B}{2}(-y, x, 0)$

On the lattice we use the compact formulation. Constant magnetic field $\equiv$ constant magnetic flux $\phi = a^2(eB)$ through all (x, y) plaquettes.

On the links define $U(1)$ elements

coupled to quark fields in lattice covariant derivative.

$$V_x(\bar{r}, \tau) = e^{-i\phi y/2}$$
$$V_y(\bar{r}, \tau) = e^{i\phi x/2}$$
$$V_x(N_s, y, z, \tau) = e^{-i\phi(N_s+1)y/2}$$
$$V_y(x, N_s, z, \tau) = e^{i\phi(N_s+1)x/2}$$

Flux will be quantized: $\quad\quad$ $\phi = \frac{2\pi N_b}{N_S^2}$ $\quad\quad$ $N_b = 1, 2, \ldots$

DeGrand, Toussaint '80

3. Setup, estimating the physical scale:

Two steps for studying the temperature dependence:

(A) vary β at fixed N_t: only bare quantities in (varying) lattice units,

(B) vary N_t at fixed β: "fixed-scale approach", physical while avoiding renormalization.

Lattice sizes:

(A) $16^3 \times 6 \quad (24^3 \times 6)$, **(B)** $32^3 \times 10, 8, 6, 4$.

Setting the scale: $T = 0$ **calculations with** $16^3 \times 32$

Lattice scale unit $a(\beta)$ fixed from scale parameter r_0 [R. Sommer, '94], assumed to be the same as in real world (QCD):

Compute static $\overline{Q}Q$ force $F(r) = dV_{\overline{Q}Q}/dr$
and use data from phenomenological charmonium potential $V_{\overline{C}C}$:

$$F(r_0)\ r_0^2 \ \equiv\ 1.65 \quad \leftrightarrow \quad r_0 \simeq 0.5 \text{ fm} \qquad \Longrightarrow\ a\ [\text{fm}]$$

Then pseudo-scalar meson correlator $\Longrightarrow$ **pion mass** $m_\pi\ [MeV]$.

For $T = 0, \quad B = 0, \quad \beta = 1.80$ **we obtain**

$$am = 0.01 \implies a = 0.170(5)\ \mathrm{fm}, \quad m_\pi = 330(10)\ \mathrm{MeV},$$

$$am = 0.0025 \implies a = 0.168(4)\ \mathrm{fm}, \quad m_\pi = 175(4)\ \mathrm{MeV}.$$

For $T > 0$**:**

$$\beta = 1.80, \quad N_t = 6 \iff T = 193(6)\ \mathrm{MeV} \gtrsim T_c(B=0)\,,$$

$$N_t = 4 \iff T > T_c\,,$$

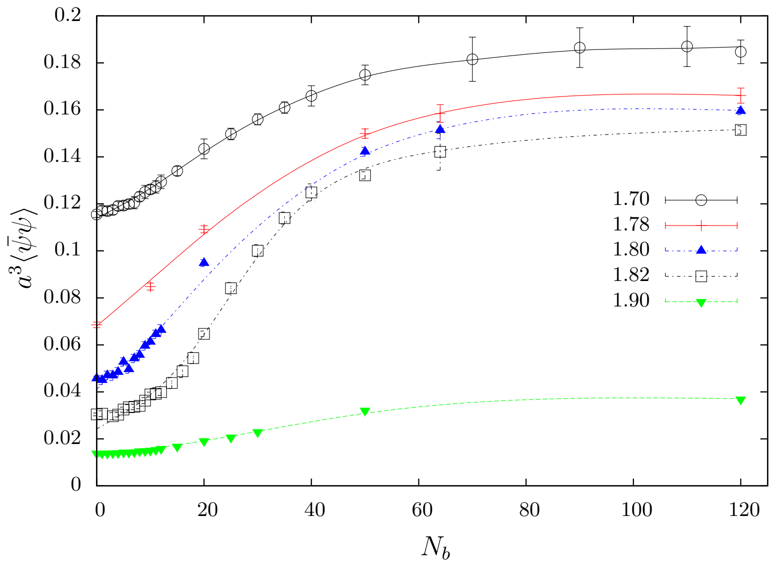
$$N_t = 8, 10 \iff T < T_c\,.$$

4. Polyakov loop and chiral condensate vs. T and eB

Ilgenfritz, Kalinowski, M.-P., Petersson, Schreiber, Phys.Rev. D85 (2012) 114504 [arXiv:1203.3360]

Technical remark: saturation behavior for various β (because V_μ periodic in ϕ)

