Continuum BCS and pair transfer reactions with weakly bound nuclei

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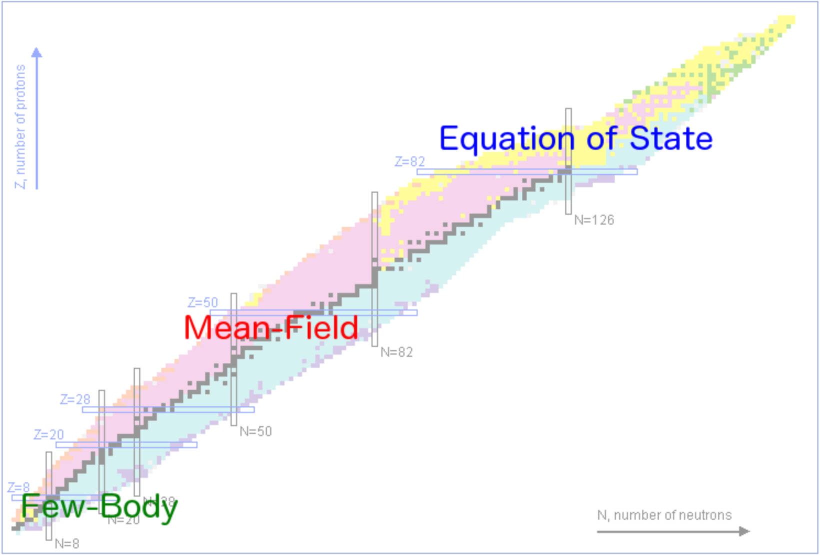
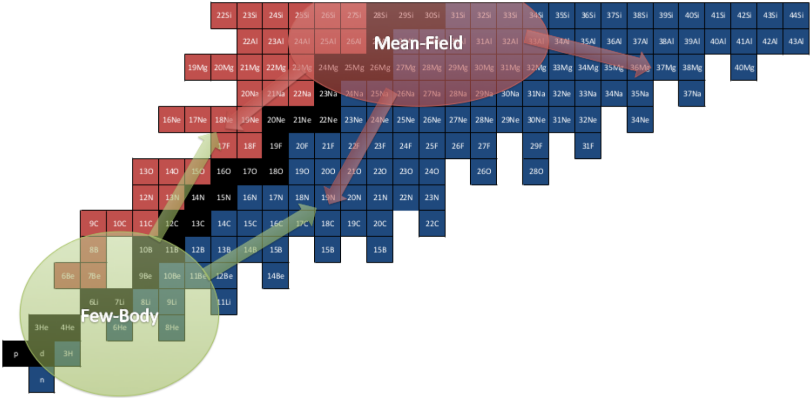
Trento, 14th May 2015


Z, number of protons
Equation of State
Z=82
N=126
Z=50
Mean-Field
N=82
Z=28
Z=20
N=50
Z=8
Few-Body
N=8
N, number of neutrons


Mean-Field
Few-Body
22Si 23Si 24Si 25Si 26Si 27Si 28Si 29Si 30Si 31Si 32Si 33Si 34Si 35Si 36Si 37Si 38Si 39Si 40Si 41Si 42Si 43Si 44Si
22Al 23Al 24Al 25Al 26Al 31Al 32Al 33Al 34Al 35Al 36Al 37Al 38Al 39Al 40Al 41Al 42Al 43Al
19Mg 20Mg 21Mg 22Mg 23Mg 24Mg 25Mg 26Mg 27Mg 28Mg 29Mg 30Mg 31Mg 32Mg 33Mg 34Mg 35Mg 36Mg 37Mg 38Mg 40Mg
20Na 21Na 22Na 23Na 24Na 25Na 26Na 27Na 28Na 29Na 30Na 31Na 32Na 33Na 34Na 35Na 37Na
16Ne 17Ne 18Ne 19Ne 20Ne 21Ne 22Ne 23Ne 24Ne 25Ne 26Ne 27Ne 28Ne 29Ne 30Ne 31Ne 32Ne 34Ne
17F 18F 19F 20F 21F 22F 23F 24F 25F 26F 27F 29F 31F
13O 14O 15O 16O 17O 18O 19O 20O 21O 22O 23O 24O 26O 28O
12N 13N 14N 15N 16N 17N 18N 19N 20N 21N 22N 23N
9C 10C 11C 12C 13C 14C 15C 16C 17C 18C 19C 20C 22C
8B 10B 11B 12B 13B 14B 15B 15B 15B
6Be 7Be 9Be 10Be 11Be 12Be 14Be
6Li 7Li 8Li 9Li 11Li
3He 4He 6He 8He
p d 3H
n

