Three easy exercises in off-shell string-inspired methods (OSSIM)

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Exercise 1. How do we know that off-shell string-inspired methods are gauge-invariant?

- The easy answer: OSSIM is always done in the background field method (BFM) Feynman gauge, equivalent to the gauge-invariant Pinch Technique (PT)—nothing left to do.
- Nevertheless, modifying a remark of Feynman (made about QED):

> *It might be worthwhile to spend one's time expressing [the PT and OSSIM] in every physical and mathematical way possible.*

- So: We generalize OSSIM to arbitrary gauges and apply PT principles.
- The **intrinsic PT** may be related to **ambiguities** occurring in perturbative loops in all covariant gauges except the BFM Feynman gauge. These arise from **discontinuities** in the worldline current.

Exercise 2. Using Feynman-parameter technology directly

- Much of the motivation for OSSIM is to have a compact form for Feynman-parameter integrals, after momentum integrations are done.
- Simplicity and compactness are maximized with the string-inspired choice of Feynman parameters.
- Long ago general rules were given for giving the Feynman-parameter form of any Feynman graph with any momentum-dependent numerator, after momentum integrations, but never explored for non-Abelian gauge theory [JMC and Tiktopoulos, 1973].
- It turns out that, at least at one-loop level, these old methods—with the string-inspired Feynman parameters—are as simple and compact as OSSIM in the BFM Feynman gauge, and moreover easily accommodate gauge-dependent terms coming from outside this gauge.

Exercise 3: How is adjoint string breaking described?

- The adjoint string breaks; how is this described?
  1. For quarks, a well-known and straightforward extension of the old Schwinger calculation.
  2. More complicated for gluons: Non-perturbative at every stage; must begin with the concept of a gluon mass in order to describe the critical chromoelectric field, and ensure gauge invariance.
  3. All this amounts to a generalization of OSSIM to one-loop graphs of infinite order.

A quick review of the PT

- The PT is a method for combining Feynman graphs for off-shell Green's functions, such as the gluon propagator, with parts of other graphs (pinch parts), so as to construct a gauge-invariant Green's function.
- Papavassiliou and Binosi have shown that to all orders of perturbation theory the PT is equivalent to calculating off-shell Green's functions in the background field method (BFM) Feynman gauge.
- The original PT starts from S-matrix elements and identifies pinch parts (missing a key propagator) through Ward identities; the intrinsic PT starts from the off-shell Green's function and throws out parts that are identified with the original PT.

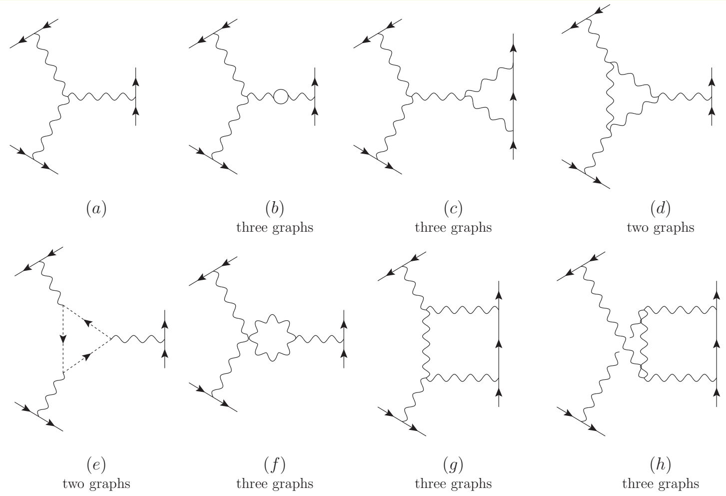

Figure: The original PT for the three-gluon vertex, using a six-quark S-matrix element. Graphs (c), (g), and (h) have pinch parts (missing the quark propagator) that contribute to this vertex.

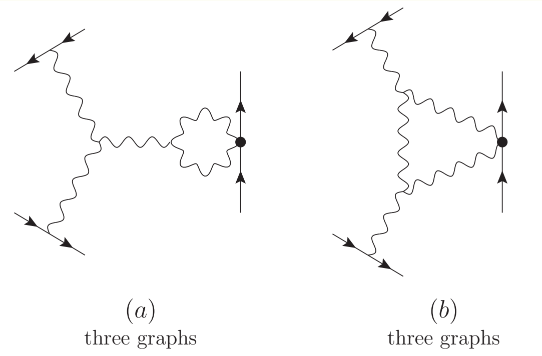

Figure: Pinch parts come from longitudinal gluon momenta triggering Ward identities that replace quark lines by ± 1. Note that these are contributions to the three-gluon vertex, and, interpreted as proper vertex parts, they have a factor of an inverse gluon propagator $\Delta^{-1}(p_1 + p_2)$.