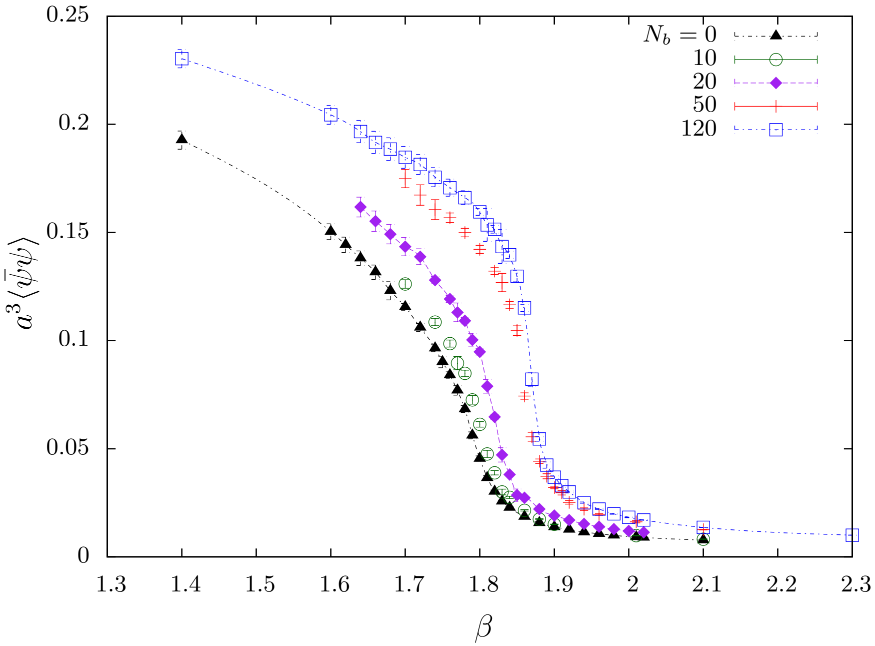
$16^3 \times 6, \quad am = 0.01.$



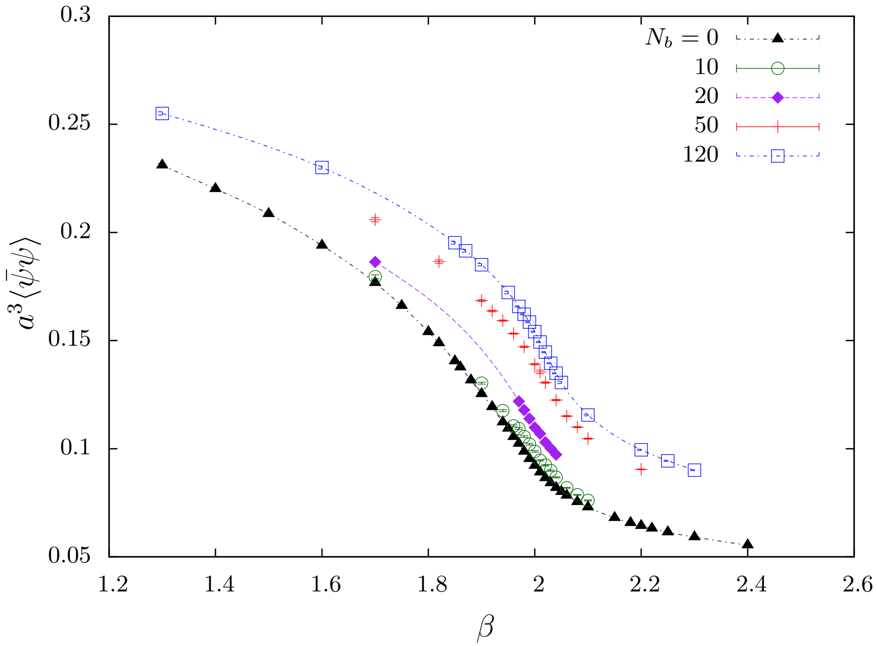
Bounds: $\phi \le \pi$ **or** $N_b \le N_s^2/2 \;=\; 128$**,** $\qquad \sqrt{2\pi}\frac{N_t}{N_s} \le \frac{\sqrt{eB}}{T} \le \sqrt{\pi}N_t.$

In practice: $N_b \le 50 \;\leftrightarrow\; eB \lesssim 1.69\ \mathrm{GeV}^2$ **for** $\beta = 1.80$**.**

