Gluon Sivers Function and Quarkonium Single Spin Asymmetries in ep Collisions

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- Sivers Effect and Sivers Function for the Gluon
- Single spin asymmetry in heavy quark pair production
- Color Evaporation model for J/ψ production
- Models for the gluon Sivers function used
- Results

New Observables in Quarkonium Production; Trento, February 29 - March 4, 2016

Sivers Effect and gluon Sivers function

- Distribution of unpolarized quarks and gluons in a transversely polarized nucleon is not left-right symmetric with respect to the plane defined by the momentum and spin : this asymmetry shows up in Sivers effect

- Sivers function of quarks/gluons gives the distortion of the distribution of unpolarized partons in a transversely polarized nucleon

- Burkardt sum rule : sum of the average transverse momentum of quarks and gluons in a transversely polarized nucleon should be zero: means that sum of the Sivers functions of quarks and gluons, integrated over x vanishes

Burkardt, PRD (2004)

- Fits to the SIDIS data at $Q^2 = 2.4 GeV^2$ almost saturates the sum rule with u and d quarks, however, contribution from gluons can still be about 30 % compared to the valence quarks

Anselmino *et al*, EPJA39, 89 (2009)

- ep collider : excellent tool to study the gluon TMDs at higher $\sqrt{S}$ and low x, where these are important

Gluon Sivers Function

- Sivers function satisfies the positivity bound
- Sivers function is process dependent due to the gauge link
- As gluon correlators contain two gauge links (in the fundamental representation), gluon Sivers function has more involved color flow dependence
- In fact the gluon Sivers function that can be probed in ep collisions is different from that probed in pp collisions : both measurements are thus important
- The gluon Sivers function in any process can be expressed as a linear combination of two universal Sivers functions multiplied by process dependent, and calculable gluonic pole factors : one of them is C-even, the other is C-odd

$$f_{1T}^{\perp,g[U]}(x,k_\perp^2) = \sum_{c=1}^{2} C_{G,c}^{[U]} f_{1T}^{\perp,g(c)}(x,k_\perp^2)$$

Buffing, Mukherjee, Mulders, PRD88, 054027 (2013)

- Burkardt sum rule constraints the C-even function

Boer, Lorce, Pisano, Zhou, Adv. High Energy Phy. (2015), 371396.

Single Spin Asymmetry in heavy quark pair production

F. Yuan, PRD 78,014024 (2008)

Heavy quark production in ep and pp collisions at low transverse momentum : sensitive to intrinsic transverse momentum and probe for gluon TMDs

Heavy quark pair production using non-relativistic limit

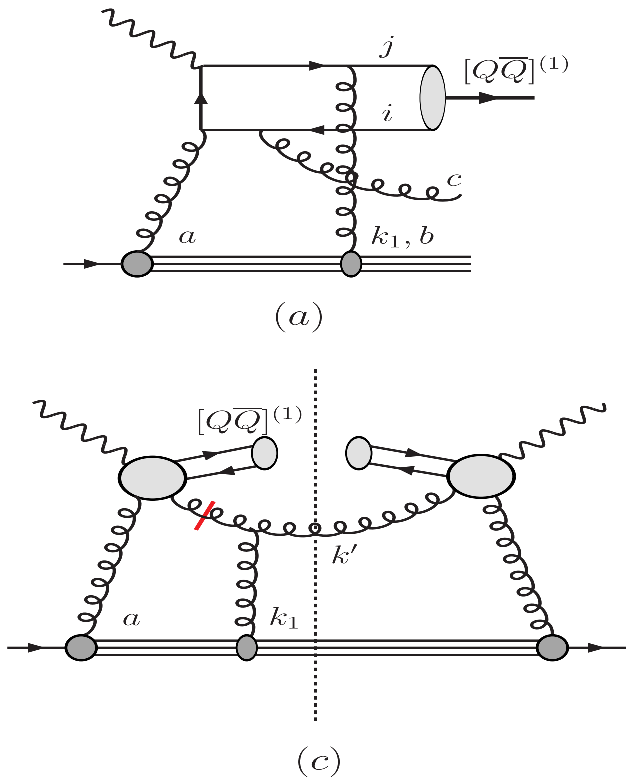
As the lepton/photon is colorless, there is no initial state interaction contributing to SSA in ep scattering

Final state interaction with the quark pair both in color singlet and color octet : contribution to Sivers effect

SSA will have to be zero when the heavy quark pair is produced in the color singlet state in ep collision

Non-zero SSA if the heavy quark pair is produced in color octet state

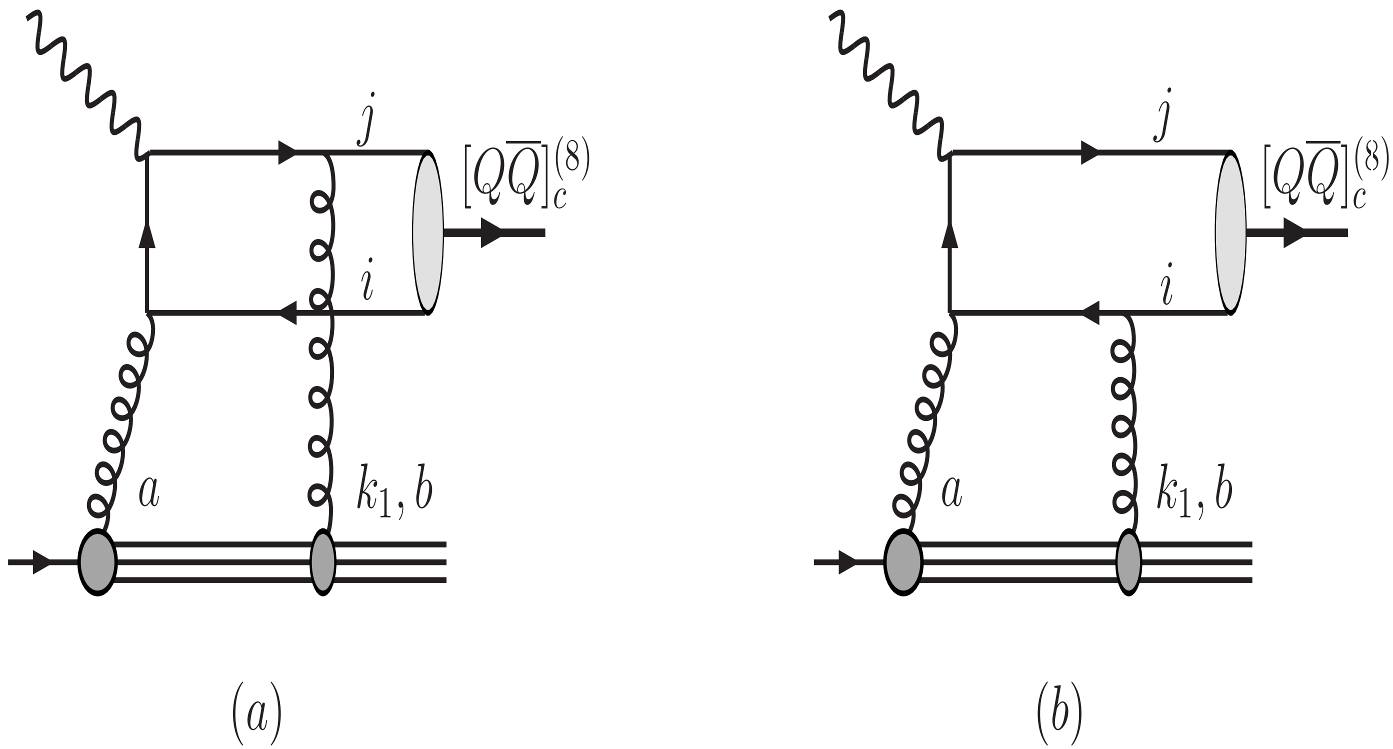
When the heavy quark pair is produced in color singlet state