Mean Field plus Continuum

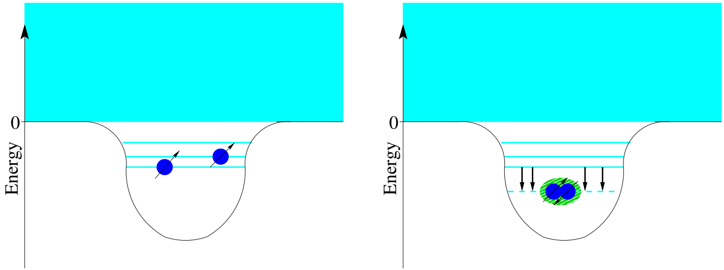
⇒ Several (more complex) options: HFB, CSM, GSM...

BCS is the simplest model to evaluate pairing

- ✓ Look for new features
- ? How far can we go within a simple pairing description?

Superfluidity and pairing

⇒ Neutrons tends to form strongly correlated pairs:



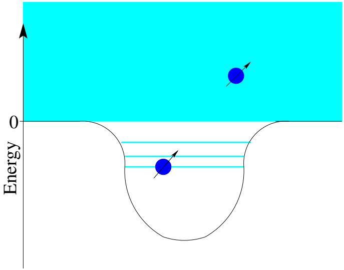
The general Hamiltonian in second quantization:

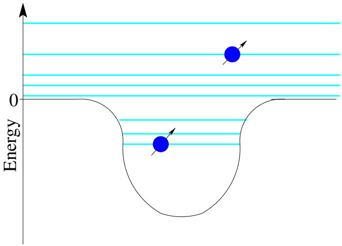
$$\mathcal{H} = h_{s.p.} + \sum_{klk'l'} v_{klk'l'} a_k^\dagger a_l^\dagger a_{k'} a_{l'}$$

Discretizing the Continuum

⇒ First we calculate the eigenvalues and eigenstates of $h_{s.p.}$:

$$\sum_{n}^{N} |n\rangle\langle n| \approx 1 \Rightarrow h_{s.p.} \approx \sum_{n,m} |n\rangle\langle n|h_{s.p.}|m\rangle\langle m|$$

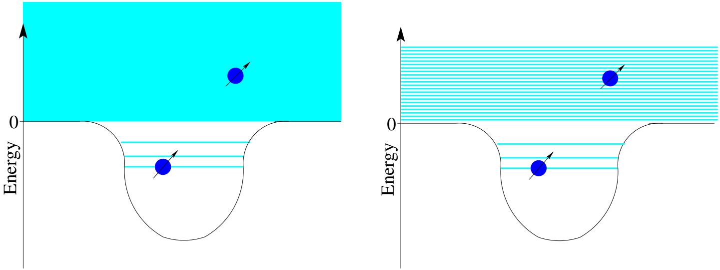




Discretizing the Continuum

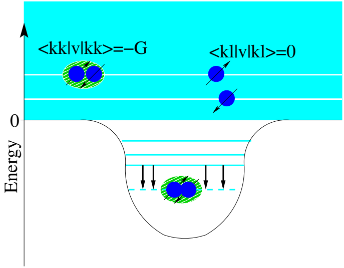
⇒ First we calculate the eigenvalues and eigenstates of $h_{s.p.}$:

$$\sum_{n}^{N\to\infty} |n\rangle\langle n| = 1 \Rightarrow h_{s.p.} = \sum_{n,m} |n\rangle\langle n|h_{s.p.}|m\rangle\langle m|$$