β-dependence ($\equiv$ T dependence) of the bare chiral condensate



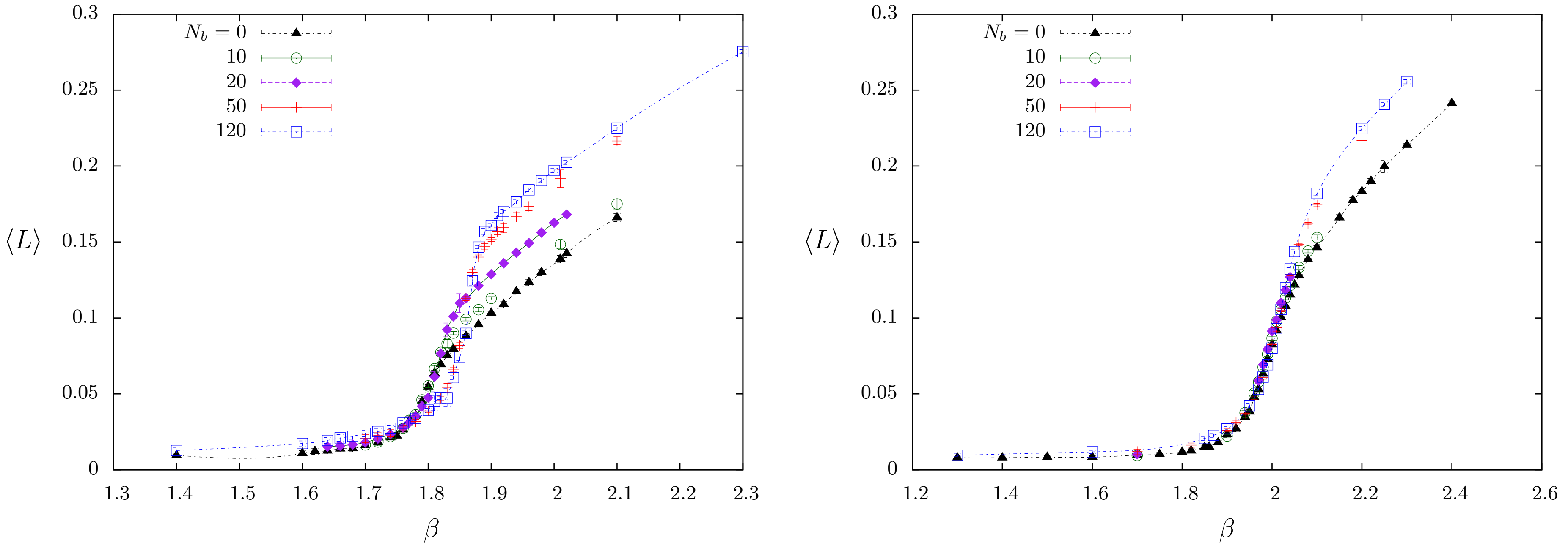
$ma = 0.01$



$ma = 0.1$

$< \bar{\psi}\psi >$ increases with B for all β $\Longrightarrow$ T_c increases monotonously (?)