F. Yuan, PRD 78,014024 (2008)

(a) and (b) cancel, final state interaction among unobserved particles cancel out among each other

When the heavy quark pair is produced in the color octet state



F. Yuan, PRD 78,014024 (2008)

Final state interaction with quark and antiquark do not cancel out

Phenomenological study of Single Spin Asymmetry in J/ψ production

- Single spin asymmetry in photoproduction (i.e. low virtuality electroproduction) of charmonium by scattering unpolarized electrons off transversely polarized proton

- SSAs involving the transverse momentum dependent pdfs and fragmentation functions: very often two or more of these functions contribute to the same physical observable

- In this process that we are considering, at LO, there is contribution only from a single partonic subprocess $\gamma g \rightarrow c\bar{c}$: can be used as a clean probe of gluon Sivers function

- May throw some light on the charmonium production mechanism as well

- Model considered : Color evaporation model : first proposed by

  Halzen and Matsuda (1978), H. Fritsch (1977)

- In CEM, it is assumed that the heavy quark pair is produced perturbatively with definite spin and color quantum numbers which can be calculated upto a desired order in α_s

- Then the heavy quark pair radiates soft gluons to evolve into any physical color neutral quarkonium state with quantum numbers different than that of initial heavy quark pair : nonperturbative

Model for charmonium production considered

"color evaporation", since the color of the initial $Q\bar{Q}$ pair does not affect the final quarkonium state

According to CEM, the cross section of quarkonium state is some long distance factor times the cross section of the $Q\bar{Q}$ pair with invariant mass below the threshold mass. These long distance factors are considered to be universal which are determined by fitting the heavy quark pair cross section with experimental data

Cross section for charmonium production is proportional to the rate of production of $c\bar{c}$ pair integrated over the mass range $2m_c$ to $2m_D$

$$\sigma = \frac{1}{9} \int_{2m_c}^{2m_D} dM_{c\bar{c}} \frac{d\sigma_{c\bar{c}}}{dM_{c\bar{c}}}$$

where m_c is the charm quark mass and $2m_D$ is the $D\bar{D}$ threshold; $1/9$ is the statistical probability for the production of a color singlet state; $M^2_{c\bar{c}}$ is the squared invariant mass of $c\bar{c}$
Phenomenological non-perturbative factor does not affect the asymmetry

Cross section for charmonium production

Cross section for low-virtuality electroproduction of J/ψ :

$$\sigma^{ep\to e+J/\psi+X} = \int_{4m_c^2}^{4m_D^2} dM^2_{c\bar{c}} \int dr\, dx\, f_{\gamma/e}(r)\, f_{g/p}(x)\, \frac{d\hat{\sigma}^{\gamma g\to c\bar{c}}}{dM^2_{c\bar{c}}}.$$

The photon flux in the electron is approximated by the distribution (Weizsacker-Williams approximation)

$$f_{\gamma/e}(r,E) = \frac{\alpha}{\pi}\{\frac{1+(1-r)^2}{r}\left(ln\frac{E}{m}-\frac{1}{2}\right)+\frac{r}{2}\left[ln\left(\frac{2}{r}-2\right)+1\right] + \frac{(2-r)^2}{2r}ln\left(\frac{2-2r}{2-r}\right)\}.$$

Kniehl (1991)

where r is the energy fraction of electron carried by the photon, E is the energy of the electron and m is the mass

Single Spin Asymmetry

Generalization of CEM expression by taking into account the transverse momentum dependence

$$\frac{d\sigma^{e+p^\uparrow\to e+J/\psi+X}}{dM^2} = \int dx_\gamma\, dx_g\, [d^2\boldsymbol{k}_{\perp\gamma} d^2\boldsymbol{k}_{\perp g}]\, f_{g/p^\uparrow}(x_g,\boldsymbol{k}_{\perp g}) f_{\gamma/e}(x_\gamma,\boldsymbol{k}_{\perp\gamma}) \frac{d\hat{\sigma}^{\gamma g\to c\bar{c}}}{dM^2}$$

where $M^2 \equiv M^2_{c\bar{c}}$

Single spin asymmetry for a transversely polarized target is defined as

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

Number density of partons inside a proton with transverse polarization $\mathbf{S}$ and momentum $\boldsymbol{P}$ is parameterized as

$$f_{a/p^\uparrow}(x_a,\boldsymbol{k}_{\perp a},\boldsymbol{S}) \equiv f(x_a,k_{\perp a}) + \frac{1}{2}\Delta^N f_{a/p^\uparrow}(x_a,k_{\perp a})\, \hat{\boldsymbol{S}}\cdot(\hat{\boldsymbol{P}}\times\hat{\boldsymbol{k}}_{\perp a})$$

