Glueball Masses from the Lattice

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Motivations

- The existence of gauge-invariant bound states of gluons implied by confinement
- However, glueball states have not yet been convincingly identified in experiments
- Glueball-like states can play a key role also in strongly interacting dynamics beyond the standard model
- A calculation from first principles using lattice techniques can serve as a guidance to theoretical models and experimental searches

Outline

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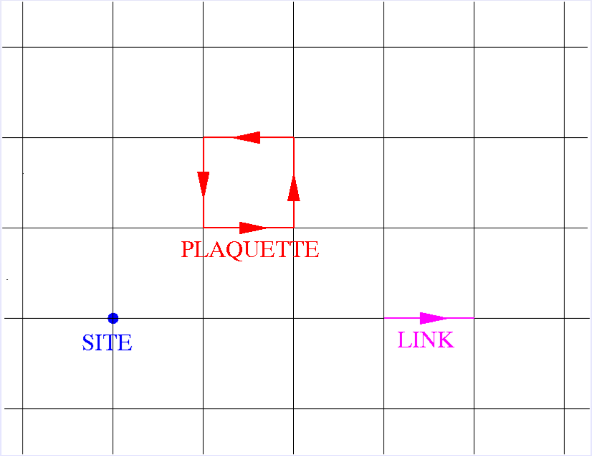
The Lattice



Lattice action for full QCD

Path integral

$$Z = \int \left(\mathcal{D}U_\mu(i)\right) \left(\det M(U_\mu)\right)^{N_f} e^{-S_g(U_{\mu\nu}(i))}$$

with

$$U_\mu(i) = Pexp\left(ig \int_i^{i+a\hat{\mu}} A_\mu(x) \mathrm{d}x\right)$$

and

$$U_{\mu\nu}(i) = U_\mu(i) U_\nu(i+\hat{\mu}) U^\dagger_\mu(i+\hat{\nu}) U^\dagger_\nu(i)$$

Gauge part

$$S_g = \beta \sum_{i,\mu} \left(1 - \frac{1}{N} \mathcal{R}\mathrm{e}\ \mathrm{Tr}(\mathrm{U}_{\mu\nu}(\mathrm{i}))\right) \quad , \quad with\ \beta = 2N/g_0^2$$

Masses from correlators

Trial operators $\Phi_1(t), \ldots, \Phi_n(t)$ with the quantum numbers of the state of interest

$$
\begin{aligned}
C_{ij}(t) &= \langle 0| \left(\Phi_i(0)\right)^\dagger \Phi_j(t)|0\rangle \\
&= \langle 0| \left(\Phi_i(0)\right)^\dagger e^{-Ht}\Phi_j(0)e^{Ht}|0\rangle \\
&= \sum_n \langle 0| \left(\Phi_i(0)\right)^\dagger |n\rangle\langle n|e^{-Ht}\Phi_j(0)e^{Ht}|0\rangle \\
&= \sum_n e^{-\Delta E_n t}\langle 0| \left(\Phi_i(0)\right)^\dagger |n\rangle\langle n|\Phi_j(0)|0\rangle \\
&= \sum_n c^*_{in} c_{jn} e^{-\Delta E_n t} = \delta_{ij} \sum_n |c_{in}|^2 e^{-a m_n t} \underset{t\to\infty}{\rightarrow} \delta_{ij}|c_{i1}|^2 e^{-a m_1 t}
\end{aligned}
$$

Variational principle

1. Find the eigenvector v that minimises

$$am_1(t_d) = -\frac{1}{t_d} \log \frac{v_i^* C_{ij}(t_d) v_j}{v_i^* C_{ij}(0) v_j}$$