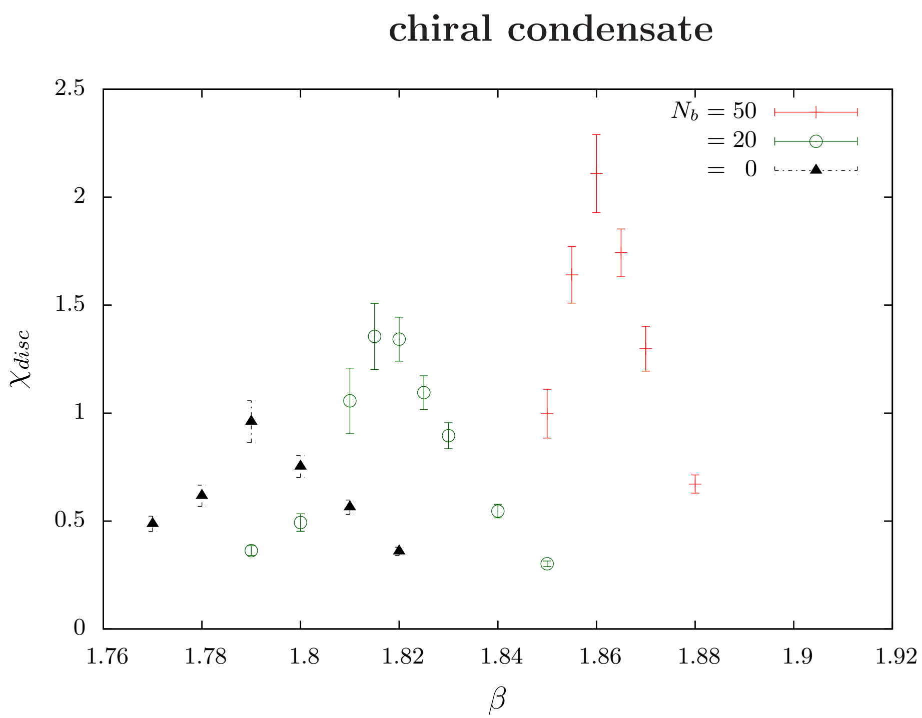
Bare Polyakov loop



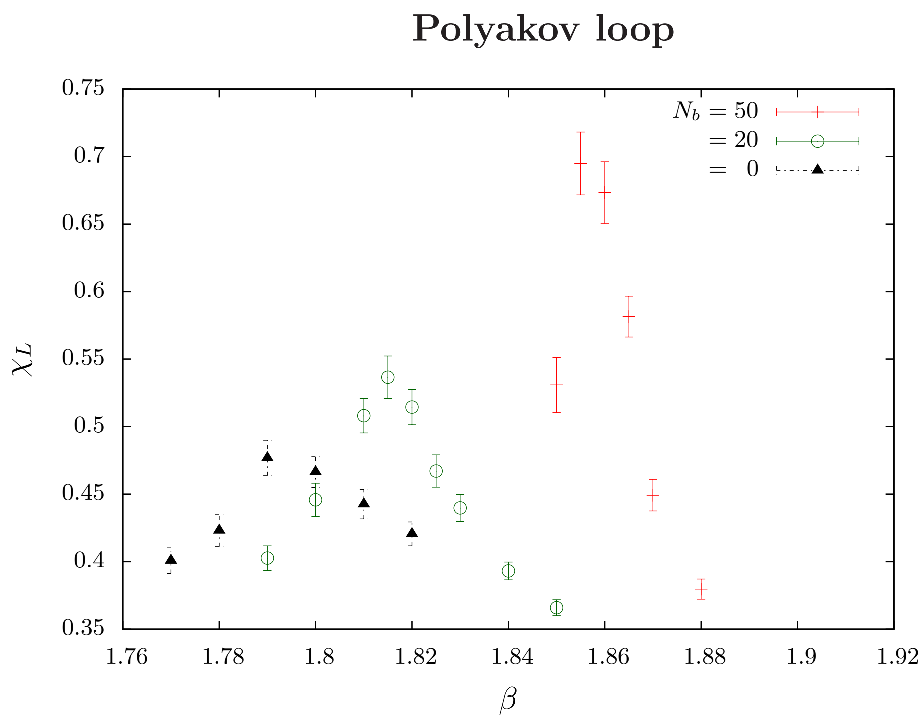
$ma = 0.01$ $ma = 0.1$

Polyakov loop may behave non-monotonously in transition region

Susceptibilities



$ma = 0.01$

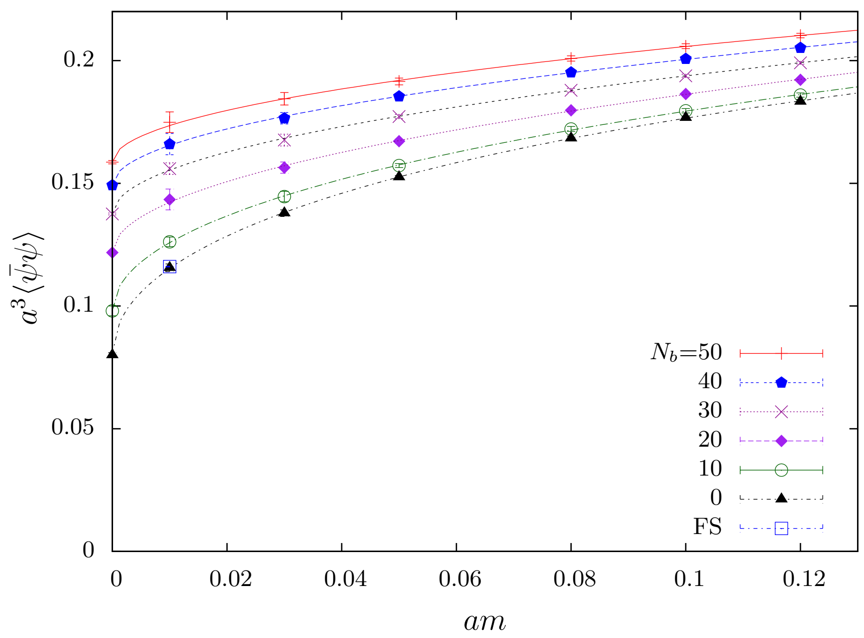


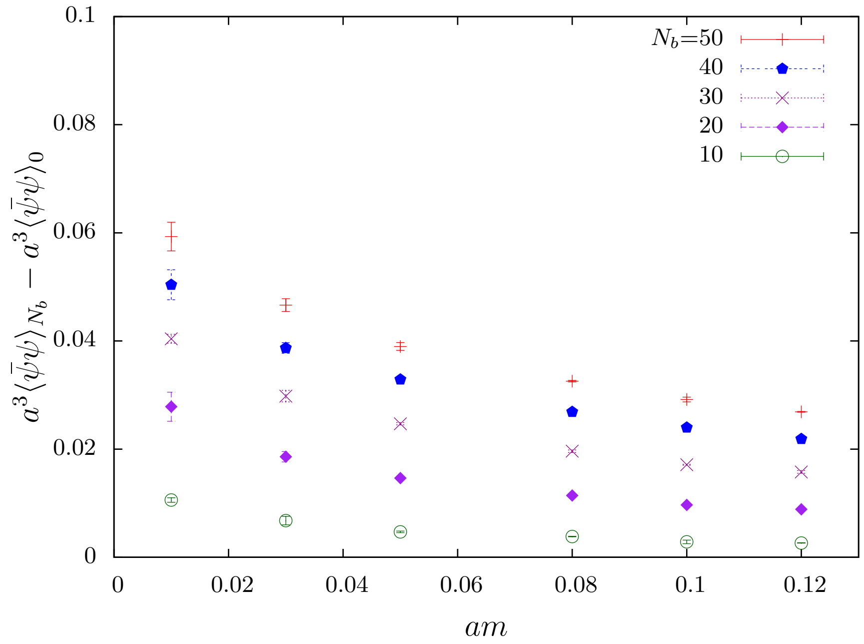
$ma = 0.01$

- $B \nearrow \;\Rightarrow\; T_c \nearrow$ **for** am **fixed, i.e. varying** $a(\beta)$. **Proper Renormalization ?**
- **Transition seen at the same** β_c (T_c) **for chiral condensate as for Polyakov loop.**

