Recoil corrections in low-energy antikaon-deuteron scattering

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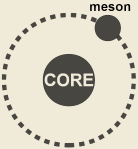
MOTIVATION

- The $\bar{K}N$ system as a testing ground for low energy $SU(3)$ meson-baryon dynamics
- Two fundamental quantities: $a_{I=0}$ and $a_{I=1}$

Two experiment(s):



- $K^{-}p$: energy shift and width of the $1s$ level for kaonic hydrogen in SIDDHARTA[1]@ DAΦNE

- $K^{-}d$: X-ray yield of kaonic deuterium derived. Important for:
  - planned upgrade to SIDDHARTA-2
  - Kaon implantation experiment @ J-PARC

[1] **Bazzi et al. (2011)**

MOTIVATION

Experiment $\leftrightarrow$ Theory:

1. "Unitarized ChPT" for meson-baryon scattering
   a) Extract a_{K^-p} from modified Deser-type formula[2]
   b) Construct a unitary amplitude from chiral potential[3], adjust free parameters
2. Three-body Faddeev equation:
   a) Assume some NN and $\bar{K}N$ potential
   b) Determine the $\bar{K}NN$ amplitude numerically

	$\bar{K}N$ system	$\bar{K}d$ system
1.	a_0, a_1 calculated ✓	A not addressed X
2.	potential assumed X	A predicted ✓

Explicit relation btw. a_1, a_0 and A?

[2] **Meißner, Raha, Rusetsky (2004)**
[3] **Ikeda, Hyodo, Weise (2012); MM, Meißner (2012); Borasoy, Nißler, Weise (2006);...**

MOTIVATION

- Multiple-scattering series $\rightarrow$ poor convergence
- Resummation of the series $\hat{=}$ static approximation $(m_N \rightarrow \infty)$ to Faddeev Equations: *Brueckner-type formula*[4]

$$A_{st} = \langle |\Psi(r)|^2 \frac{4r\tilde{a}_0\tilde{a}_1 + r^2(\tilde{a}_0 + 3\tilde{a}_1)}{2r^2 + r(\tilde{a}_0 - \tilde{a}_1) - 2\tilde{a}_1\tilde{a}_0} \rangle_r$$

$\tilde{a} = (1+\xi)a,\ \xi = \frac{M_K}{m_N}$

! Nucleon recoil corrections start with $\sqrt{\xi} \sim 0.7$
But numerically:

→ at NLO: 15% effect in double scattering[5]
→ numerical solutions of Faddeev eqn. suggest $\sim 15\%$ effect[6]

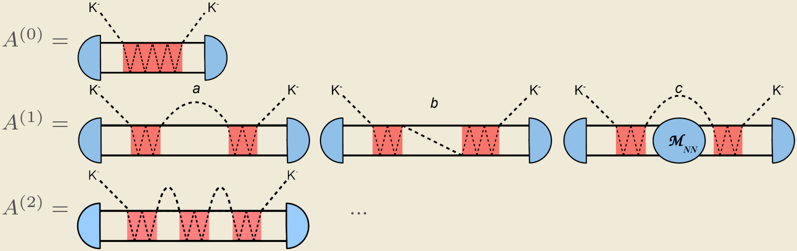
[4] **Kamalov, Oset, Ramos (2001)**
[5] **Baru, Epelbaum, Rusetsky (2009)**
[6] **Gal (2008)**

IDEA

- Systematic analysis of recoil from non-rel. eff. Lagrangian:
  Assume a single interaction channel, specified by scattering length a

$$\begin{aligned} A = & \ \tilde{a} + \tilde{a}^2 g + \tilde{a}^3 g^2 + \cdots \\ = & \ \{\tilde{a} + \tilde{a}^2 g_{\mathsf{st}} + \cdots\} + \{\tilde{a} + \tilde{a}^2 g_{\mathsf{st}} + \cdots\} \ (\Delta g) \ \{\tilde{a} + \tilde{a}^2 g_{\mathsf{st}} + \cdots\} + \cdots \\ = & \ A_{\mathsf{st}} + A^{(1)} + A^{(2)} + \ldots, \end{aligned}$$

  where $g := g_{\mathsf{st}} + \Delta g$ and $g_{\mathsf{st}} = 1/r$ are the full and static kaon propagators.
- Set of Feynman diagrams in a realistic case:



IDEA

- One insertion ⇒ *Brueckner formula*
- Two inserions ⇒ three diagramms:

$$A^a = \langle f(p,l)\Psi(r)e^{-i(\vec{p}+\vec{l}/2)\cdot\vec{r}} X_a(r,r',a_1,a_0)\Psi(r')e^{-i(\vec{p}+\vec{l}/2)\cdot\vec{r}'}\rangle_{p,l,r,r'}$$
$$A^b = \langle f(p,l)\Psi(r)e^{-i(\vec{p}+\vec{l}/2)\cdot\vec{r}} X_b(r,r',a_1,a_0)\Psi(r')e^{-i(\vec{p}-\vec{l}/2)\cdot\vec{r}'}\rangle_{p,l,r,r'}$$
$$A^c = \langle d(p,l)d(q,l)M_{NN}(p,q,l)\Psi(r)e^{-i(\vec{p}+\vec{l}/2)\cdot\vec{r}} X_c(r,r',a_1,a_0)\Psi(r')e^{-i(\vec{q}+\vec{l}/2)\cdot\vec{r}'}\rangle_{p,q,l,r,r'}$$

$$d(p,l) := \frac{1}{l^2(1+\xi/2)+2\xi(p^2+m_N\epsilon_d)^2}\,, \quad f(p,l) := d(p,l) - \frac{1}{(1+\xi)l^2}$$

- Higher order terms: two, three, ... insertions can be calculated analogously
- Computational challenge: rising number of integrals → one insertion is done, two is in preparation

IS THIS A GOOD APPROACH?