$\Delta^N f_{a/p^\uparrow}(x,k_{\perp a})$ is the Sivers function

Single Spin Asymmetry

$$\frac{d^4\sigma^{\uparrow}}{dy dM^2 d^2\boldsymbol{q}_T} - \frac{d^4\sigma^{\downarrow}}{dy dM^2 d^2\boldsymbol{q}_T} = \frac{1}{2}\int [dx_\gamma d^2\boldsymbol{k}_{\perp\gamma} dx_g d^2\boldsymbol{k}_{\perp g}] \Delta^N f_{g/p^\uparrow}(x_g, \boldsymbol{k}_{\perp g})$$
$$\times f_{\gamma/e}(x_\gamma, \boldsymbol{k}_{\perp\gamma}) \delta^4(p_g + p_\gamma - q)\, \hat{\sigma}_0^{\gamma g \to c\bar{c}}(M^2)$$

where $q = p_c + p_{\bar{c}}$ and the partonic cross section

$$\hat{\sigma_0}^{\gamma g \to c\bar{c}}(M^2) = \frac{1}{2} e_c^2 \frac{4\pi\alpha\alpha_s}{M^2}[(1 + \gamma - \frac{1}{2}\gamma^2) \ln \frac{1 + \sqrt{1-\gamma}}{1 - \sqrt{1-\gamma}} - (1+\gamma)\sqrt{1-\gamma}]$$

$\gamma = \frac{4m_c^2}{M^2}$ and $M^2 \equiv \hat{s} = xS$, S is the CM energy squared of the $\gamma - p$ system.
$\Delta^N f_{g/p^\uparrow}(x_g, \boldsymbol{k}_{\perp g})$ is related to the gluon Sivers function $\Delta^N f_{g/p^\uparrow}(x, k_{\perp g})$ by

$$\Delta^N f_{g/p^\uparrow}(x_g, \boldsymbol{k}_{\perp g}) = \Delta^N f_{g/p^\uparrow}(x_g, k_\perp)\, \hat{\boldsymbol{S}} \cdot (\hat{\boldsymbol{P}} \times \hat{\boldsymbol{k}}_{\perp g}).$$

Models for WW Function

For $k_{\perp g}$ dependence of the unpolarized pdf's, we use a simple factorized and Gaussian form

$$f_{g/p}(x_g, k_\perp) = f_{g/p}(x_g)\frac{1}{\pi\langle k_{\perp g}^2\rangle}e^{-k_{\perp g}^2/\langle k_{\perp g}^2\rangle}.$$

we have used two choices for WW function for the photon

1) A simple Gaussian form as above :

$$f_{\gamma/e}(x_\gamma, k_{\perp\gamma}) = f_{\gamma/e}(x_\gamma)\frac{1}{\pi\langle k_{\perp\gamma}^2\rangle}e^{-k_{\perp\gamma}^2/\langle k_{\perp\gamma}^2\rangle}$$

2) Second form :

$$f_{\gamma/e}(x_\gamma, k_{\perp\gamma}) = f_{\gamma/e}(x_\gamma)\,\frac{1}{2\pi}\,\frac{N}{k_{\perp\gamma}^2 + {k_0}^2}$$

Models for Sivers Function

For the Sivers function we use two models

M. Anselmino, M. Boglione, U. D'Alesio, S. Melis, F. Murgia and A. Prokudin (2009)

M. Anselmino, M. Boglione, U. D'Alesio, E. Leader and F. Murgia (2004)

Model I :

$$\Delta^N f_{g/p^\uparrow}(x_g, \boldsymbol{k}_{\perp g}) = \Delta^N f_{g/p^\uparrow}(x_g) \frac{1}{\pi \langle k^2_{\perp g} \rangle} \, h(k_{\perp g}) \, e^{-k^2_{\perp g}/\langle k^2_{\perp g} \rangle} \cos(\phi_{k_\perp})$$

where the gluon Sivers function, $\Delta^N f_{g/p^\uparrow}(x_g)$ is defined as

$$\Delta^N f_{g/p^\uparrow}(x_g) = 2 \, \mathcal{N}_g(x_g) \, f_{g/p}(x_g)$$

$$h(k_{\perp g}) = \sqrt{2e} \, \frac{k_{\perp g}}{M_1} \, e^{-k^2_{\perp g}/M_1^2}$$

Models for Sivers Function

Parametrization for quark Sivers function

$$\mathcal{N}_f(x) = N_f x^{a_f} (1-x)^{b_f} \frac{(a_f+b_f)^{(a_f+b_f)}}{{a_f}^{a_f} {b_f}^{b_f}} \; .$$

For gluons we use (Boer and Vogelsang PRD69, 094025 (2004))