5. The chiral limit well below, above, and at the phase transition

Confined phase, $\beta = 1.7$



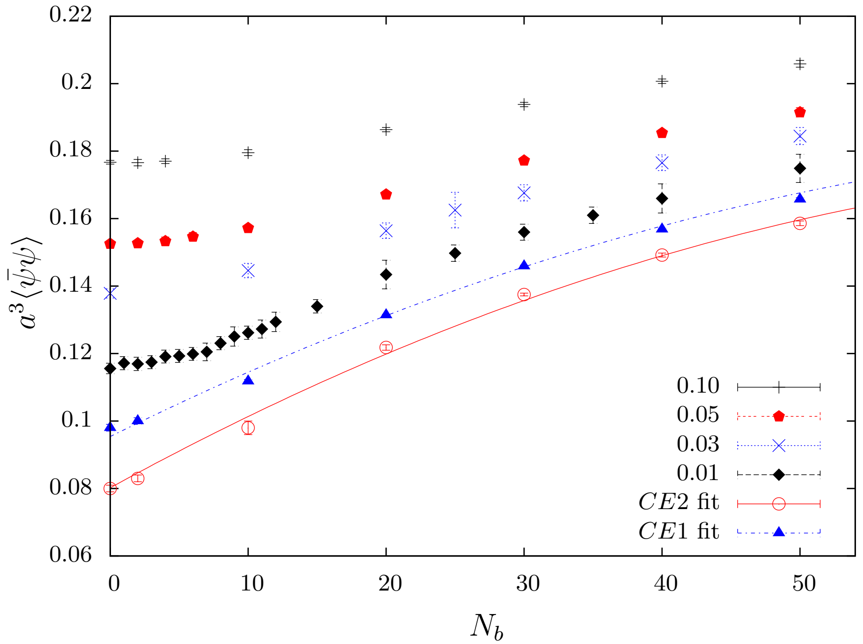


Fit: $a^3 < \bar\psi\psi >= a_0 + a_1\sqrt{ma} + a_2 ma$ **[A. Bazavov et al., '12]**

(Alternative $T = 0$ fit: $a^3 < \bar\psi\psi >= b_0 + b_1 ma \log ma + b_2 ma$**)**

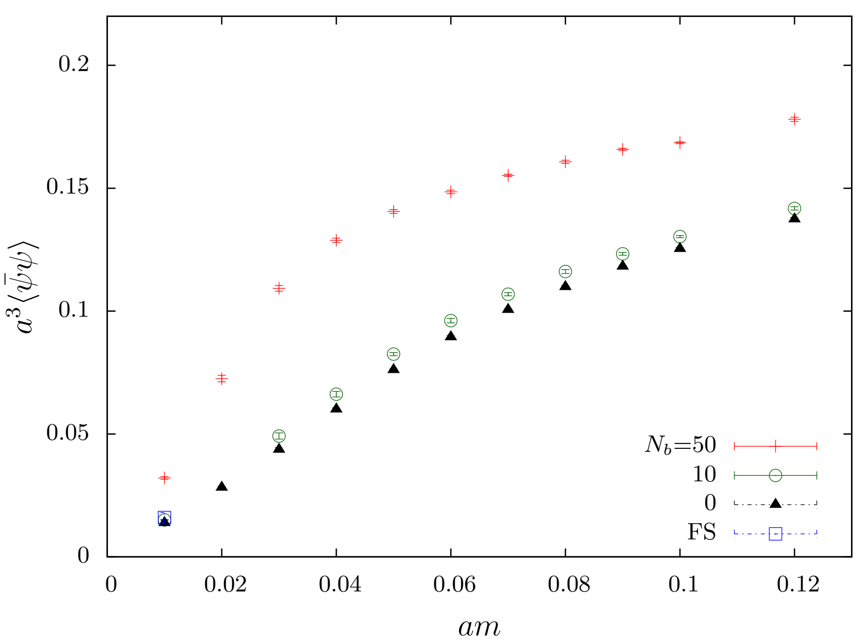
FS = check for finite-size effects with $24^3 \times 6$.

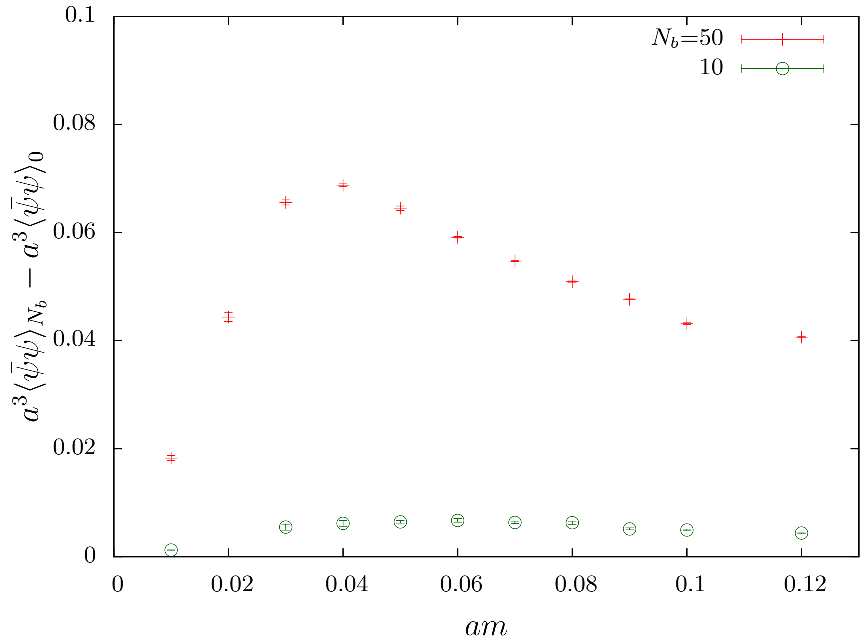
The chiral condensate as a function of the flux for various values of ma