BCS

$$\mathcal{H} = h_{s.p.} - \sum_{kk'} G P_k^\dagger P_{k'}$$

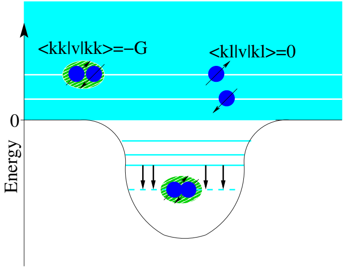


$$\Delta = \frac{1}{4}\sum_k \frac{(2j+1)G\Delta}{\sqrt{(\varepsilon_k - E_F)^2 + \Delta^2}}$$

$$\Delta = 0 \rightarrow |g.s.\rangle = 100\%(S_j)^n$$
$$\Delta \neq 0 \rightarrow |g.s.\rangle = a(S_j)^n + \ldots$$

BCS

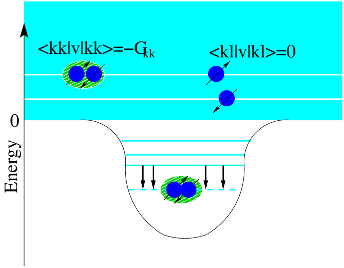
$$\mathcal{H} = h_{s.p.} - \sum_{kk'} G P_k^\dagger P_{k'}$$



- **x** Infinite number of continuum states $\Rightarrow \langle \mathcal{H} \rangle \to -\infty$

Generalized BCS

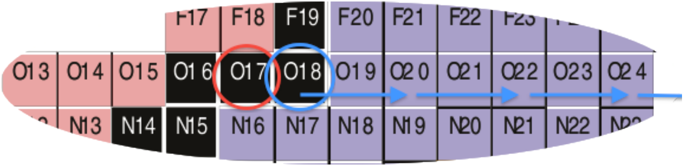
$$\mathcal{H} = h_{s.p.} - \sum_{kk'} G_{kk'} P_k^\dagger P_{k'}$$



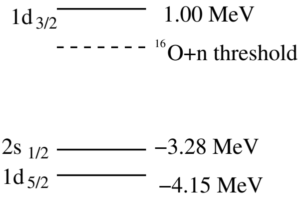
- ⇒ S.p. Continuum discretized with a HO/THO basis (N real functions)
- ⇒ $G_{kk'}$ calculated with a Density Dependent Delta Interaction

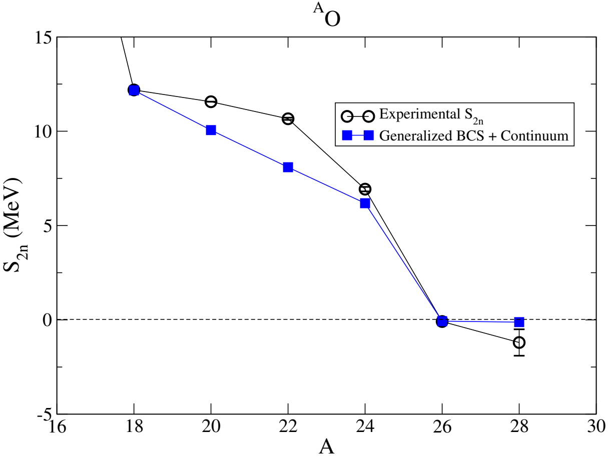
- ✓ $N \uparrow \Rightarrow G_{kk'} \downarrow$
- ✓ All states treated naturally
- ✗ $\Delta \rightarrow \Delta_k$: System of N (non-linear) equations

