IR-conformal gauge theories and composite Higgs

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Over the last several years a lot of effort has been devoted to exploring the possible existence and structure of IR fixed points (FP) in gauge theories with varying fermion flavor content and color representation.

Apart from their intrinsic QFT interest, a major motivation for these studies is potential application to physics beyond the SM (e.g., walking TC ideas).

One should note that so far only a small subset of possible IR-conformal gauge theories, i.e. theories whose long distance behavior is governed by an IR FP, has been explored.

Recall: 2-loop PT beta function:

$$\beta(g) = -\left[b_0 \frac{g^3}{(4\pi)^2} + b_1 \frac{g^5}{(4\pi)^4}\right]$$

For N_f fermion flavors in representation R_F of simple color group G:

$$\begin{aligned} b_0 &= \frac{11}{3}C_2(G) - \frac{4}{3}\kappa T(R_F)N_f \\ b_1 &= \frac{34}{3}[C_2(G)]^2 - \kappa[\frac{20}{3}C_2(G) + 4C_2(R_F)]T(R_F)N_f \end{aligned}$$

with

$$t^a_R t^a_R = C_2(R)\mathbf{1}\,, \qquad t^a_{adj} t^a_{adj} \equiv C_2(G)\mathbf{1}\,, \qquad \mathrm{tr}[t^a_R t^b_R] = T(R)\delta^{ab}$$

and

$\kappa = 1(1/2)$ for 4-component (2-component) fermions

b_0 reverses sign at: $N_f^* = \frac{11}{4\kappa}\frac{C_2(G)}{T(R_f)}$. e.g. for $SU(N_c)$: $N_f^* = \frac{11}{2}N_c$

b_1 reverses sign at: $N_f^{**} = N_f^* \frac{34}{55}[1 + \frac{3}{5}C_2(R_F)/C_2(G)]^{-1} < N_f^*$

So for:

$$N_f^{**} < N_f < N_f^*$$

one has $b_0 > 0$, $b_1 < 0$, and $\beta(g)$ has a zero at

$$g^* = -(4\pi)^2 \frac{b_0}{b_1}$$

If $(N_f^* - N_f) << 1$ then this g^* is within PT validity regime, and exitence of FT can be trusted (Banks-Zaks IR FT).

Actual value (non-perturbative) of lower bound N_f^{**}, i.e. true extent of the “Conformal Window" (CW) an open issue in each case.

Outline

Existence of IR FT's and presence or absence of CSB in space of N_f, color group(s), bare coupling g, relevant parameters ...

1. Recent finding of CS restoration at strong coupling with many fermion flavors
2. Phase diagram in g, N_f, N_c, m_q and open issues
3. Application to composite Higgs models
4. Conclusions

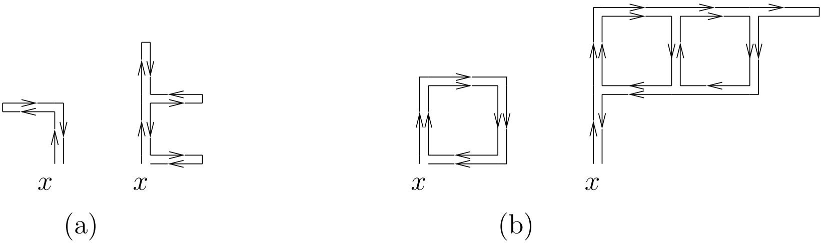
CS at strong coupling

It has been commonly assumed that, for any given number of fermion flavors, chiral symmetry will be eventually broken provided the coupling is taken strong enough.