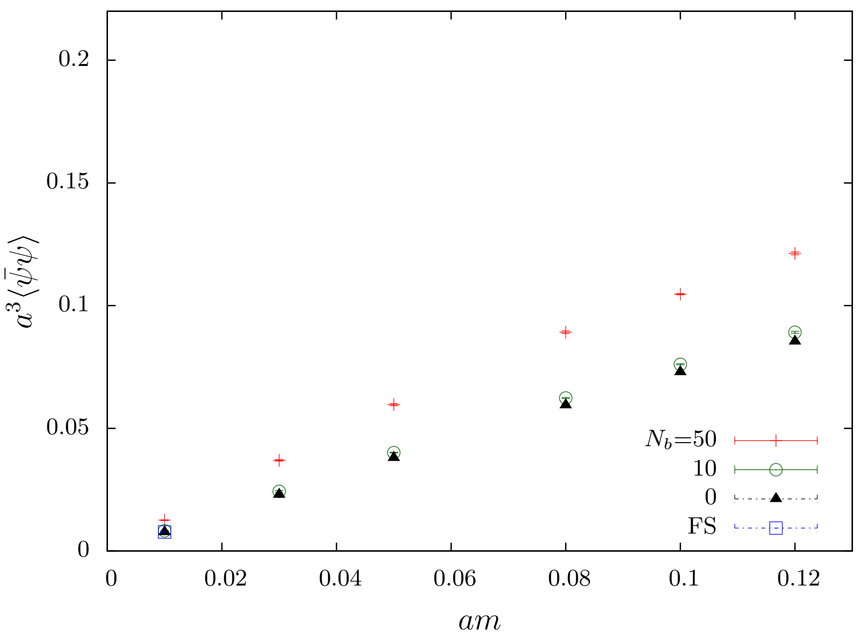
The slope at $ma = 0$ can be compared with chiral model $\Rightarrow F_\pi \approx 60\text{MeV}$

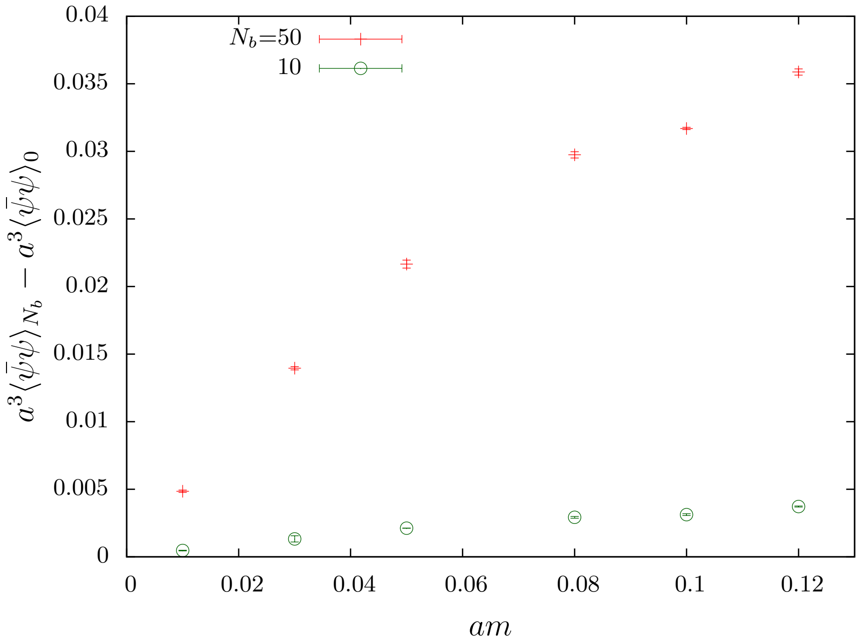
The chiral condensate, transition region, $\beta = 1.9$





The chiral condensate, deconfined phase, $\beta = 2.1$





6. Fixed-scale approach: inverse magnetic catalysis ?

Aim: at smallest available fixed mass
study the temperature and eB dependence at fixed lattice scale a.

Claim by Regensburg-Wuppertal group: "Inverse catalysis"
(see also Bruckmann, Endrödi, Kovacs, arXiv:1303.3972):

With rising eB:

- $\langle\bar{\psi}\psi\rangle$ $(\langle L\rangle)$ may decrease (increase),
- correspondingly T_c decreases.

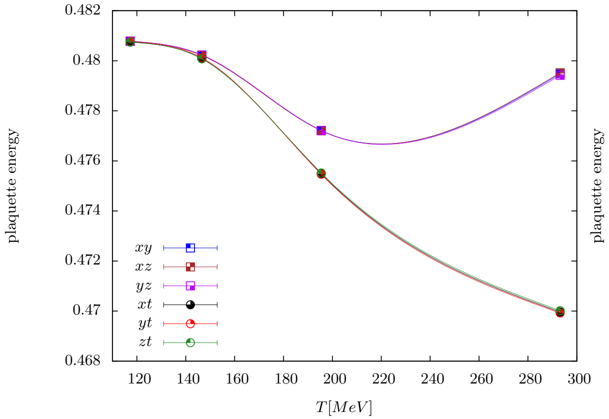
Explained with the opposite behavior of valence and sea quark contributions to the path integral average $\langle\bar{\psi}\psi\rangle$.