Application to oxygen isotopes

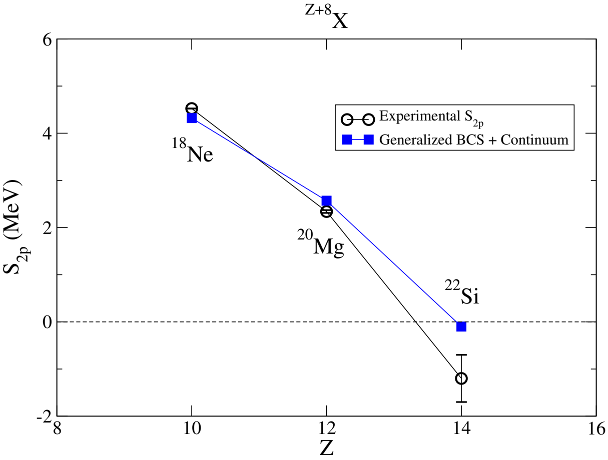


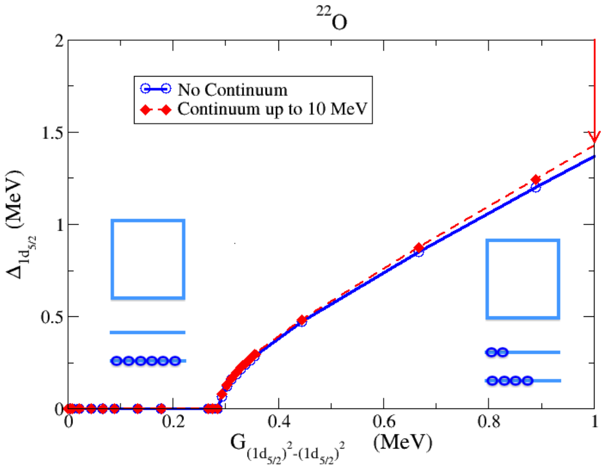
- ⇒ Single particle continuum fixed to $^{17}O=^{16}O+n$
- ⇒ Pairing strength fixed to $^{18}O=^{16}O+2n$
- ⇒ Adding pairs of neutrons towards the drip line and beyond



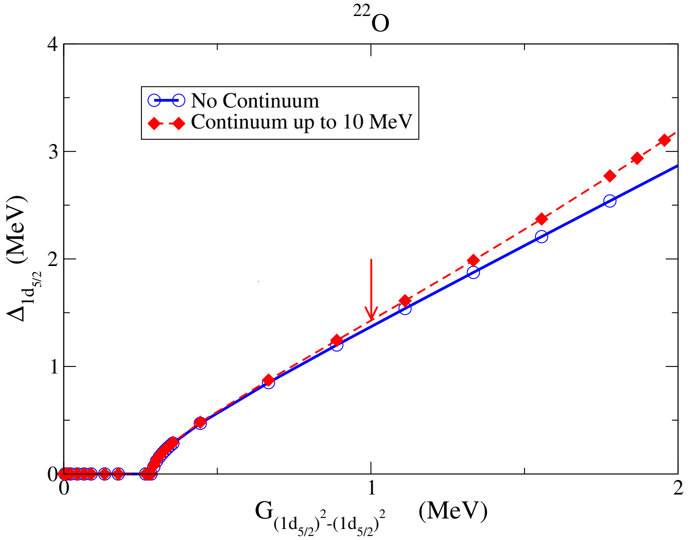


^{A}O
15
10
5
0
-5
S_{2n} (MeV)
16
18
20
22
24
26
28
30
A
Experimental S_{2n}
Generalized BCS + Continuum

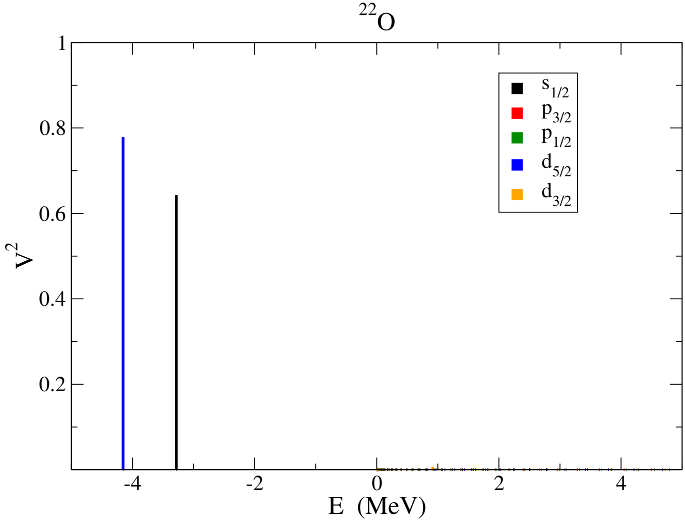

Z+8X
6
4
2
0
-2
8
10
12
14
16
Z
S2p (MeV)
Experimental S2p
Generalized BCS + Continuum
18Ne
20Mg
22Si