(a) $\mathcal{N}_g(x) = (\mathcal{N}_u(x) + \mathcal{N}_d(x))/2$,

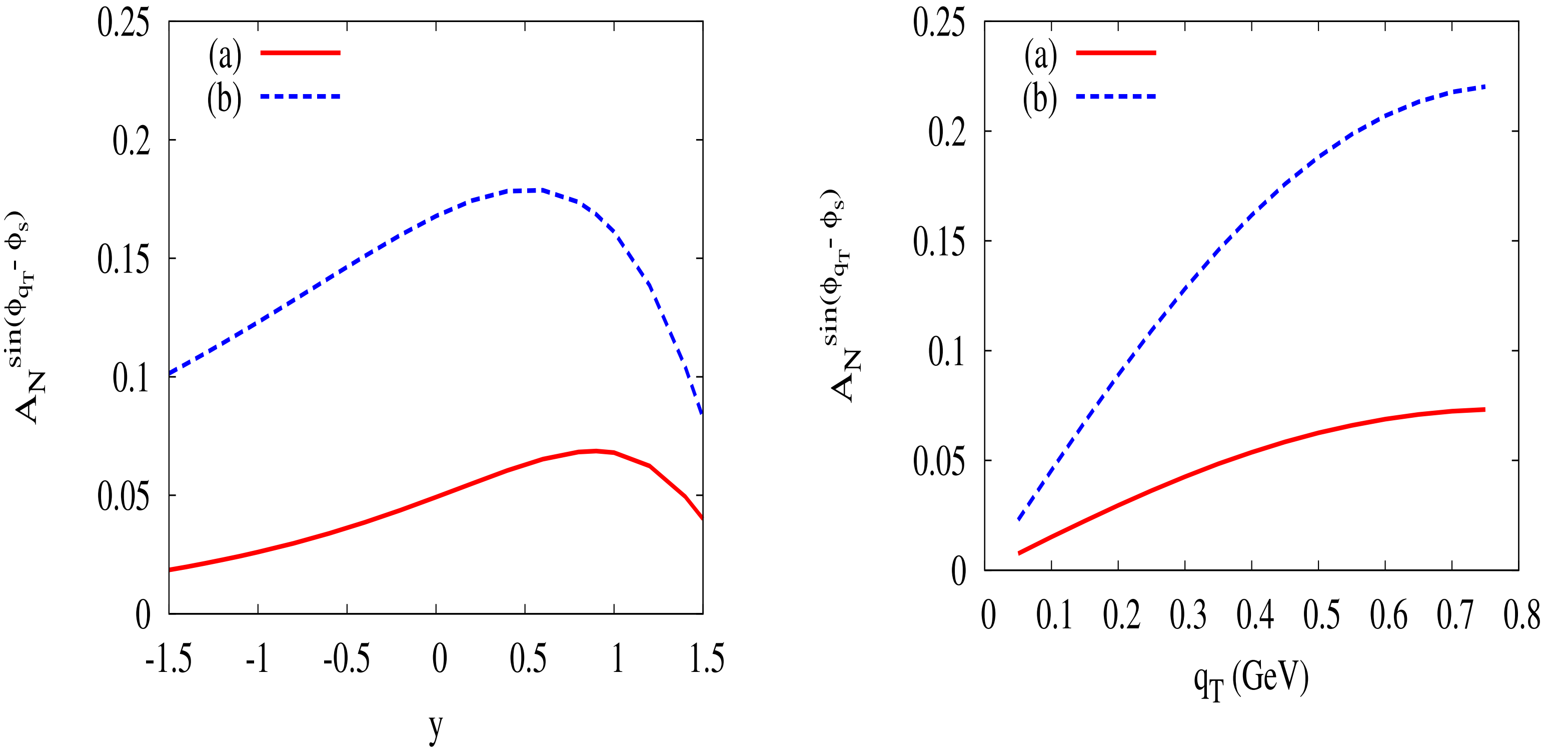
(b) $\mathcal{N}_g(x) = \mathcal{N}_d(x)$.

Model II :

$$\Delta^N f_{g/p^\uparrow}(x_g, \boldsymbol{k}_{\perp g}) = \Delta^N f_{g/p^\uparrow}(x_g) \frac{1}{\pi \langle k_{\perp g}^2 \rangle} e^{-k_{\perp g}^2/\langle k_{\perp g}^2 \rangle} \frac{2 k_{\perp g} M_0}{k_{\perp g}^2 + M_0^2} \cos(\phi_{k_\perp}),$$

and $M_0 = \sqrt{\langle k_{\perp g}^2 \rangle}$ where the gluon Sivers function is given as in Model I

SSA for J/ψ production at COMPASS

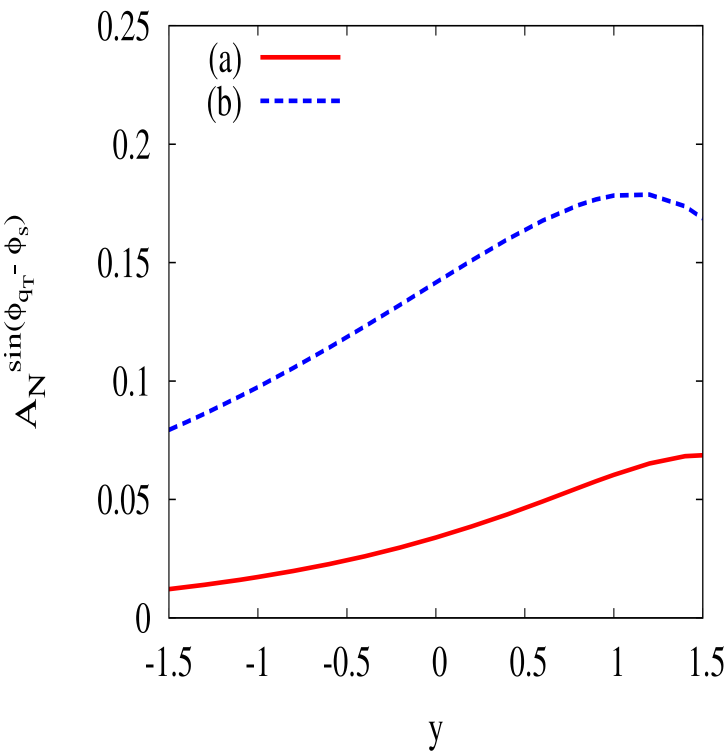


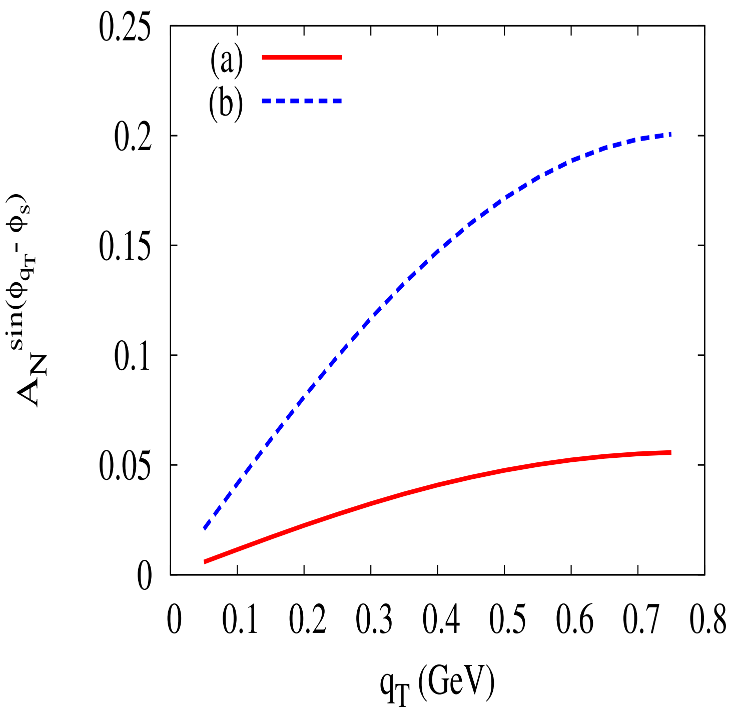
Godbole, Misra, Mukherjee, Rawoot, PRD85, 094013 (2012)