Our results:

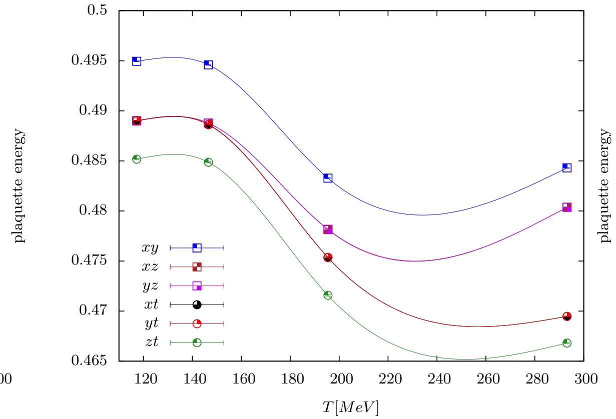
A. Schreiber, Master thesis, HU Berlin; Schreiber, M.-P., Petersson to be published.

Spatial anisotropy of plaquette energies vs. T

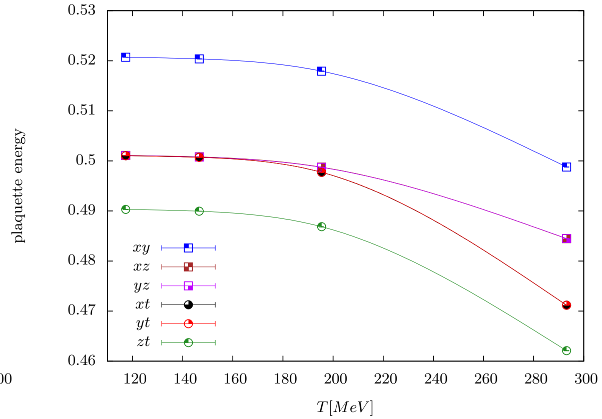
$eB = 0$



$eB = 0.67\ \mathrm{GeV}^2$

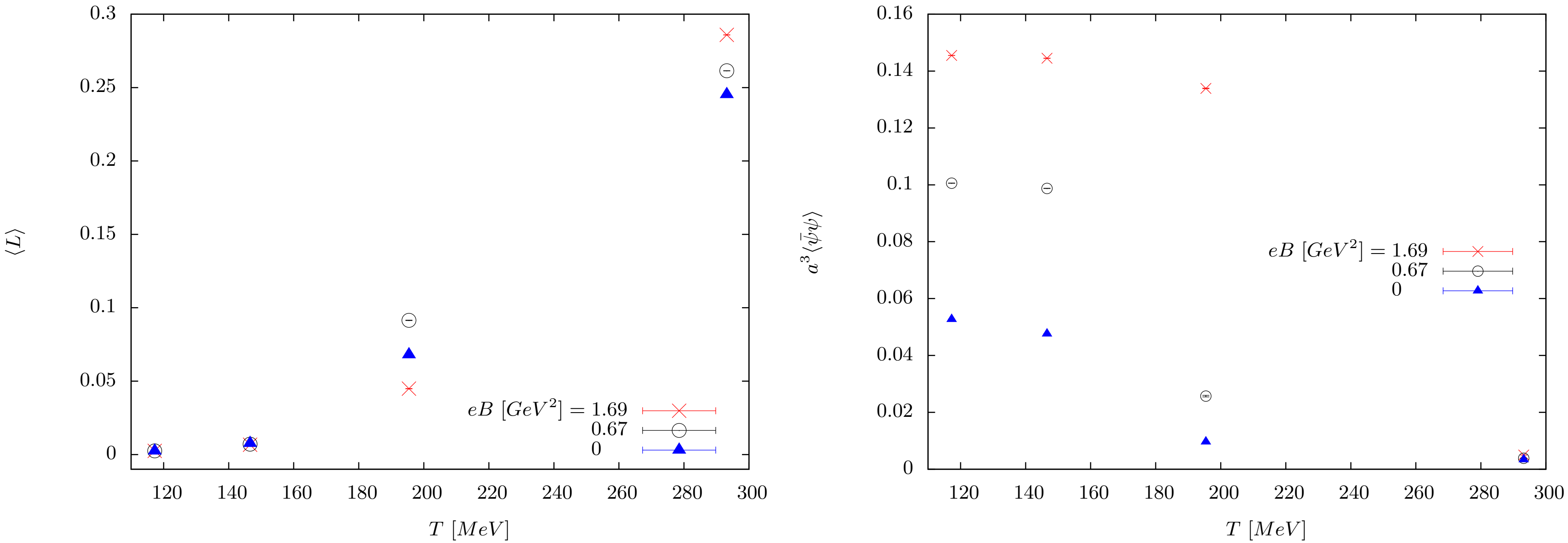


$eB = 1.69\ \mathrm{GeV}^2$



Polyakov loop and chiral condensate vs. T

$am = 0.0025, \quad 32^3, \quad \beta = 1.80$



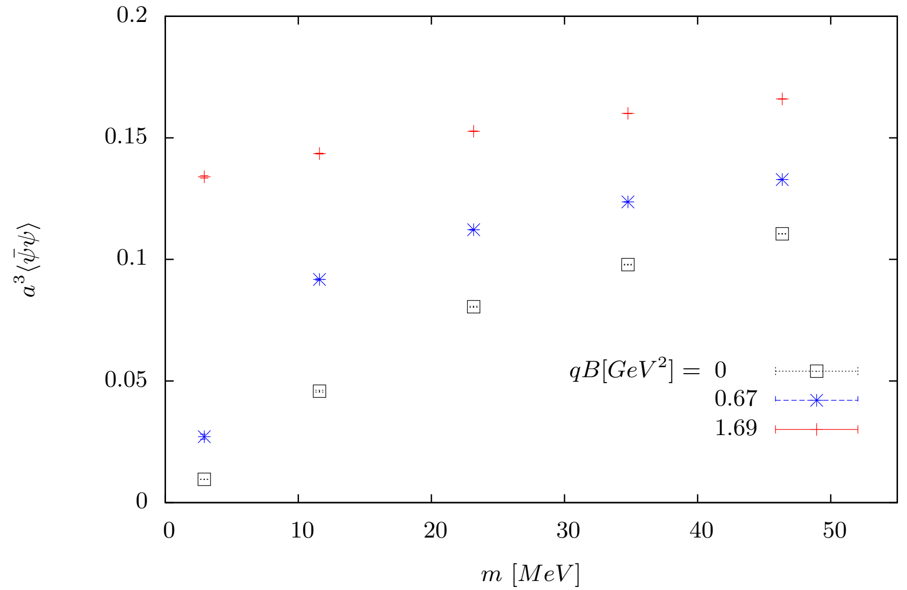
$\Longrightarrow$ $\langle L \rangle$ **rises with small** B **(i.e.** T_c **may decrease, inverse catalysis ?).**

$\Longrightarrow$ $\langle L \rangle$ **lowered with larger** B **(i.e.** T_c **increases, normal catalysis).**

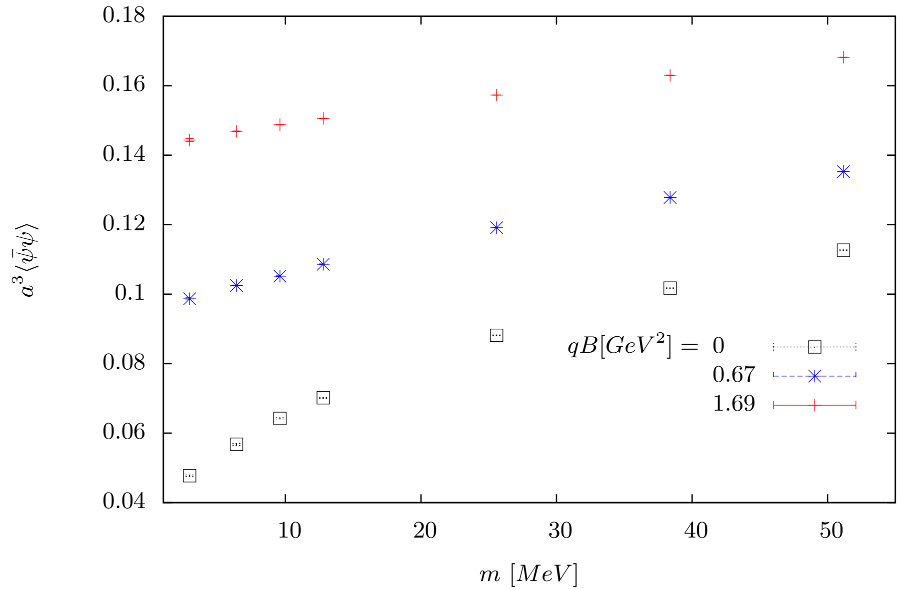
$\Longrightarrow$ $\langle \overline{\psi}\psi \rangle$ **rises monotonously with** B **at fixed (small) mass. Chiral limit?**

Chiral condensate vs. quark mass for various eB

$T = 193$ MeV $\gtrsim T_c(B=0)$



$T = 147$ MeV



$T = 193$ MeV: **at small B – chirally restored phase (T_c may decrease, inverse catalysis (?)),**

at larger B – chiral symmetry broken (i.e. T_c increases, normal catalysis).

$T = 147$ MeV: **for all B – chirally broken phase.**

7. Conclusions

- We have investigated how a finite temperature system reacts to a constant external magnetic field, in two-colour QCD with equally charged fermions.