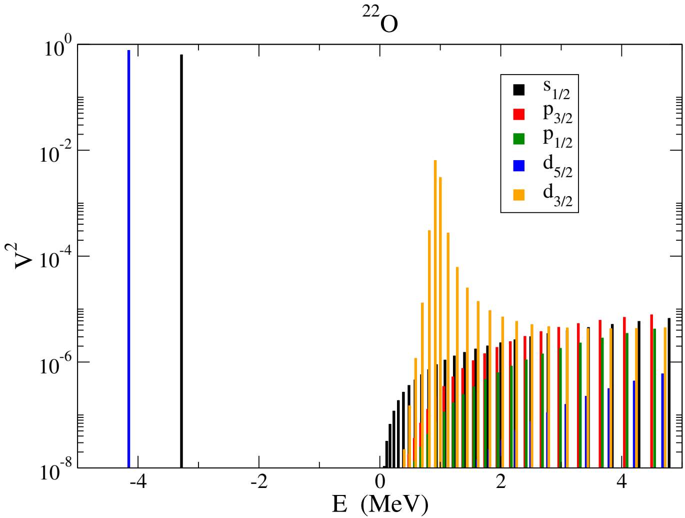


^{22}O
No Continuum
Continuum up to 10 MeV
$\Delta_{1d_{5/2}}$ (MeV)
$G_{(1d_{5/2})^2-(1d_{5/2})^2}$ (MeV)
0
0.2
0.4
0.6
0.8
1
0.5
1.5
2

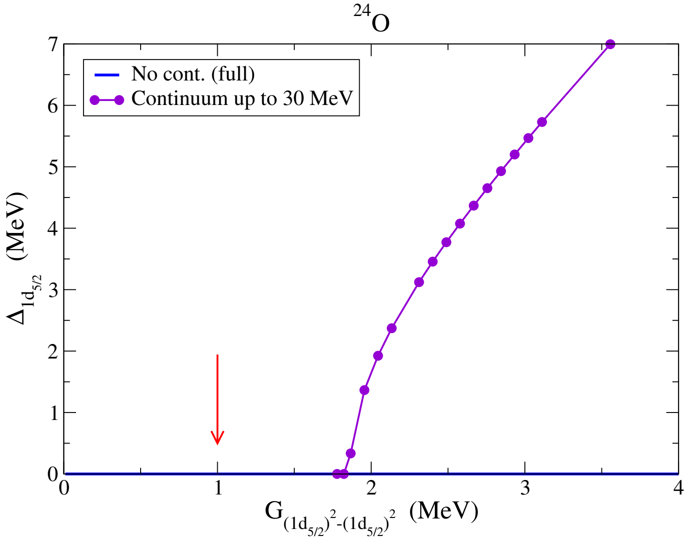


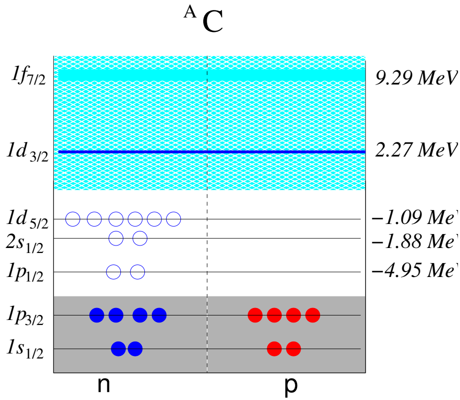
^{22}O
No Continuum
Continuum up to 10 MeV
$\Delta_{1d_{5/2}}$ (MeV)
$G_{(1d_{5/2})^2-(1d_{5/2})^2}$ (MeV)