IF “YES“, WHAT DOES IT PREDICT?

TEST OF THE FRAMEWORK

- Convergence of $A^{(n)}$ series in $\sqrt{\xi}$

  $\hookrightarrow$ Sign for a good counting scheme of $A = A_{\mathsf{st}} + A^{(1)} + A^{(2)} + \ldots$

- Uniform expansion[7] of $A^{(n)}$:

  - Independent of the regularization procedure
  - Applicable to any Feynman diagram

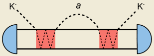
  Recipe:

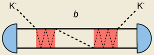
  a) Identify the momentum scales, e.g. small scale λ, large scale Λ.
  b) Expand the integrand $f(\lambda, q, \Lambda)$ in the low-, high- and intermediate momentum regime, i.e $\lambda \sim q \ll \Lambda$, $\lambda \ll q \sim \Lambda$ and $\lambda \ll q \ll \Lambda$.
  c) $\int_q f(\lambda, q, \Lambda) = \int_q f_l(\lambda, q, \Lambda) - \int_q f_i(\lambda, q, \Lambda) + \int_q f_h(\lambda, q, \Lambda)$.

[7] **Baru, Epelbaum, Rusetsky (2009) [double scattering]**

Expansion in powers of $\sqrt{\xi}$

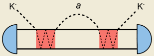
- Isospin (NN interm. state) decomposition reveals cancellation pattern:

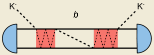
  1) I=0 (S=1,L=1): 

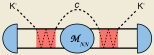
  – 

  $\hookrightarrow$ cancels exactly at $\mathcal{O}(\sqrt{\xi})$

  2) I=1 (S=1,L=0):

  \+ 

  \+ 

  $\hookrightarrow$ cancels exactly at $\mathcal{O}(\sqrt{\xi})$, if X=const.

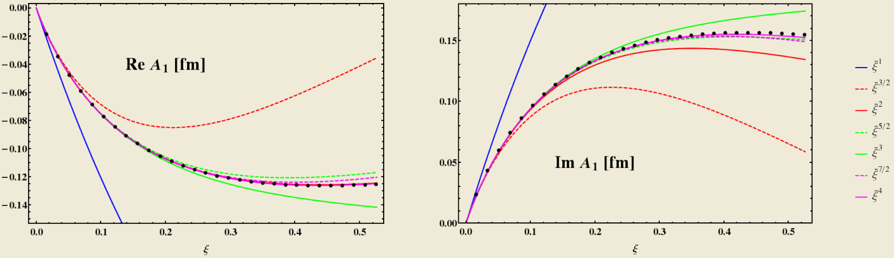
  orthogonality of bound state and continuum w.f.

  $\hookrightarrow$ in general screened by X

- NN-interaction parametrized (*for convergence test!*) by Hulthen potential ($\beta = 1.4 fm^{-1}$):

$$V_{NN}(p,q) = \lambda g(p) g(q)\,, \quad g(p) = \frac{1}{\beta^2 + p^2}\,, \quad \lambda = 32\pi m_N \beta\,(\beta+\gamma)^2$$

Results of the expansion, I=1

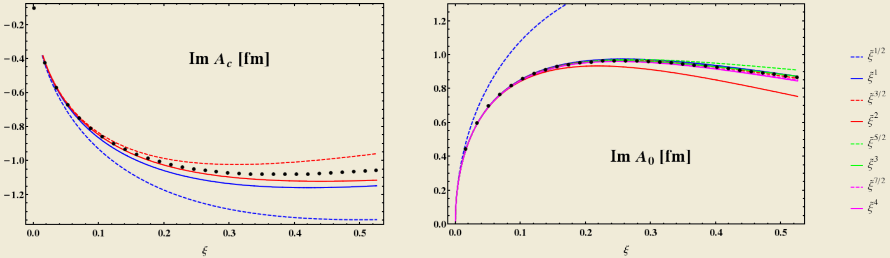


- Expansion in $\tilde{\xi} := \xi/(1+\xi/2)$ yields:

$$A_1 \quad = \frac{8\pi}{(1+\xi/2)^2}\left(c_{1,1}\tilde{\xi} + c_{1,1}\tilde{\xi}^2 + \ldots + b_{1,1}\tilde{\xi}^{3/2} + b_{1,2}\tilde{\xi}^{5/2} + \ldots\right)$$