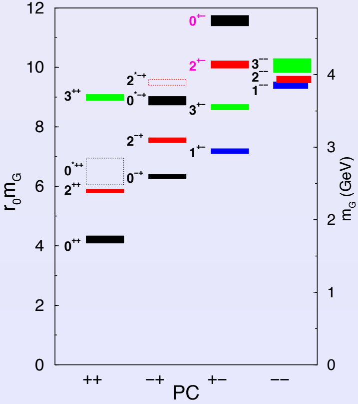
   for some t_d
2. Fit $v(t)$ with the law $Ae^{-m_1 t}$ to extract m_1
3. Find the complement to the space generated by $v(t)$
4. Repeat 1-3 to extract $m_2, \ldots, m_n$

Sources of systematics

- Need a good overlap of the eigenvectors with the state of interest
- Need a large variational basis including all possible states overlapping with the required one
- Need to keep under control finite size and lattice artefacts
- Care should be taken in assigning the spin

Outline

Glueballs in the quenched approximation



(From Morningstar and Peardon, hep-lat/9901004)

QCD at large N

Generalisation of QCD: SU(N) gauge theory (possibly enlarged with N_f fermions in the fundamental representation)

Taking the limit $g^2 \to 0$, $N \to \infty$, $\lambda = g^2 N$ fixed simplifies the theory and one can see that:

- Quark loop effects $\propto 1/N \Rightarrow$ The $N = \infty$ limit is quenched
- Mixing glueballs-mesons $\propto 1/\sqrt{N} \Rightarrow$ No mixing between glueballs and mesons at $N = \infty$
- Meson decay widths $\propto 1/N \Rightarrow$ mesons do not decay at $N = \infty$
- OZI rule $\propto 1/N \Rightarrow$ OZI rule exact at $N = \infty$

$\hookrightarrow$ The simpler large N phenomenology can explain features of QCD phenomenology in a *quenched* setup that removes most of the practical computational difficulties for QCD (and SU(3))

Large N limit on the lattice

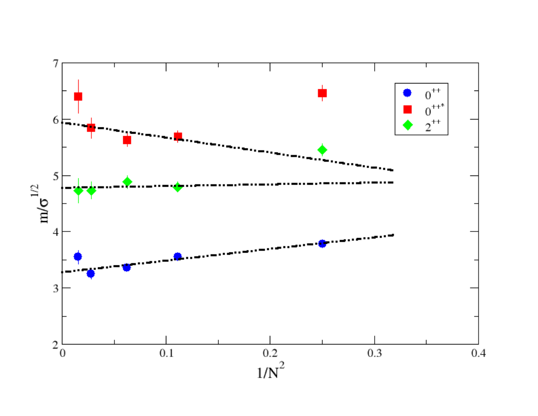
The lattice approach allows us to go beyond perturbative and diagrammatic arguments. For a given observable

1. Continuum extrapolation
   - Determine its value at fixed a and N
   - Extrapolate to the continuum limit
   - Extrapolate to $N \to \infty$ using a power series in $1/N^2$
2. Fixed lattice spacing
   - Choose a in such a way that its value in physical units is common to the various N
   - Determine the value of the observable for that a at any N
   - Extrapolate to $N \to \infty$ using a power series in $1/N^2$

Study performed for various observables both at zero and finite temperature for

$2 \leq N \leq 8$

Glueball masses at large N



[BL, Teper and Wenger, JHEP 0406 (2004) 012]