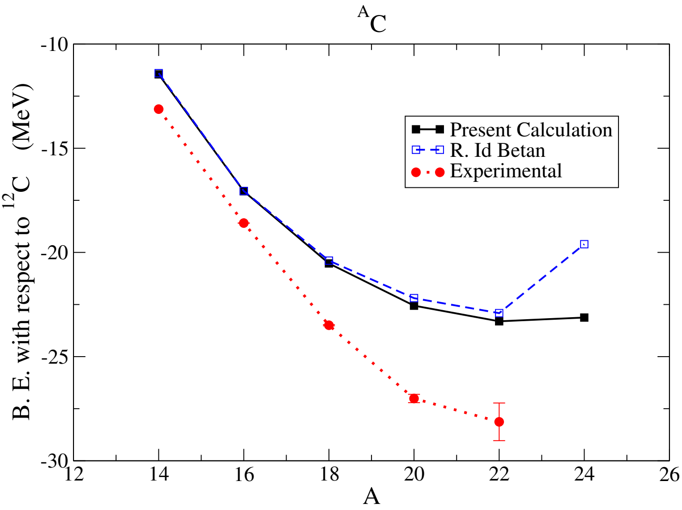

22O
1
0.8
0.6
0.4
0.2
V2
-4
-2
0
2
4
E (MeV)
s1/2
p3/2
p1/2
d5/2
d3/2



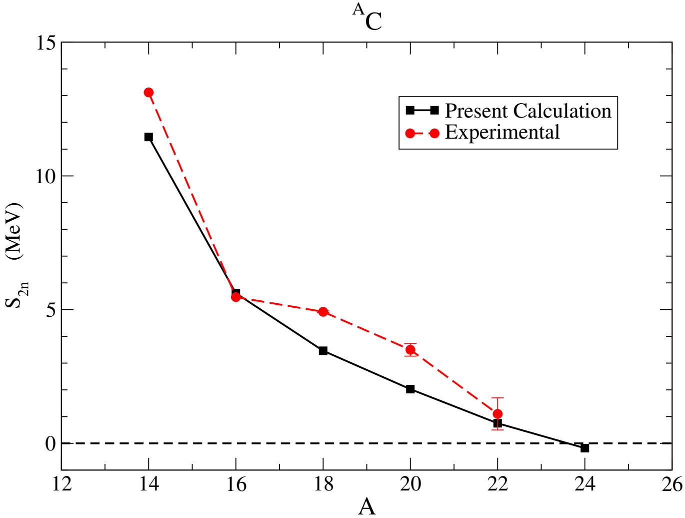
^{22}O
V^2
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
-4
-2
0
2
4
E (MeV)
$s_{1/2}$
$p_{3/2}$
$p_{1/2}$
$d_{5/2}$
$d_{3/2}$


^{24}O
No cont. (full)
Continuum up to 30 MeV
$\Delta_{1d_{5/2}}$ (MeV)
$G_{(1d_{5/2})^2-(1d_{5/2})^2}$ (MeV)
0
1
2
3
4
5
6
7
0
1
2
3
4

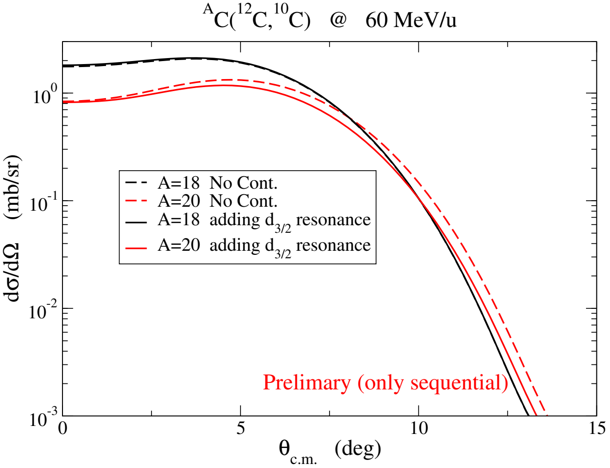

^{A}C
$1f_{7/2}$
9.29 MeV
$1d_{3/2}$
2.27 MeV
$1d_{5/2}$
−1.09 Me
$2s_{1/2}$
−1.88 Me
$1p_{1/2}$
−4.95 Me
$1p_{3/2}$
$1s_{1/2}$
n
p