Kinematic preliminaries and OSSIM

- Both the PT and OSSIM use polarization vectors for **off-shell** background gluons of momentum p , satisfying $\epsilon(p) \cdot p = 0$ even though $p^2 \neq 0$ (for example, any gluon in Fig. 1(a)). Example: A background gluon of momentum p couples to $\bar{u}(p+q)\gamma_\mu u(q) \equiv \epsilon_\mu(p;q)$ where the spinors are on-shell.
- After pinching, the same type of polarization vectors are generated for Figs. 1(c), (g), (h).
- We always work in Euclidean space (spacelike momenta) for the off-shell Green's functions; this is assured by using equal-mass Minkowski-space spinors.

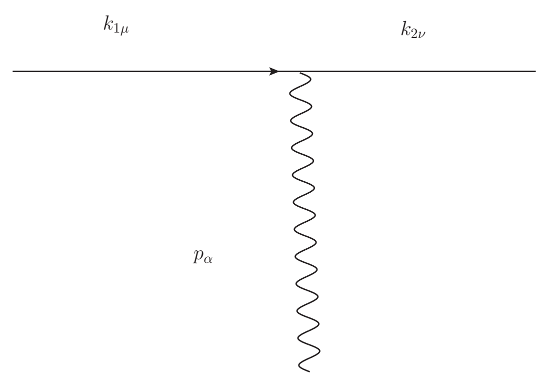

Figure: The basic three-gluon vertex. The solid lines are quantum gluons; the wiggly line is a background gluon. We choose the momenta on the quantum lines to go in the same direction, so that $p + k_1 = k_2$.

Return

The three-gluon bare vertex is the sum of a convective, a spin, and a pinch part

- The standard three-gluon vertex:

$$\Gamma_{\alpha\mu\nu}(p, k_1, k_2) = (k_1 + k_2)_\alpha \delta_{\mu\nu} - (k_2 + p)_\mu \delta_{\alpha\nu} + (p - k_1)_\nu \delta_{\alpha\mu}.$$

- Decompose it into convective, spin, and pinch terms:

$$\Gamma_{\alpha\mu\nu}(p, k_1, k_2) = \Gamma^C_{\mu\nu\alpha} + \Gamma^S_{\alpha\mu\nu} + \Gamma^P_{\alpha\mu\nu};$$

$$\Gamma^C_{\alpha\mu\nu} = (k_1 + k_2)_\alpha \delta_{\mu\nu},\ \Gamma^S_{\alpha\mu\nu} = -2p_\mu \delta_{\nu\alpha} + 2p_\nu \delta_{\mu\alpha};$$

$$\Gamma^P_{\alpha\mu\nu} = -k_{2\nu}\delta_{\mu\alpha} - k_{1\mu}\delta_{\nu\alpha}.$$

- Γ^P does all the pinching; the Feynman rules for the BFM Feynman gauge are to set $\Gamma^P = 0$ and $\Gamma \to \Gamma^F_{\alpha\mu\nu} \equiv \Gamma^C_{\alpha\mu\nu} + \Gamma^S_{\alpha\mu\nu}$.

Longitudinal gluon momenta trigger Ward or ST identities

- *Background gluon Ward identity:*

$$p_\alpha \Gamma^F_{\alpha\mu\nu}(p, k_1, k_2) = \Delta^{-1}_{\mu\nu}(k_2) - \Delta^{-1}_{\mu\nu}(k_1).$$

(Γ^S gives zero.)

- *Quantum gluon Slavnov-Taylor identity:*

$$k_{1\mu}\Gamma_{\alpha\mu\nu}(p, k_1, k_2) = p^2 P_{\alpha\nu}(p) - k_2^2 P_{\alpha\nu}(k_2)$$

$$P_{\alpha\nu}(q) = \delta_{\alpha\nu} - \frac{q_\alpha q_\nu}{q^2}$$

- Inverse propagators on background lines signal terms that come from pinch parts and that should be dropped (intrinsic PT).

Ward identities in Feynman-parameter space

- In momentum space, Ward identities have an easily-recognized structure, but not so after momentum integrations have been done.
- It is not hard to guess that a Ward identity for a graph after momentum integrations have been done is a **total derivative** in Feynman parameters.
- Equivalently, with OSSIM it is a total derivative in proper times.
- **Warning:** There are total derivatives in OSSIM that are unrelated to the Ward identities used in the PT.