- Convergence after a few orders in $\tilde{\xi}^{1/2}$

Results of the expansion, I=0



- Expansion in $\tilde{\xi} := \xi/(1+\xi/2)$ yields:

$$A_0 = \frac{8\pi}{(1+\xi/2)^2}\left(c_{0,1}\tilde{\xi} + c_{0,1}\tilde{\xi}^2 + ... + b_{0,0}\tilde{\xi}^{1/2} + b_{0,1}\tilde{\xi}^{3/2} + ...\right)$$

$$A_c = \frac{8\pi}{(1+\xi/2)^2}\left(C_1\tilde{\xi} + C_2\tilde{\xi}^2 + ... + B_1\tilde{\xi}^{1/2} + B_2\tilde{\xi}^{3/2} + ...\right)$$

- Convergence after a few orders in $\tilde{\xi}^{1/2}$
- LO sizable cancellations: $b_{0,0} = -0.047 + i0.154 \leftrightarrow B_1 = +0.047 - i0.132$

IS THIS A GOOD APPROACH? ✓

WHAT DOES IT PREDICT?

ONE INSERTION

- NN interaction: Hulthen and PEST[8] potential (short range physics)
- $\bar{K}N$: ($a_1 = -1.62 + i0.78$ fm, $a_0 = +0.18 + i0.68$ fm)[9]

Hulthen potential

A_{st}	$-1.49 + i1.19$	
$A^{(1)}$	A_1	$-0.13 + i0.16$
	$\Delta A_{\text{st},1}$	$+0.12 - i0.20$
	A_0	$-0.27 + i0.87$
	$\Delta A_{\text{st},0}$	$-0.12 + i0.33$
	$A^{(c)}$	$+0.35 - i1.06$
	Sum:	$-0.03 + i0.09$
$A_{\text{st}} + A^{(1)}$	**-1.52+i1.27**	

PEST potential

A_{st} [fm]	$-1.55 + i1.25$	
$A^{(1)}$ [fm]	A_1	$-0.13 + i0.18$
	$\Delta A_{\text{st},1}$	$+0.13 - i0.22$
	A_0	$-0.29 + i0.97$
	$\Delta A_{\text{st},0}$	$-0.11 + i0.34$
	$A^{(c)}$	$+0.36 - i1.19$
	Sum:	$-0.04 + i0.08$
$A_{\text{st}} + A^{(1)}$	**-1.59+i1.32**	

[8] **Zankel et al. (1983)**
[9] **Shevchenko (2012)**

HIGHER ORDERS? *(preliminary)*

- Do we need the two, three, ... recoil insertions corrections?
- Formally they start at $(\xi^{1/2})^2$, $(\xi^{1/2})^3$, ...
- First estimation of two insertion corrections (Hulthen):

A_{st} [fm]		$A^{(1)}$ [fm]		$A^{(2)}$ [fm]	
		I		I	
		1	$-0.00 - i0.04$	11	$+0.01 - i0.01$
		0	$-0.03 + i0.13$	00	$+0.04 + i0.09$
				10	$+0.01 - i0.00$
$\sum$	$-1.49 + i1.19$	$\sum$	$-0.03 + i0.09$	$\sum$	$+0.06 + 0.07$

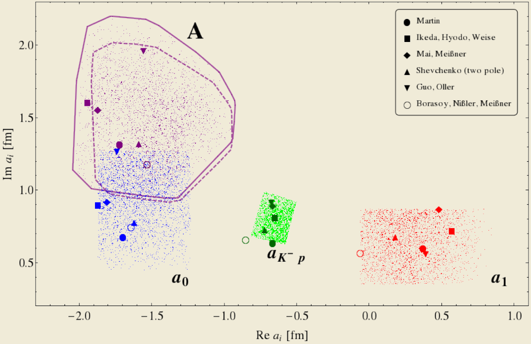
- Estimate two recoil corrections: $\xi^{1/2} Im(A_1^{(1)}) = -0.03$ fm, $\xi^{1/2} Im(A_0^{(1)}) = 0.09$ fm

⇝ Estimate three recoil corrections: $\xi Im(A_0^{(1)}) = 0.07 \text{ fm} \approx 6\% A_{st}$

⇝ Further cancellations might reduce the size of recoil corrections

FULL RESULTS

- Randomly distributed $\bar{K}N$ scattering lengths
  Range: Literature values; Restriction: SIDDHARTA exp.



- $(A_{\rm st} + A^{(1)})$ depends strongly on the choice of $\bar{K}N$ s.l.

$\Rightarrow$ ***Precise exp. data on $\bar{K}d$ system will restrict a_0 and a_1 significantly!***

Conclusion

- ✓ Analytic formulas for multiple insertion corrections
- ✓ Expansion of $A^{(1)}$ in powers of ξ converges
- ✓ Large cancellations at LO in one insertion corr.
- ✓ One insertion corr.: $7-8\%$ of the static result $\Rightarrow$ Good news!
- ✓ A is sensitive to a_0 and a_1 $\Rightarrow$ Good news! for future experiment on kaonic deuterium

- ! Finite range/relativistic corrections
- ! Investigation of results for two insertion correction

... *in progress*

GRAZIE!