Integration ranges are $(0 \leq q_T \leq 1)$ GeV and $(0 \leq y \leq 1)$

Plot for Model I at COMPASS ($\sqrt{s} = 17.33$ GeV)

SSA for J/ψ production at eRHIC



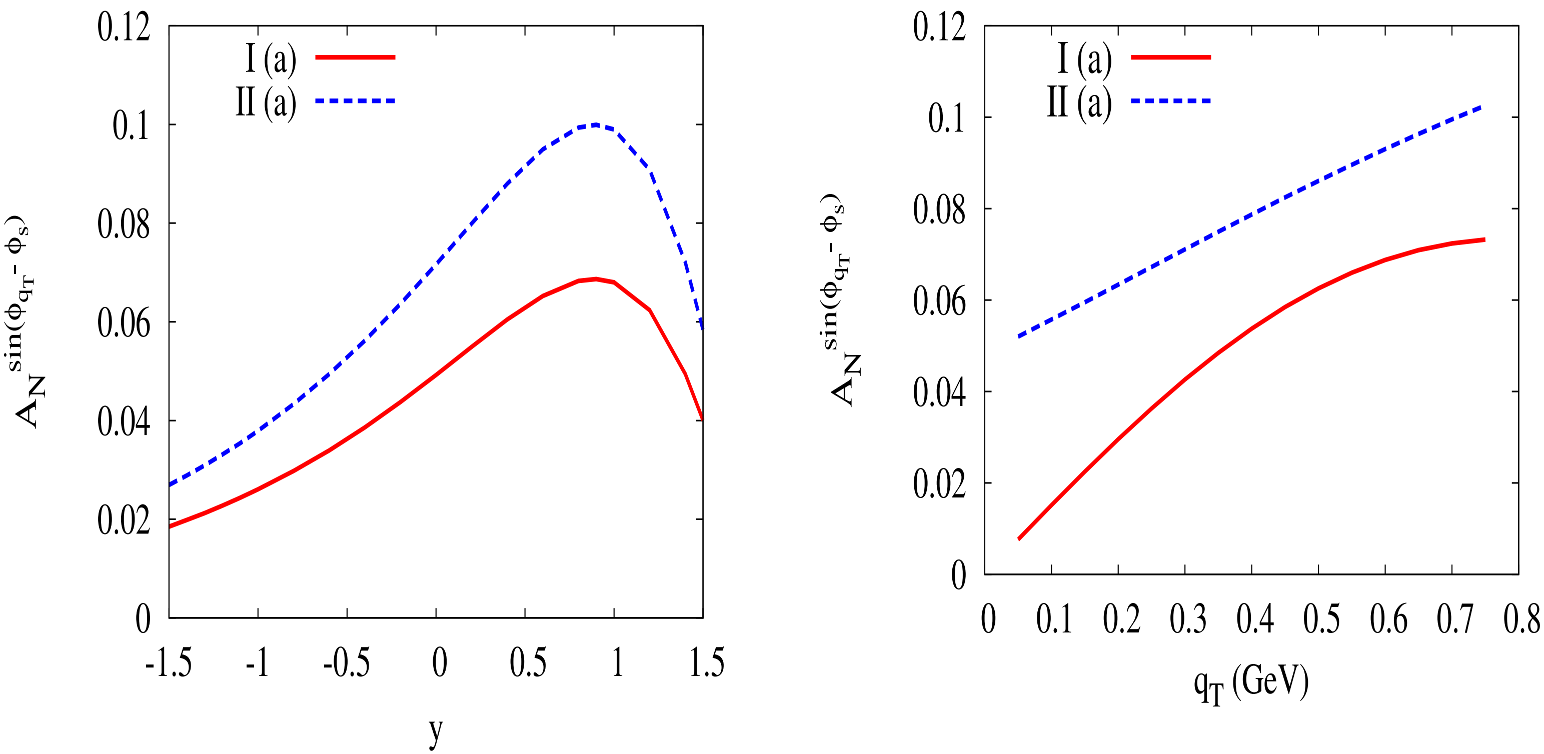


Godbole, Misra, Mukherjee, Rawoot, PRD85, 094013 (2012)

Integration ranges are $(0 \leq q_T \leq 1)$ GeV and $(0 \leq y \leq 1)$

Plot for Model I at eRHIC ($\sqrt{s} = 31.6$ GeV)