OSSIM today

- OSSIM yields expressions for off-shell Feynman graphs after momentum-space integrations, leaving Feynman-parameter integrations whose integrands are described by special algorithms.
- OSSIM results can be found directly in string theory in the limit $\alpha' \to 0$; the result is in the BFM Feynman gauge. There is no natural way to use any other gauge, which would be **complicated.**
- OSSIM can also be found directly in field theory, using Feynman-Schwinger proper-time methods. For simplicity, and for agreement with string theory, BFM Feynman gauge is always used.
- To my knowledge, OSSIM from field theory has only been applied at one loop, but with (in principle) arbitrarily many off-shell **background** gluons attached to a single **quantum** loop.

Proper-time OSSIM

- The one-loop effective action for a scalar is:

$$\Gamma_S\{B_\mu\} = -\frac{1}{2}\mathrm{Tr}\log\Delta_F.$$

Here B_μ is the background potential, a sum of plane waves.

- OSSIM always uses Feynman gauge, which greatly simplifies the index structure for quantum gluons.
- The building block is the Schwinger-Feynman proper-time propagator for a **scalar** quantum field:

$$\Delta_F(x-y) = \mathrm{P}\int_0^\infty \mathrm{d}s e^{-m^2 s}\int_x^y \{\mathrm{d}z_\mu\}\exp\{-\int_0^s \mathrm{d}\tau[\frac{\dot{z}_\mu^2}{4} - ig\dot{z}\cdot B(z)]\}$$

where P means color/proper-time ordering (which we now suppress).

- The coupling $\dot{z}\cdot B$ induces one source in the path integral for each plane-wave insertion. **It is the same as using Γ^C, the convective vertex, in Feynman graphs.**

Spin is a complication

- Use Grassmann variables to account for the spin vertex (world-line supersymmetry); the scalar result is multiplied by a complicated factor in the x_{ij}.
- For any spin, delta-function terms arise from $\ddot{G}_B$; the rule is to integrate by parts and ignore surface terms.
- The final result of OSSIM is that the Feynman-parameter integrals are cleverly organized.
- **But how do we know the result is gauge-invariant?**

OSSIM with quantum gluons

- Use Feynman gauge; add a spin coupling by writing

$$\Delta_F^{-1} = D^2 - \Sigma_{\mu\nu} B^{\mu\nu}$$

(D_μ is the background covariant derivative).

- The standard choice for $\Sigma_{\mu\nu}$ is

$$\Sigma_{\mu\nu}^{(\alpha\beta)} = i[\delta_\mu^\alpha \delta_\nu^\beta - \delta_\mu^\beta \delta_\nu^\alpha]$$

which **couples the quantum and background gluons as in the BFM Feynman gauge. In Feynman graphs this amounts to the same coupling as given by the spin vertex Γ^S. There is no pinch vertex Γ^P.**

Integration over the worldline equals momentum-space integration

- The path integral for the convective terms yields for a **scalar** quantum field a sum of terms:

$$\Gamma\{B_{\mu_i}\} = \frac{(ig^2)^N \mathrm{Tr}(t_{a_N} \dots t_{a_1})}{16\pi^2} \int_0^\infty \frac{ds}{s^{3-N}} \int [\mathrm{d}x_i] \exp\{s \sum_{i<j} p_i \cdot p_j G_B(x_{ij})\}$$

$$\times \exp[\sum_{i<j}^{N} (-i(p_i \cdot \epsilon_j - p_j \cdot \epsilon_i)\dot{G}_B(x_{ij}) + \epsilon_i \cdot \epsilon_j \ddot{G}_B(x_{ij})]$$

saving only the term multi-linear in the polarization vectors.

- Here $x_i = \tau_i / s$ are Feynman parameters, $x_{ij} \equiv x_i - x_j$, and the $G_B(x_{ij}) = |x_{ij}| - (x_{ij})^2$ are bosonic Green's functions on the circle.

The path-integral current for worldline variables is a sum of discontinuous pieces

- For perturbation theory:

$$B_\mu(z) = \sum_{i=1}^{N} t_{a_i} \epsilon_{i\mu}(p_i) e^{ip_i\cdot z}$$

- For a given ordering of the t_{a_i} (and τ_i) the color trace factors out, and the source for $z_\mu(\tau)$ is **discontinuous:**

$$K_\mu(\tau) = \sum^{N} \delta(\tau - \tau_i)(\epsilon_{i\mu}\partial_{\tau_i} + ip_{i\mu})$$

where the plane waves act at proper times τ_i.