Masses at $N = \infty$

0^{++} $$\frac{m}{\sqrt{\sigma}} = 3.28(8) + \frac{2.1(1.1)}{N^2}$$

0^{++*} $$\frac{m}{\sqrt{\sigma}} = 5.93(17) - \frac{2.7(2.0)}{N^2}$$

2^{++} $$\frac{m}{\sqrt{\sigma}} = 4.78(14) + \frac{0.3(1.7)}{N^2}$$

Accurate $N = \infty$ value, normal $\mathcal{O}(1/N^2)$ correction

Glueball spectrum at $aT_c = 1/6$

Glueballs

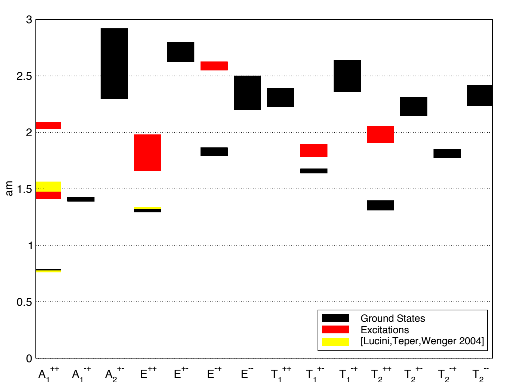
Biagio Lucini

Lattice setup

Configing theories

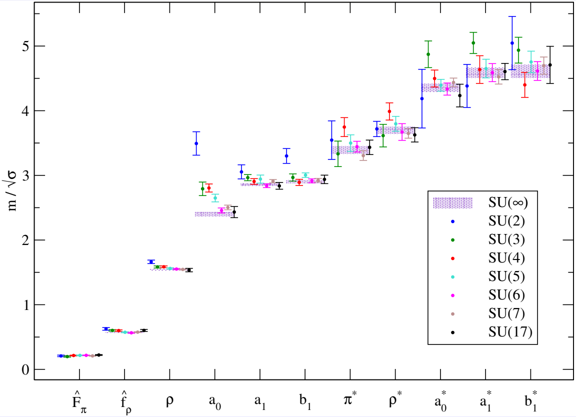
(Near-) conformal theories

Conclusions and outlook



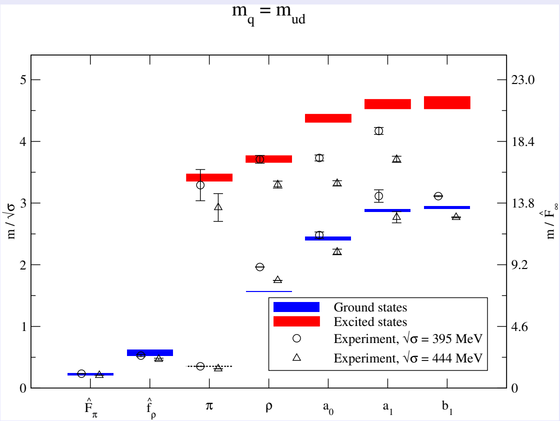
[BL, Rago and Rinaldi, JHEP 1008 (2010) 119]

Meson spectrum (at fixed a)



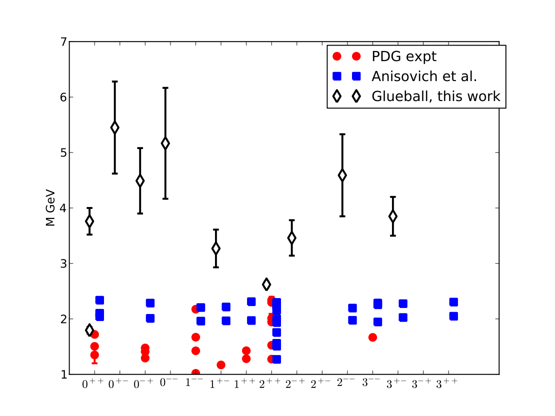
[Bali *et al.*, JHEP 06 (2013) 071]

Large N vs. experiments



[Bali *et al.*, JHEP 06 (2013) 071]

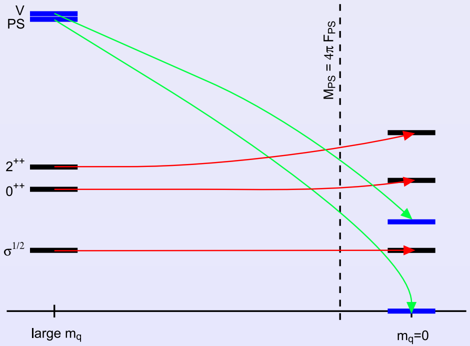
Back to QCD



[Gregory *et al.*, JHEP 1210 (2012) 170]