It was recently found that this is in fact incorrect:

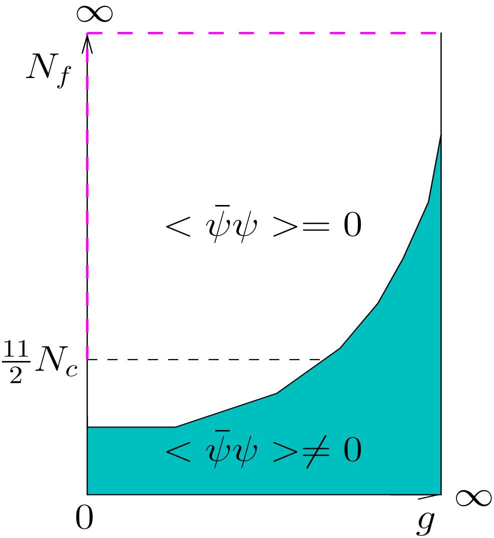
- MC simulations for $N_c = 3$ at $\beta = 0$ and small β show that chiral symmetry is restored via a first-order transition above a critical number of flavors ($\sim$ 52 continuum).
  Ph. de Forcrand, S. Kim and W. Unger, 10.1007 JHEP (2013) 051 [arXiv:1208.2148 [hep-lat]]
- The same result is arrived at by resummation of the hopping expansion in the strong coupling limit: the familiar CSB solution abruptly disappears above a critical N_f/N_c.
  TT, PRD 87, 034513 (2013) [arXiv:1211.4842 [hep-lat]]

At strong coupling limit ($\beta = 0$):



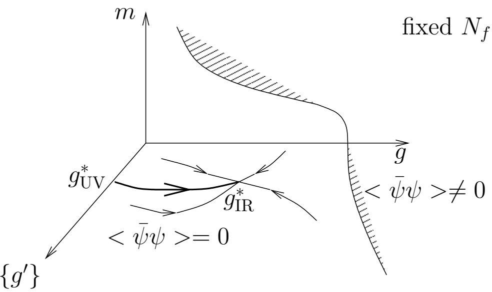
- At large N_c and fixed N_f: “tree" graphs (a) are dominant whereas “loop graphs" (b) are subdominant Resummation of all tree graphs and taking the limit $m \longrightarrow 0$ gives CSB solution for condensate. (Blairon, Brout, Englert, Greensite; Martin, Siu).
- At large N_f/N_c: “loop graphs" (b) are dominant and “tree" graphs (a) become subdominant. Resumation (now quite more involved) and $m \longrightarrow 0$ limit gives above result.

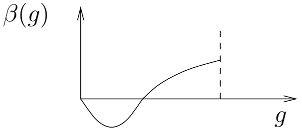
Phase diagram - Putting available info together:

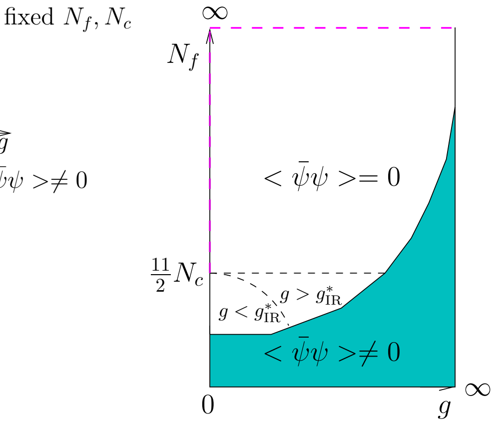


1. Weak coupling PT, T. Banks and A. Zaks, NP B196, 189 (1982).
2. Strong coupling: Ph. de Forcrand, S. Kim and W. Unger, 10.1007 JHEP (2013) 051 [arXiv:1208.2148 [hep-lat]];
TT, PRD 87, 034513 (2013) [arXiv:1211.4842 [hep-lat]].
3. Conformal window - extensive lattice investigations recent years (DelDebbio, LAT10, Neil, LAT11 reviews)
4. Conformal window - CSB first order phase transition with bare coupling increase:
P. H. Damgaard, U. M. Heller, A. Krasnitz and P. Olesen, Phys. Lett. B **400**, 169 (1997) [arXiv:hep-lat/97071008]; A. Cheng, A. Hasenfratz and D. Schaich, Phys. Rev. **D 85**, 094509 (2012) [arXiv:1111.2317 [hep-lat]]; A. Deuzeman, M. P. Lombarto, T. N. da Silva and E. Pallante, PL B720, 358 (2013) [arXiv:1209.5720 [hep-lat]].