- In the limit $N \to \infty$, $k_{i+1} - k_i = \mathcal{O}(1/N)$ the current is smooth.

Exercise 1: Adding pinch vertices, to go beyond BFM Feynman gauge, generates ambiguities

- The results of adding pinch vertices will be gauge-dependent.
  - Apply the intrinsic PT. Of course, the result is Green's functions in the BFM Feynman gauge, which is just the Feynman gauge with $\Gamma^P = 0$.
  - **The Γ^P terms generate terms that are ambiguous, when translated into worldline langauge.**
  - Perhaps application of the PT in the worldline formalism has something to do with regularizing the ambiguities.

Extending OSSIM: Pinch vertices in worldline language

- The convective vertex $i \overleftrightarrow{\partial}_\alpha$ for a background gluon of momentum p_α is $(k_1 + k_2)_\alpha$ where k_1, k_2 are the momenta on either side of the vertex and $p + k_1 = k_2$.
- The gradient of a classical on-orbit action S with respect to its endpoints, $\partial_\mu S$, yields $\pm P_\mu$, the canonical momentum at the endpoint, so

$$\partial_{1\mu}\Delta_{F0}(x_1 - x_2) = \langle -\frac{\dot{z}_\mu}{2}(\tau_1 = 0; x_1, x_2, s)\rangle = -ik_{1\mu}.$$

- **The $\dot{z}_\mu$ form is ambiguous; we need to specify k_1 or k_2 by $\dot{z}_\mu(\tau_1 \pm \epsilon)$.**
- We also need terms quadratic in $\dot{z}_\mu$ (*e.g.*, k_i^2 coming from Slavnov-Taylor identities).

Ward identities are ambiguous too

- By momentum conservation, one should be able to replace the convective vertex $\Gamma^C_\alpha = (k_1 + k_2)_\alpha$ by, *e.g.*, $(2k_1 + p)_\alpha \to 2k_{1\alpha}$ (after multiplying by $\epsilon_\alpha(p)$). Go to Vertex
- But now the Ward identity $p \cdot \Gamma^C = k_2^2 - k_1^2$ only works if $p^2 = 0$! And this is not required, either in OSSIM or in the PT.
- The proper-time positions on either side of a vertex are continuous, but the velocities are **discontinuous**.
- **Using Γ^C resolves the ambiguity by effectively averaging the velocities at the discontinuity (cf. Fourier transforms).**

Maybe resolving ambiguities is as simple as dropping the pinch vertex

- With Feynman graphs, a pinch term is perfectly unambiguous and ordinarily would be kept.
- With proper-time methods, every perturbative background gluon causes a jump in $\dot{z}_\mu(\tau)$ and a potential ambiguity.
- The convective vertex Γ^C is unambiguous and symmetric in the quantum gluons on either side of the background-gluon vertex, but **not so** for the pinch vertex Γ^P.

Ambiguities and the intrinsic PT for proper-time methods

- A typical Slavnov-Taylor identity for the intrinsic PT:

$$-k_1\cdot(k_2+p)\delta_{\nu\alpha}+\cdots=(p-k_2)\cdot(p+k_2)\delta_{\nu\alpha}+\cdots=(p^2-k_2^2)\delta_{\nu\alpha}+\ldots$$

- Replace k_i by $(-i/2)\dot{z}_i(\tau_i \pm \epsilon)$, note that $\dot{z}(\tau_1 \pm \epsilon)\cdot\dot{z}(\tau_1 \pm \epsilon)$is ambiguous.
- By momentum conservation, $k_1 \cdot k_2 = \frac{1}{2}(k_1^2 + k_2^2 - p^2)$.
- **The intrinsic PT demands that p^2 be dropped.** Does this have anything to do with regularizing ambiguities in proper time, for example, $[\dot{z}(\tau_1)]^2 \rightarrow \frac{1}{2}[\dot{z}(\tau_1-\epsilon)^2+\dot{z}(\tau_1+\epsilon)^2]$?

Exercise 2: Extending OSSIM the old-fashioned way

- Use old topological rules for expressing an arbitrary Feynman graph, with momentum-dependent numerators, *after* momentum-space integrations are done, plus string-inspired Feynman parameters.
- At least at one loop and in the BFM Feynman gauge this is certainly no more complex than standard OSSIM methods. For example, a term linear in the integration momentum k is replaced by

$$k \to \sum_j x_{ij} p_j,$$

  the power of which is that this replacement holds for **any** value of the index i (with momentum conservation plus $x_{ij} = x_i - x_j$).
- Using this formulation amounts to resumming some exponential forms in standard OSSIM.