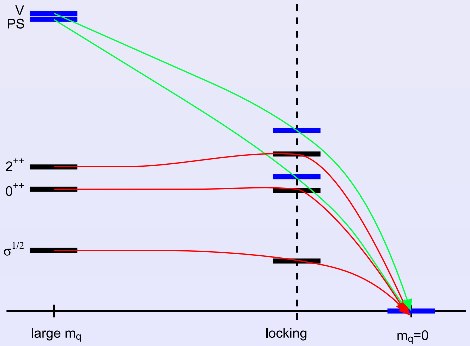
Outline

The spectrum for a QCD-like theory



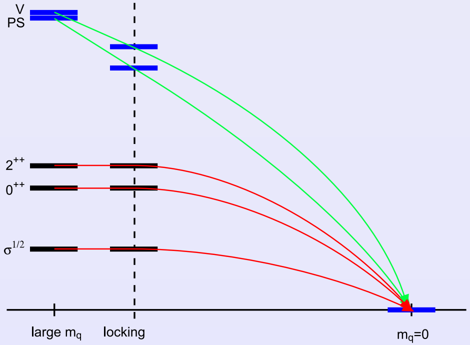
- At high fermion masses the theory is nearly-quenched
- At low fermion masses the relevant degrees of freedoms are the pseudoscalar mesons

Locking



All spectral mass ratios depend very mildly on m below the locking scale

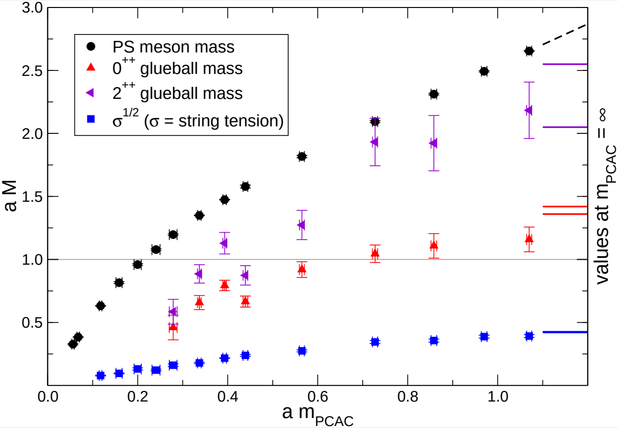
Locking near the Banks-Zaks point



If this scenario is valid beyond BZ, at all scales $\ll \Lambda$

$$m_V/m_{PS} \simeq 1+\epsilon$$
$$m_{PS} \gg \sigma^{1/2}$$
$$m_G/\sigma^{1/2} \simeq \left[m_G/\sigma^{1/2}\right]^{(YM)}$$

Spectrum in SU(2) $N_f = 2$ Adj



[Del Debbio, BL, Patella, Pica and Rago, Phys. Rev. D80 (2009) 074507]

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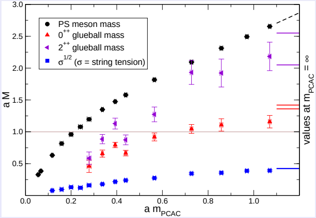

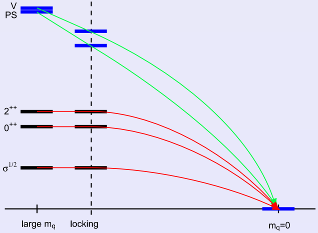

The pseudoscalar is always higher in mass than the 0^{++} glueball, as predicted by the locking scenario at high fermion mass

These theories naturally provide a light scalar $\Rightarrow$ is this a route to understanding the mechanism of electroweak symmetry breaking?

Outline

Conclusions and outlook

- Lattice calculations are an (increasingly more) useful tool to understand the fate of glueballs in QCD
  $\hookrightarrow$ Need to control better mixing with scattering and meson states
- Valuable information can be provided by lattice calculations in the large N limit
  $\hookrightarrow$ Need to take the continuum limit
- The dynamics of glueball is heavily influenced by the proximity to the conformal window
  $\hookrightarrow$ New classes of theories need to be explored in order to see if strongly interacting BSM dynamics is a viable mechanism of electroweak symmetry breaking