SSA for J/ψ production at COMPASS

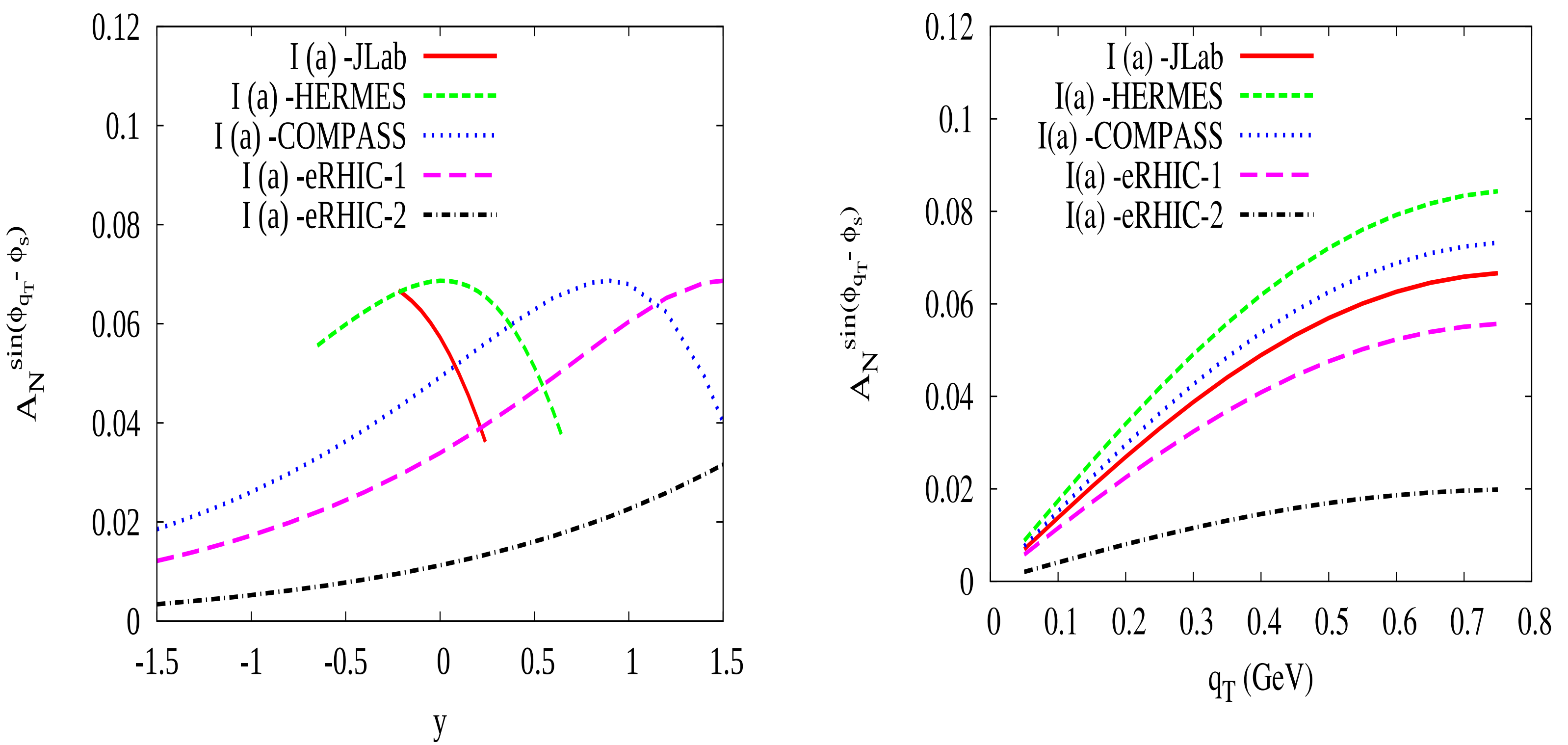


Godbole, Misra, Mukherjee, Rawoot, PRD85, 094013 (2012)

Integration ranges are $(0 \leq q_T \leq 1)$ GeV and $(0 \leq y \leq 1)$

Plot for COMPASS ($\sqrt{s} = 17.33$ GeV) : Comparison of two models

Compare



Godbole, Misra, Mukherjee, Rawoot, PRD85, 094013 (2012)

Asymmetry in model I with parameterization (a) compared for JLab ($\sqrt{s} = 4.7$ GeV) (solid red line), HERMES ($\sqrt{s} = 7.2$ GeV) (dashed green line), COMPASS ($\sqrt{s} = 17.33$ GeV) (dotted blue line), eRHIC-1 ($\sqrt{s} = 31.6$ GeV) (long dashed pink line) and eRHIC-2 ($\sqrt{s} = 158.1$ GeV) (dot-dashed black line)

TMD Evolution

Anselmino, Boglione, Melis, PRD86, 014028 (2012)

Analytic method to implement the TMD evolution

In this formalism, the Q^2 evolution of the $k_\perp$ dependent distribution function is given by

$$\widehat{f}_{q/p}(x, k_\perp; Q) = f_{q/p}(x, Q_0)\; R(Q, Q_0)\; \frac{e^{-k_\perp^2/w^2}}{\pi\, w^2}\;,$$

where $f_{q/p}(x, Q_0)$ is the usual integrated PDF evaluated at the initial scale Q_0 and $w^2 \equiv w^2(Q, Q_0)$ is the “evolving” Gaussian width

$$w^2(Q, Q_0) = \langle k_\perp^2 \rangle + 2\, g_2 \ln \frac{Q}{Q_0}\;.$$

$R(Q, Q_0)$ is the limiting value of a function $R(Q, Q_0, b_T)$ that drives the Q^2 evolution of TMDs in coordinate space

TMD Evolution

$$R(Q,Q_0,b_T) \equiv \exp\left\{\ln\frac{Q}{Q_0}\int_{Q_0}^{\mu_b}\frac{\mathrm{d}\mu'}{\mu'}\gamma_K(\mu') + \int_{Q_0}^{Q}\frac{\mathrm{d}\mu}{\mu}\gamma_F\left(\mu,\frac{Q^2}{\mu^2}\right)\right\}$$

Aybat, Rogers, PRD83, 114042 (2011)

where b_T is the parton impact parameter and

$$\mu_b = \frac{C_1}{b_*(b_T)}\,, \qquad b_*(b_T) \equiv \frac{b_T}{\sqrt{1+b_T^2/b_{\max}^2}},$$

with $C_1 = 2e^{-\gamma_E}$ and $\gamma_E = 0.577$

In the limit $b_T \to \infty$, $R(Q,Q_0,b_T) \to R(Q,Q_0)$ and $b_* \to b_{max}$. γ_F and γ_K are anomalous dimensions

$$\gamma_F(\mu;\frac{Q^2}{\mu^2}) = \alpha_s(\mu)\,\frac{C_F}{\pi}\left(\frac{3}{2} - \ln\frac{Q^2}{\mu^2}\right)$$