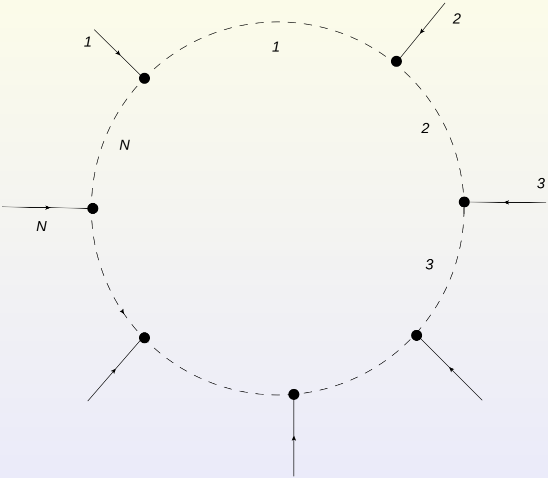

Figure: The natural Feynman parameters α_i are related to the Koba-Nielsen variables x_i by $[i < j]: \; x_{ij} \equiv x_i - x_j = \sum_{k=i}^{j-1} \alpha_k$.

Exercise 3: The nature of the gluon mass and adjoint string breaking

- Lattice, continuum studies yield a gluon mass—not just an ordinary mass, because the gluon can't propagate indefinitely.
- Conventional PT Schwinger-Dyson equations give an ordinary mass, because there is no adjoint string breaking (infinite order in g).
- The signature of an unconventional mass on the lattice is compromised by other effects:
  - Finite-distance effects
  - Minkowski regime inaccessible
  - Mass is momentum-dependent

A model of adjoint string breaking

1. Use PT-OSSIM to study the generalized Schwinger[1] instability for **massive** quantum gluons.
2. Use a background electric gauge field for the adjoint string from flux tube (or gluon-chain model).
3. Use as input for a study (to be done) of gluon propagation.

[1] Not the Nielsen-Olesen instability, which is cured by a gluon mass.

Semi-phenomenological proper-time methods for massive gluons

- Replace the massless OSSIM propagator, Δ_F^{-1}, by adding a **constant** mass.

$$\Delta_F^{-1} = D^2 + m^2 - \Sigma_{\mu\nu} B^{\mu\nu}$$

  because
  1. Ward identities (the basic PT tool) are preserved.
  2. The Brodsky-Binger supersymmetry relation, relating three-gluon vertices with spin 0, 1/2, and 1 in the loop, is preserved, with a common mass for all spins.
  3. A lengthy investigation (unpublished) confirms the utility of this phenomenology
- **Result:** An adjoint string decay width $\Gamma = \frac{2m}{\pi^2}(\frac{g^2}{4\pi})^2 \exp[-2\pi^2/g^2] \approx 0.0015m$ at $\frac{g^2}{4\pi} \approx \frac{1}{2}$;$\approx m$ at $\frac{g^2}{4\pi} \approx \pi$.

The way it's done

- Do it the Schwinger way, with a few changes for group-theoretic numbers.
- Or do a simple tunneling estimate that gives the same answer, based on:
  1. $\sigma_A \approx 2\sigma_F \approx m^2$ based on extended gluon-chain model [Trento 2009].
  2. Assume constant chromoelectric field ε across a distance $1/m$:

$$\mathrm{Flux:}\quad (\frac{\pi}{m^2})\varepsilon = g \;\; \mathrm{plus}\; \sigma_F = (\frac{\pi}{m^2})\frac{\varepsilon^2}{2} = m^2(\frac{g^2}{2\pi}) \rightarrow \frac{g^2}{4\pi} \approx \frac{1}{2}$$

.

  3. Maximum tunneling rate for zero-momentum gluon pairs $\rightarrow$ adjoint string breaks at length $\ell = \frac{2}{m} \approx .7$ Fm.
  4. All these numbers are nicely consistent with known values: $\sigma_F \approx 0.19$ GeV^2, $m \approx 0.6$ GeV, $\frac{\bar{g}^2(m^2)}{4\pi} \approx \frac{1}{2}$.
- Seeing effects associated with adjoint string breaking might well be within the grasp of lattice simulations.

Summary

1. We extend proper-time OSSIM to include gauge-variant terms, to which the intrinsic PT can apply.
2. We suggest that the intrinsic PT for extended perturbative OSSIM is equivalent to an algorithm for **smoothing** path integrals in the presence of a discontinuous source.
3. We reformulate and extend OSSIM with old and so far little-used topological rules for Feynman parameter integrals.
4. We use a phenomenological extension of OSSIM to massive gluons, leading to a picture of adjoint string breaking.