^{A}C
-10
-15
-20
-25
-30
12
14
16
18
20
22
24
26
A
B. E. with respect to ^{12}C (MeV)
Present Calculation
R. Id Betan
Experimental



^{A}C
15
10
5
0
S_{2n} (MeV)
12
14
16
18
20
22
24
26
A
Present Calculation
Experimental



$\langle ^{12}C | ^{10}C \rangle : (1p_{3/2})^4$ should increase the effect of the $d_{3/2}$ resonance
Lay, Fortunato, Vitturi PRC89 (2014) 034618

	No Cont.	Adding $d_{3/2}$ resonance
$\sigma_{18\to20}$	0.106 mb	0.107 mb
$\sigma_{20\to22}$	0.080 mb	0.070 mb

The ratio should vary with:

$\Rightarrow$ the presence of the Continuum

$\Rightarrow$ the stregnth of the pairing

Conclusions

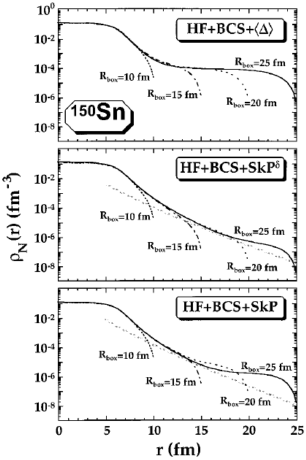
- It is possible to consistently include PS into a Generalized BCS scheme
- Occupancies of the continuum states follows the expected level density
- We applied this scheme to oxygen isotopes reproducing S_{2n} trend
- Interesting features but for unnatural pairing strengths

Next Steps

- Apply to more and more cases
- Look for an isotopic chain with s.p. energies closer to the threshold
- Calculate 2n-transfer cross sections (2^{nd} order DWBA g.s.$\rightarrow$g.s.)

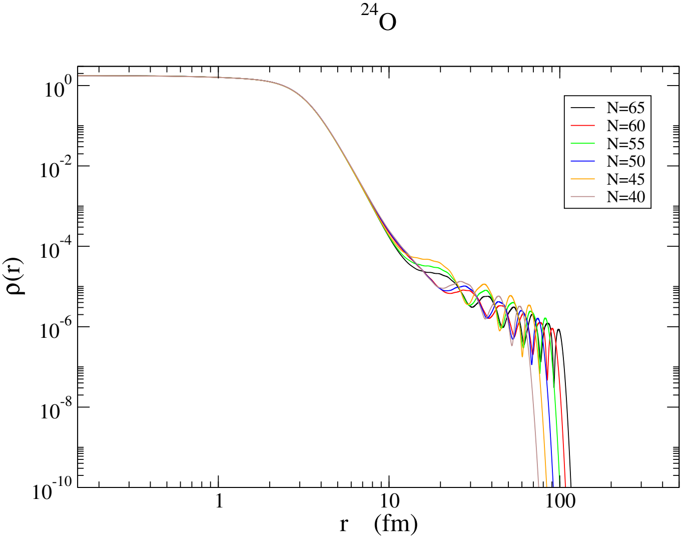
Thank you!!

