Nuclear structure effects in atomic spectra

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Beyond the Coulomb potential

- finite nuclear mass, well visible in the isotope shift
- finite nuclear size: $\delta E_{\rm fs} = \frac{2\pi}{3} Z\alpha \langle \sum_a \delta^{(3)}(r_a)\rangle\, r_{\rm ch}^2$
  - this formula is universal for all light atoms
  - the energy shift is proportional to the mean square charge radius $r_{\rm ch}^2$
  - additional corrections are usually small (for electronic atoms)
- nuclear polarizability effects are in general quite small, can be “significant” only for weakly bound nuclei
- one can determine $r_{\rm ch}$ from comparison of measured and calculated transition frequencies, like that of r_p
- how come r_p from (electronic) H differs by 4% from that of μH ?
- no universal formula for nuclear structure effects in hfs

Measurements of atomic properties

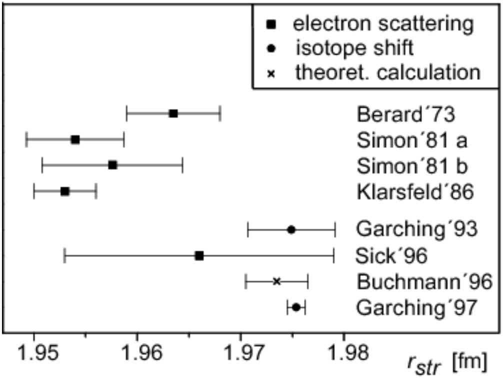
Measurement of transition frequencies can be very accurate [Garching, 2013]

- $\nu(1S - 2S)_{\mathrm{H}} = 2466\,061\,413\,187\,018(11)\ \mathrm{Hz}$
- sensitive to the nuclear size and to the nuclear polarizability
- from $\nu(1S - 2S)_{\mathrm{H-D}}$: $r_d^2 - r_p^2 = 3.820\,07(65)\ \mathrm{fm}^2$

Another example: electron mass from the g-factor measurement in hydrogen-like C, [Sturm, et al, Nature 2014]

- $m_e = 0.000\,548\,579\,909\,067(14)(9)(2)$ au

r_D from H–D isotope shift



Phys. Rev. Lett. **80**, 468 (1998)
Deuteron polarizability $\sim$ 20 kHz

Hyperfine splitting in D

- $\nu_{\rm exp} = 327\,384.352\,522\,2(17)$ kHz
- $\nu_F = -\frac{2}{3}\,\langle\psi|\vec{\mu}\cdot\vec{\mu}_e\,\delta^3(r)|\psi\rangle\,2\,\pi\,\hbar$
- $\frac{\nu_{\rm exp}-\nu_{QED}}{\nu_F} = 138$ ppm: due to the nuclear structure effects
- Can the hyperfine splitting be used to obtain magnetic properties of nuclei, especially for the halo nuclei such as ^{11}Be ?

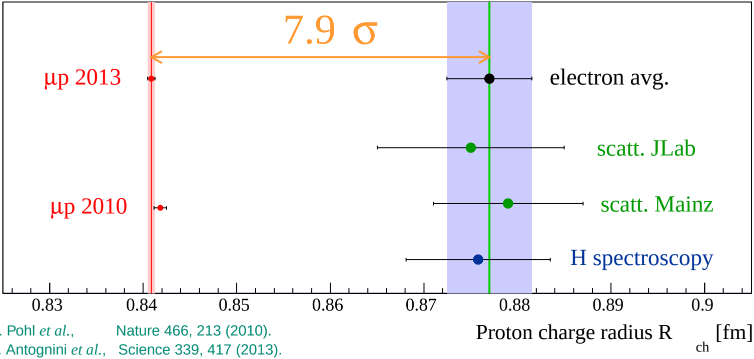
  Project at RIKEN: K. Okada *et al*, Phys. Rev. Lett. **101**, 212502 (2008)

The proton radius puzzle

The proton rms charge radius measured with

electrons: 0.8770 ± 0.0045 fm

muons: 0.8409 ± 0.0004 fm



R. Pohl *et al.*, Nature 466, 213 (2010).
A. Antognini *et al.*, Science 339, 417 (2013).

Proton charge radius puzzle

- $\delta_{\rm fs} E = (2\,\pi\,\alpha/3)\,\phi^2(0)\,\langle r_p^2\rangle$
- If $e - p$ experiments and μH theory are correct the plausible solution of this puzzle is an additional interaction at the 1 fm scale

How it can be verified ?