TMD Evolution

$$\gamma_K(\mu) = \alpha_s(\mu)\,\frac{2\,C_F}{\pi}\;.$$

TMD evolved Sivers function is given by

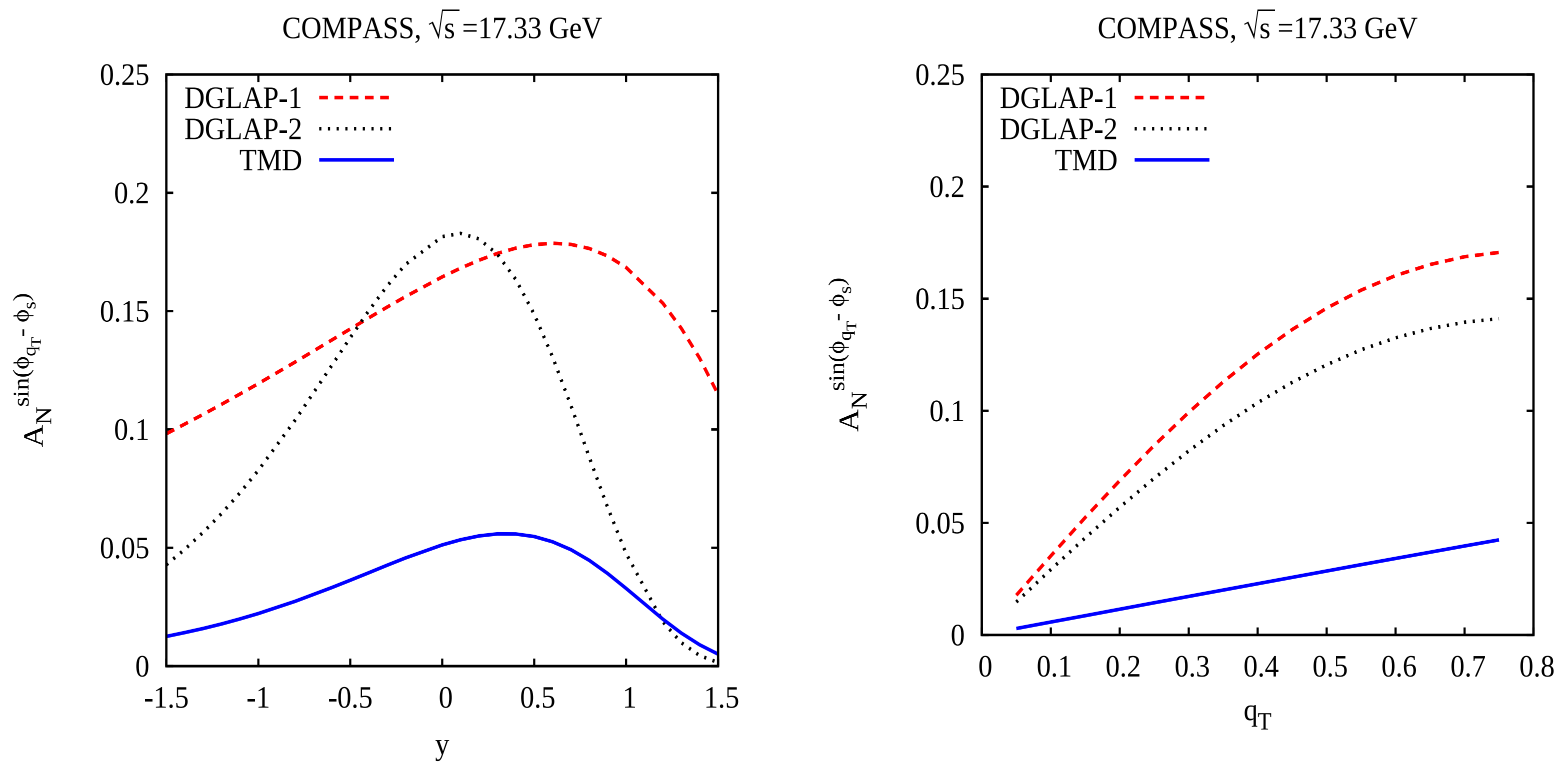
$$\Delta^N \widehat{f}_{q/p^\uparrow}(x,k_\perp;Q) = \frac{k_\perp}{M_1}\,\sqrt{2e}\,\frac{\langle k_S^2\rangle^2}{\langle k_\perp^2\rangle}\,\Delta^N f_{q/p^\uparrow}(x,Q_0)\,R(Q,Q_0)\,\frac{e^{-k_\perp^2/w_S^2}}{\pi w_S^4}\;,$$

$$w_S^2(Q,Q_0) = \langle k_S^2\rangle + 2g_2 \ln\frac{Q}{Q_0}\;; \quad \frac{1}{\langle k_S^2\rangle} = \frac{1}{M_1^2} + \frac{1}{\langle k_\perp^2\rangle}$$

$$\langle k_{\perp g}^2\rangle = 0.25\;GeV^2, \qquad g_2 = 0.68, \qquad b_{max} = 0.5\;GeV^{-1}$$