Conformal Window RG flow:

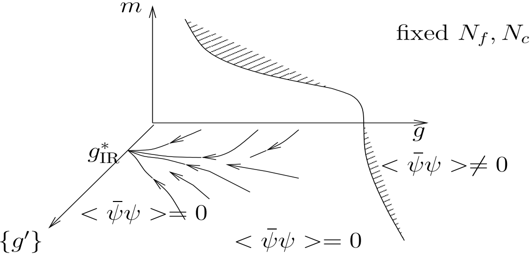


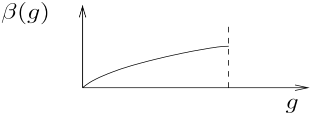


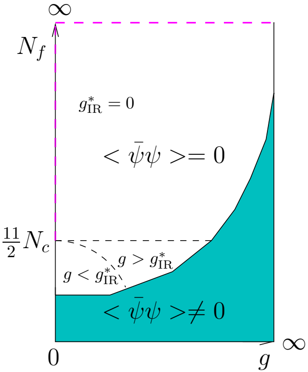


$N_f/N_c > 11/2$ ("above" CW)

Simplest possibility:





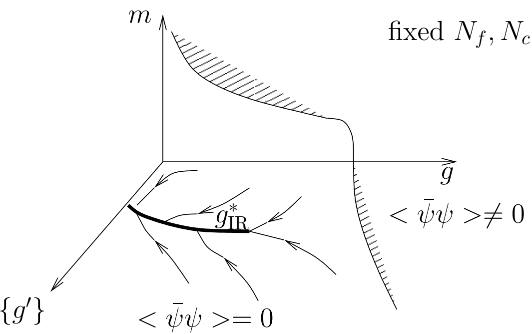


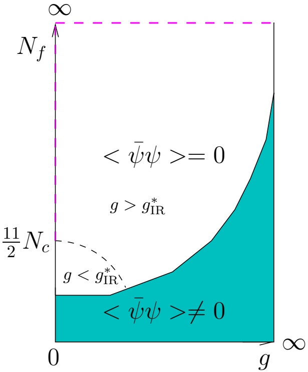
$N_f/N_c > 11/2$ ("above" putative CW)

There is, however, evidence for non-trivial IR fixed points also in this region:

- MC simulations (at strong coupling -
  de Forcrand, Kim, Unger, JHEP (2013) 051 [arXiv:1208.2148 [hep-lat]])
  1. effective coupling (defined from toleron correlator)
  2. Dirac spectrum
  3. mass spectrum
- Computation of beta functions in the large N_f expansion of $SU(N), U(1)$ gauge theories in the continuum:
  Distinct branches of beta function (belonging to different regions of the 'tHooft coupling) obtained.
  (B. Holdom, PL B694, 74 (2010), review. Cf. pure susy $SU(N)$)

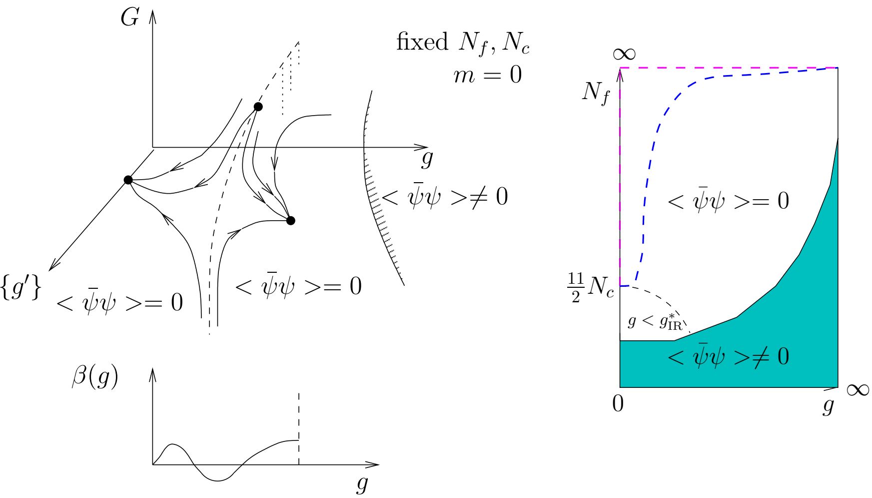
dFKU proposal:



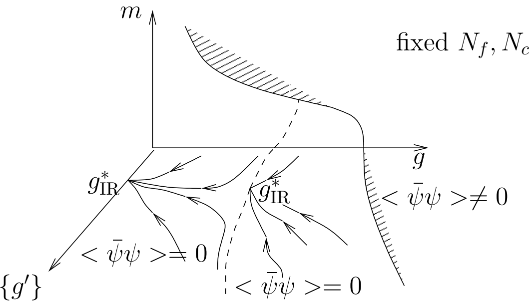


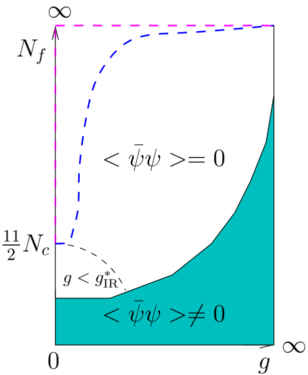
This, however, appears to contradict weak coupling PT.

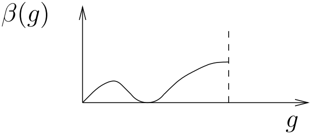
Another proposal: UV FT at strong coupling with additional relevant direction: χS 4-fermi interaction $G\,(\bar\psi\gamma_\mu\psi)^2$ (cf. massless QED_4).



As N_f becomes sufficiently large, UV and second IR FT merge:







Alternative possibility: picture above (no extra relevant direction) at all $N_f/N_c > 11/2$ with infinite order (?) line.

Wider possibilities

Note: All cases investigated so far form a small subset of possible IR-conformal gauge theories.

In particular, so far only simple color groups considered.

If color group semi-simple: $SU(N_1) \times SU(N_2) \times \cdots \times SU(N_n)$, there are n gauge couplings.
Coupled set

$$dg_i/d\mu = \beta_i(\{g_j\}), \qquad i = 1, 2, \cdots n$$

of gauge coupling evolution equations open new possibilities for IR fixed points, in particular at weak couplings, in N_i, N_f space.

Expected behavior in regime governed by IR FP g^*:

- Relevant parameters away from conformality: quark mass $\hat{m} = m/\mu = am$, or box size L, or coupling to other gauge interactions.
- Below a "locking" scale M_l (specific theory dependent) scaling regime obtains where physical mass ratios essentially constant. Hadron masses scale as : $M_{\rm H} \sim \mu \hat{m}^{1/(1+\gamma^*)}$ (Del Debbio, Zwicky, ...).
- The detailed ordering of mass spectrum is theory dependent but generally non-QCD like with 0^{++} states lowest and gluonic states below lowest meson (scalar, pseudoscalar, vector) mesons. (Miranski et al (around BZ FP); lattice spectrum computations: Del Debbio, LAT10, Neil, LAT 11 reviews, ...)

As m (or other relevant deformation parameter) $\longrightarrow 0$ spectrum collapses to "Unparticles".

But at small non-zero deformation: particle spectrum, with mass gap as above, containing a light scalar 0^{++} meson and a scalar gluball 0^{++} state plus the (somewhat heavier) rest of the meson/baryon and glueball spectrum.

This suggest the following application:
Assume theory with N_f, N_c such that IR FP present at weak coupling. At small deformation (e.g. finite box) one has spectrum of weakly coupled light scalar and other states of composites. Now couple other (weak) gauge interactions. This leads to a new setup for composite Higgs models (distinct from Higgs as NG boson or walking TC).