- muon-proton scattering
- μHe

Nuclear polarizability effects

Two photon exchange between electrons and the nucleus in the electric dipole approximation:

$$
\begin{aligned}
E_{\rm pol} &= -m\,\alpha^4 \left\langle \sum_a \delta^3(r_a) \right\rangle (m^3\,\tilde\alpha_{\rm pol}) \\
\tilde\alpha_{\rm pol} &= \frac{16\,\alpha}{3} \int_{E_T}^{\infty} dE\, \frac{1}{e^2}\, |\langle \phi_N | \vec d | E \rangle|^2 \int_0^\infty \frac{dw}{w}\, \frac{E}{E^2 + w^2} \\
&\quad \times \frac{1}{(\kappa + \kappa^\star)} \left[1 + \frac{1}{(\kappa+1)(\kappa^\star+1)} \left(\frac{1}{\kappa+1} + \frac{1}{\kappa^\star+1} \right) \right]
\end{aligned}
$$

where $\kappa = \sqrt{1 + 2\,i\,m/w}$, E_T is the excitation energy, and $\vec d$ is the nuclear dipol moment operator

cancellation of the third Zemach moment with inelastic contributions for e-atoms

Nuclear polarizability effects

If E_T is much larger than the electron mass m, one can perform a small electron mass expansion and obtain a simplified formula:

$$\begin{aligned}
\tilde{\alpha}_{\rm pol} &= \frac{19}{6}\,\alpha_{\rm E} + 5\,\alpha_{\rm Elog} \\
\alpha_{\rm E} &= \frac{2\,\alpha}{3}\,\frac{1}{e^2}\left\langle \phi_N \middle| \vec{d}\,\frac{1}{H_N - E_N}\,\vec{d}\,\middle| \phi_N \right\rangle \\
\alpha_{\rm Elog} &= \frac{2\,\alpha}{3}\,\frac{1}{e^2}\left\langle \phi_N \middle| \vec{d}\,\frac{1}{H_N - E_N}\,\ln\left(\frac{2(H_N - E_N)}{m}\right)\vec{d}\,\middle| \phi_N \right\rangle
\end{aligned}$$

where $\alpha_{\rm E}$ is the static electric dipole polarizability and $\alpha_{\rm Elog}$ is the logarithmically modified polarizability. We have tested this approximation for ^{3}He and ^{4}He isotopes, and found that numerical results differ from the exact formula by less than 0.1%.

Helium nuclear polarizabilities

	Ref.	α_{E}[fm^3]	α_{Elog}[fm^3]	$\tilde{\alpha}_{\mathrm{pol}}$[fm^3]
^{3}He		0.153(15)	0.615(62)	3.56(36)
	Navratil et al. [2009]	0.149(5)		
	Rinker [1976]	0.130(13)		
	Leidemann [2003]	0.145		
	Goeckner et al. [1991]	0.250(40)		
^{4}He		0.076(8)	0.365(37)	2.07(20)
	Navratil et al. [2009]	0.0683(8)(14)		
	Leidemann [2003]	0.076		
	Gazit [2006]	0.0655(4)		
	Friar [1997]	0.072(4)		
^{6}He		1.99(40)	4.78(96)	24.7(5.0)

Table : The electric dipole polarizability α_{E}, the logarithmically modified polarizability α_{Elog}, and the weighted polarizability $\tilde{\alpha}_{\mathrm{pol}}$ for helium isotopes.