Some limitations

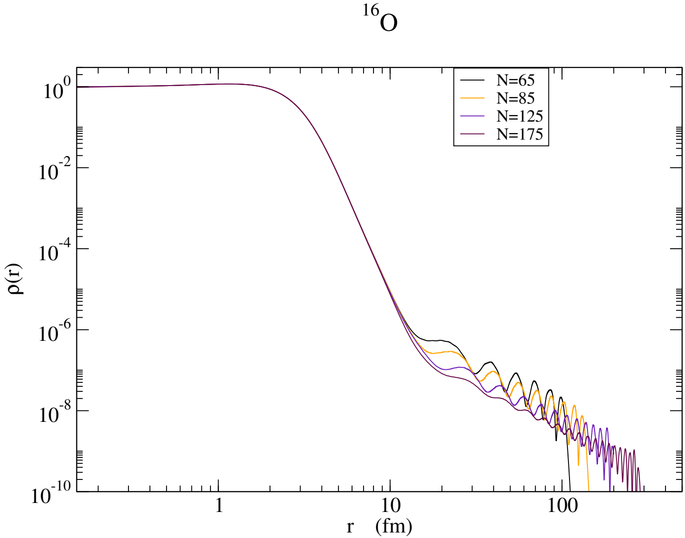


From J. Dobaczewski *et al* PRC**53**, 2809 (1996)

Some limitations



Some limitations



Some limitations

A	$0 < r < 7$ fm	7 fm$< r <$120 fm	% outside	$0 < r < 10$ fm
16	7.973	0.027	0.34%	7.985
18	9.915	0.085	0.85%	9.954
20	11.836	0.164	1.37%	11.909
22	13.722	0.278	1.99%	13.831
24	15.315	0.685	4.28%	15.473
26	15.402	2.598	-	15.568
28	15.416	4.584	-	15.583
30	15.416	6.584	-	15.584
32	15.416	8.584	-	15.584

Transformed Harmonic Oscillator basis

Analytic LST from Karataglidis *et al.*,PRC71,064601(2005)

$$s(r) = \frac{1}{\sqrt{2}b}\left[\frac{1}{\left(\frac{1}{r}\right)^m + \left(\frac{1}{\gamma\sqrt{r}}\right)^m}\right]^{\frac{1}{m}}$$

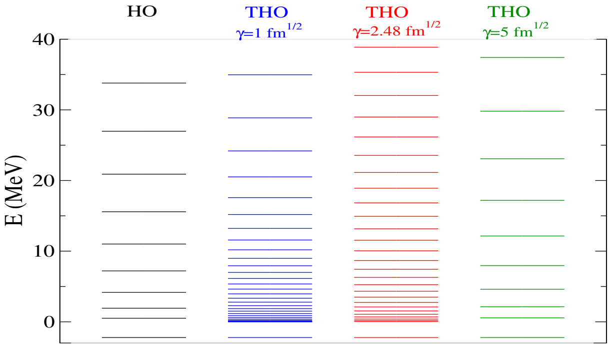
HO vs THO:

$$\phi(s) \longmapsto e^{-\left(\frac{s}{b}\right)^2} \quad \Longrightarrow \quad \phi[s(r)] \longmapsto e^{-\frac{\gamma^2}{2b^2}r}$$

- Correct asymptotic behaviour for bound states.
- Range controlled by the parameters of the LST.

THO parameters

- b is treated as a variational parameter to minimize g.s. energy
- Then $\frac{\gamma}{b}$ controls the density of states:

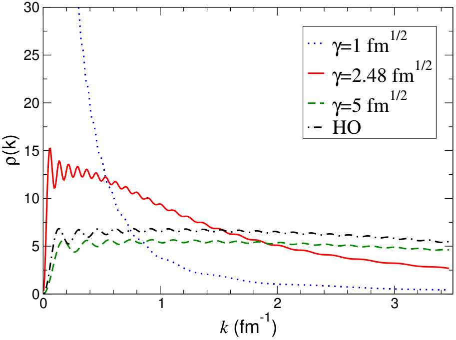


- γ can be also used to look for resonances

Energy distribution of pseudo-states

Density of states

$$\rho_\ell^{(N)}(k) = \sum_{n=1}^{N} \langle \varphi_\ell(k) | \varphi_{n,\ell}^{(N)} \rangle$$



The Smoothing Process

For any operator

$$\begin{aligned} O(\varepsilon) &= \langle k|\hat{O}|g.s.\rangle \\ &= \langle k|\sum_n |n\rangle\langle n|\hat{O}|g.s.\rangle \\ &= \sum_n \rho^{(n)}(k)\langle n|\hat{O}|g.s.\rangle \end{aligned}$$

$\Rightarrow$ A continuous distribution in energy from discrete values of S matrix, B(Eλ), cross sections...