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Example

Consider $SU(N)$ (or $U(N)$) theory with N_f flavors such that system in CS phase.
Single out just two of these flavors $Q = (U, D)$ (to be later coupled to elw $SU(2) \times U(1)$).
There are then states formed by Q and the remaining fermion flavors ψ_a, $a = 1, \ldots, N_f - 2$, such as:

$$\bar{\psi}\psi, \quad \ldots, \qquad \bar{\psi}Q, \quad \ldots, \qquad \bar{Q}Q, \quad \ldots$$

Eliminate the “mixed" sector by taking semi-simple color gauge group. Take, e.g., $SU(N_1) \times SU(N_2)$ with ψ charged under both factors, and the Q charged under only one factor.

‘Mixed’ composites such as $\bar{\psi}Q$, ψQQ, ... no longer form. Only possible color singlet mixed states are highly unstable multi-quark (tetra and higher) states if they form at all.

Now consider coupling the elw interactions to the Q fermions.

One way of doing this is as follows:
The scalar meson $\bar{Q}Q$ gives rise to the four real fields $h_i, i = 0,1,2,3$:

$$h^+ = -\bar{D}U, \quad h^- = \bar{U}D, \quad h_3 = (\bar{U}U - \bar{D}D)/\sqrt{2}, \quad h_0 = (\bar{U}U + \bar{D}D)/\sqrt{2}$$

that may be taken to form the weak scalar doublet

$$H = \begin{pmatrix} h^+ \\ (h_0 + ih_3)/\sqrt{2} \end{pmatrix} \qquad \tilde{H} = i\tau_2 H^* = \begin{pmatrix} (h_0 - ih_3)/\sqrt{2} \\ h^- \end{pmatrix}$$

In addition one of course has the other meson, baryon states $\bar{Q}\gamma_5 Q$, $\bar{Q}\gamma_k Q$, ... as pseudoscalars, vectors, etc.

The glueball states are all weak singlets (completely dark). In particular one has the 0^{++} state, which, together with the h_0, are expected to be lightest states.

They may mix, but elw interactions only couple to fermionic component.

Effective theory at low energies by matching the composites to interpolating fields. Effective potential:

$$\lambda(H^\dagger H)^2 + \lambda_1(P^\dagger P)^2 + \lambda_2|P^\dagger H|^2 + \cdots \lambda_V|V_k^\dagger V_k|^2 \\ + \cdots + \lambda_d(\Psi^\dagger\Psi)^2 + \lambda'_d(\Psi^\dagger\Psi)(H^\dagger H) + \cdots \qquad (1)$$

Features

- With IR FP assumed at weak coupling, all effective couplings in this effective potential are weak.
- The coupling to the electroweak gauge fields renters this system of (nearly) massless scalar fields unstable under the CW mechanism. Only Higgs field H condenses (Parity and Lorentz symmetry assumed preserved) - usual elw breaking.
- Conformal dilaton mechanism still applicable in this context.
- Only interaction with dark sector through the “Higgs portal". Dark matter of WIMP variety (by also introducing gauge interactions in dark sector and/or Unparticle dark sector) plus light scalar (completely) dark particle.
- SM quark masses by usual extended TC: effective 4-fermi interactions between Q and SM quarks q at high scale $\longrightarrow$ effective Yukawa couplings at IR.
  Note: No strong condensate of Q quarks at some intermediate scale is involved!

Conclusions

- Much remains unknown concerning the existence of IR FP's, especially outside what has been commonly called the conformal window. In particular:
  - Pursuing the evidence for possible non-trivial IR FP's in the region above the $11N_c/2$.
  - Extension of search for IR FP's for semi-simple color groups (FP's in the space of more than one gauge coupling).
- In the context of such wider possibilities a large class of new composite Higgs models may become available where the Higgs is not a NG boson.

  They may help us understand how the SM emerges as an effective weakly coupled theory with something like the standard Higgs being not an elementary scalar field.