^{3}He - ^{4}He isotope shift of centroid energies in kHz

Contribution	$2^3S - 2^3P$	$2^3S - 2^1S$
$m_r\,\alpha^2$	12 412 458.1	8 632 567.86
$m_r\,\alpha^2\,(m_r/M)$	21 243 041.3	−608 175.58
$m_r\,\alpha^2\,(m_r/M)^2$	13 874.6	7 319.80
$m_r\,\alpha^2\,(m_r/M)^3$	4.6	−0.30
$m_r\,\alpha^4$	17 872.8	8 954.22
$m_r\,\alpha^4\,(m_r/M)$	−20 082.4	−6 458.23
$m_r\,\alpha^4\,(m_r/M)^2$	−3.0	−1.84
$m\,\alpha^5\,(m/M)$	−60.7	−56.61
$m\,\alpha^6\,(m/M)$	−15.5 (3.9)	−2.75 (69)
Nuclear polarizability	−1.1 (1)	−0.20 (2)
HFS mixing	54.6	80.69
Total	33 667 143.2 (3.9)	8 034 065.69 (69)
Drake, Morton, 2006, 2010	33 667 146.2 (7)	8 034 067.8 (1.1)

^{3}He - ^{4}He charge radii difference

$$\delta r^2(\text{Florence 2012}, 2^3P - 2^3S) = 1.074\,(4)\,\text{fm}^2\,,$$

$$\delta r^2(\text{Shiner 1995}, 2^3P - 2^3S) = 1.066\,(4)\,\text{fm}^2\,,$$

$$\delta r^2(\text{Amsterdam 2011}, 2^1S - 2^3S) = 1.028\,(11)\,\text{fm}^2$$

$4\,\sigma$ discrepancy

^{6}Li-^{7}Li isotope shift and the charge radii diff.

$$\delta_{\rm fs} E = \frac{2\,\pi\,Z\,\alpha}{3}\left\langle \sum_a \delta^3(r_a)\right\rangle \langle r^2 \rangle$$

$$\delta r^2 = r^2(^6\mathrm{Li}) - r^2(^7\mathrm{Li}) = \begin{cases} 0.705(3)\,\mathrm{fm}^2 \\ \qquad 2P_{1/2} - 2S_{1/2},\ \mathrm{NIST}\ (2013) \\ 0.700(9)\,\mathrm{fm}^2 \\ \qquad 2P_{3/2} - 2S_{1/2},\ \mathrm{NIST}\ (2013) \\ 0.731(22)\,\mathrm{fm}^2 \\ \qquad 3S_{1/2} - 2S_{1/2},\ \text{Nörtershäuser } et\ al\ (2011) \end{cases}$$

similar results have been obtained for Li and Be isotopes including short lived ones, it needed accurate atomic masses (TRIUMF)

Li: ground state hyperfine structure

Fermi contact interaction

$$H_{\rm hfs} = \frac{2\,g_N\,Z\,\alpha}{3\,m\,M}\,\sum_a \vec{I}\cdot\vec{\sigma}_a\,\pi\,\delta^3(r_a)\,.$$

Finite nuclear size effect:

$$H_{\rm size} = -H_{\rm hfs}\,2\,Z\,\alpha\,m\,r_Z$$

where

$$r_Z = \int d^3r\,d^3r'\,\rho_E(r)\,\rho_M(r')\,|\vec{r}-\vec{r}'|$$

Li: hyperfine structure

	^{7}Li[MHz]	^{6}Li[MHz]
$A^{(4)}$	401.654 08(21)	152.083 69(11)
$A^{(5)}_{\rm rec}$	−0.004 14	−0.001 80
$A^{(6)}$	0.260 08(2)	0.098 48(1)
$A^{(7)}$	−0.010 2(13)	−0.003 9(5)
$A_{\rm the}$ (point nucleus)	401.899 8(13)	152.176 5(5)
$A_{\rm exp}$	401.752 043 3(5)	152.136 839(2)
$(A_{\rm exp} - A_{\rm the})/A_{\rm exp}$	−368(3) ppm	−261(3) ppm
r_Z	3.25(3) fm	2.30(3) fm
r_E	2.390(30) fm	2.540(28) fm

significant dependence of r_Z on the isotope

Problems to solve

- accurate description of nuclear structure effects in atomic hfs
- absolute determination of nuclear charge radii
- development of computational methods for the 5-particle systems, such as boron

Collaborators

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