- In the confined phase the chiral condensate increases with the magnetic field strength as predicted by a chiral model, also semi-quantitative agreement.

- In the deconfined region the chiral condensate in the chiral limit goes to zero for all values of the magnetic field.

- **From fixed scale approach:**
  Non-monotonic behavior observed in the transition region
  at fixed $m_\pi \simeq 175 \mathrm{MeV}$**.**
  Weak magnetic field:
  Chiral condensate increases slightly.
  Polyakov loop rises $\Rightarrow$ T_c **might decrease (inverse catalysis ?).**
  Strong magnetic field:
  Chiral condensate rises strongly.
  Polyakov loop lowers $\Rightarrow$ T_c **increases again.**
- **Other observables like magnetization, electric conductivity, ...**
  still under discussion.
- **Topology to be investigated: "Chiral magnetic effect".**
  Signatures for Kraan-van Baal calorons ?

Thank you for your attention

Magnetization

Spin contribution to magnetization

$$\mu(B,T) = -\frac{\langle \overline{\psi}\ \Sigma_{12}\ \psi \rangle_{B,T}}{\langle \overline{\psi}\psi \rangle_{B,T}}, \qquad \Sigma_{12} = \frac{1}{2i}[\gamma_1, \gamma_2]$$

computed for staggered fermions with noisy estimator technique.

Spin contribution μ to magnetization vs. T, eB

$am = 0.0025, \quad 32^3, \quad \beta = 1.80$