M_1 is a best fit parameter $Q^2 = \hat{s}$,and $Q_0^2 = 1.0$ GeV^2

SSA for J/ψ production at COMPASS : Effect of TMD Evolution

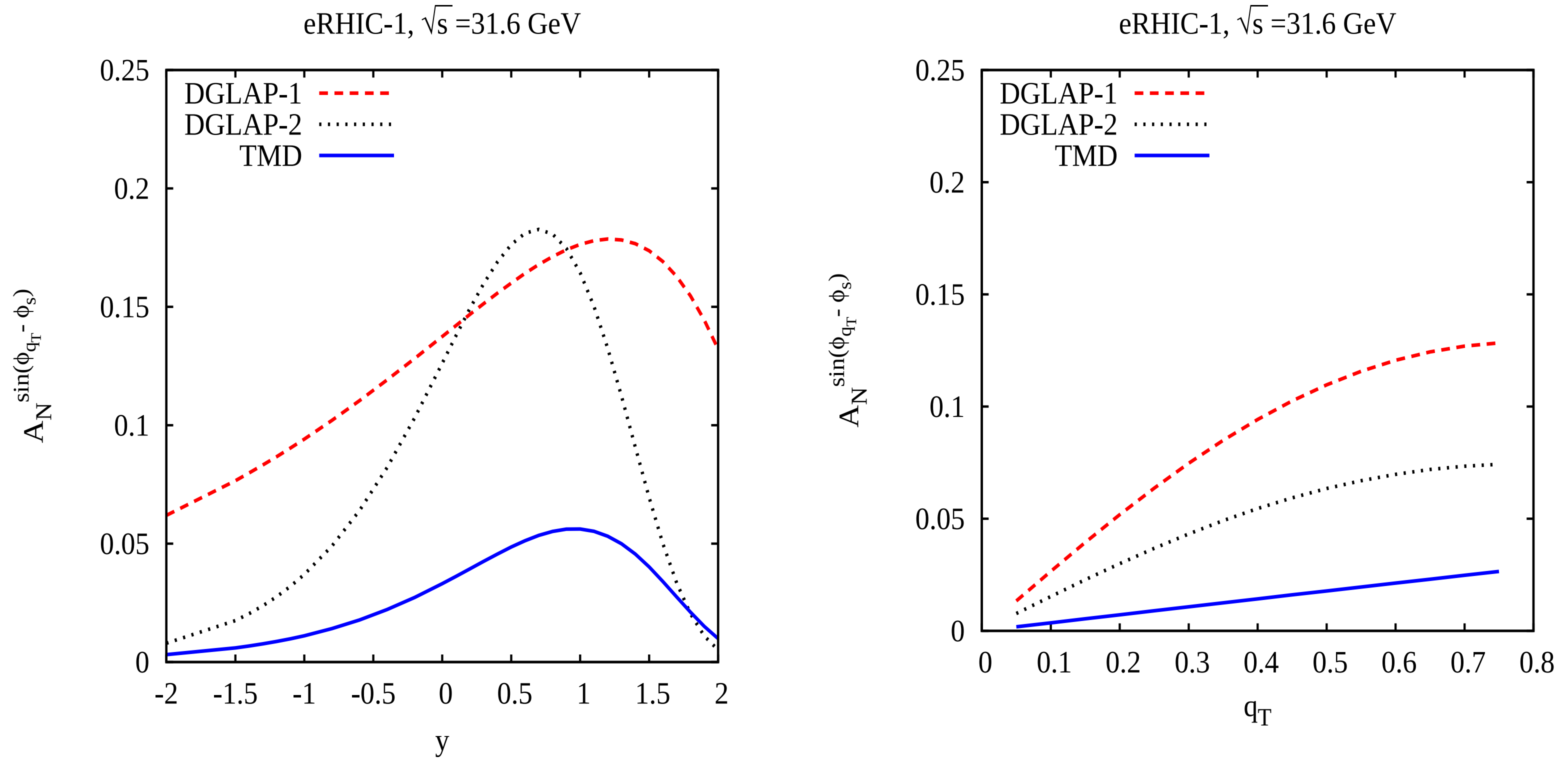


Godbole, Misra, Mukherjee, Rawoot, PRD88, 014029 (2013)

Dashed (red) and dotted (black) line corresponds to DGLAP evolution with DGLAP fit parameters at $Q_0 = \sqrt{2.4}$ GeV and $Q_0 = 1$ GeV respectively

The integration ranges are $(0 \leq q_T \leq 1)$ GeV and $(-1.5 \leq y \leq 1.5)$

SSA for J/ψ production at eRHIC : Effect of TMD Evolution

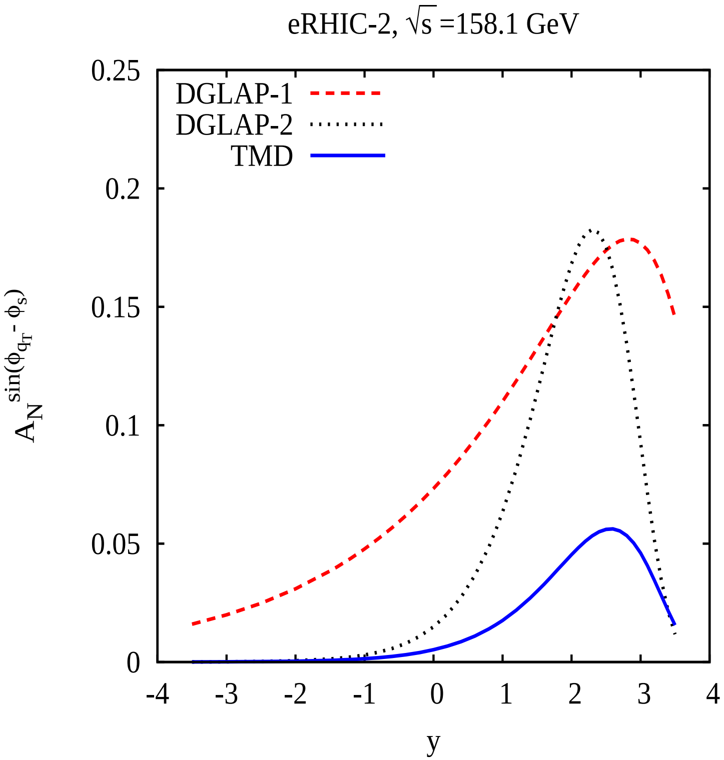


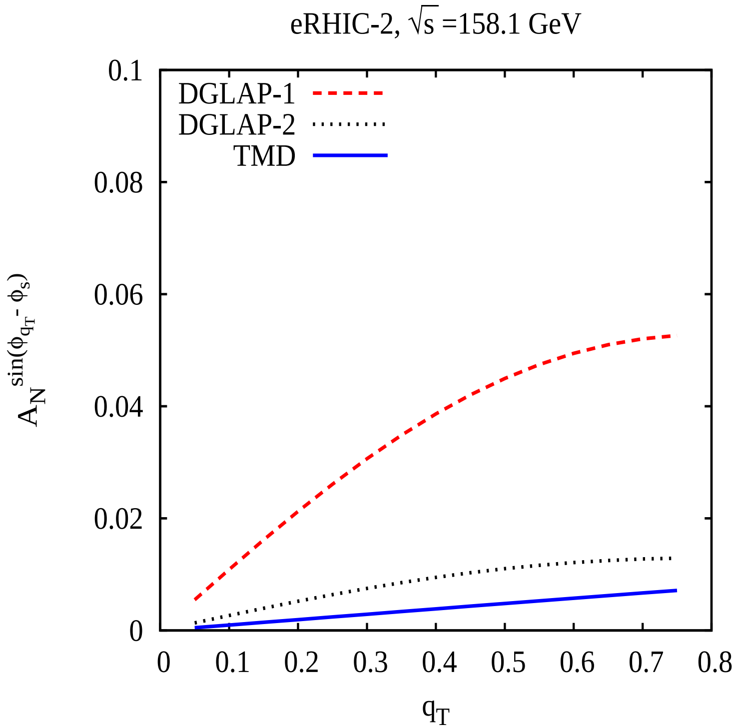
Godbole, Misra, Mukherjee, Rawoot, PRD88, 014029 (2013)

Sivers asymmetry at eRHIC ($\sqrt{s} = 31.6$ GeV).

Integration ranges are $(0 \leq q_T \leq 1)$ GeV and $(-2.1 \leq y \leq 2.1)$

SSA for J/ψ production at eRHIC : Effect of TMD Evolution





Godbole, Misra, Mukherjee, Rawoot, PRD88, 014029 (2013)

Sivers asymmetry at eRHIC ($\sqrt{s} = 158.1$ GeV).

Integration ranges are $(0 \leq q_T \leq 1)$ GeV and $(-3.7 \leq y \leq 3.7)$

Summary and Conclusion

- Gluon Sivers function can be sizable
- SSA in ep collisions give complimentary information on gluon Sivers function as compared to pp collision, as they probe different color structures
- SSA for J/ψ electroproduction : decreases if one includes TMD evolution of the Sivers function, still sizable
- SSA does not depend much on the choice of the $k^{\perp}$ dependence of the photon distribution; important to consider higher order and resolved photon contributions
- Would be interesting to see how sensitive the SSA is on the charmonium production model
- Effect of the final state interaction and color flow