Effects of resonance widths on EoS and particle distributions

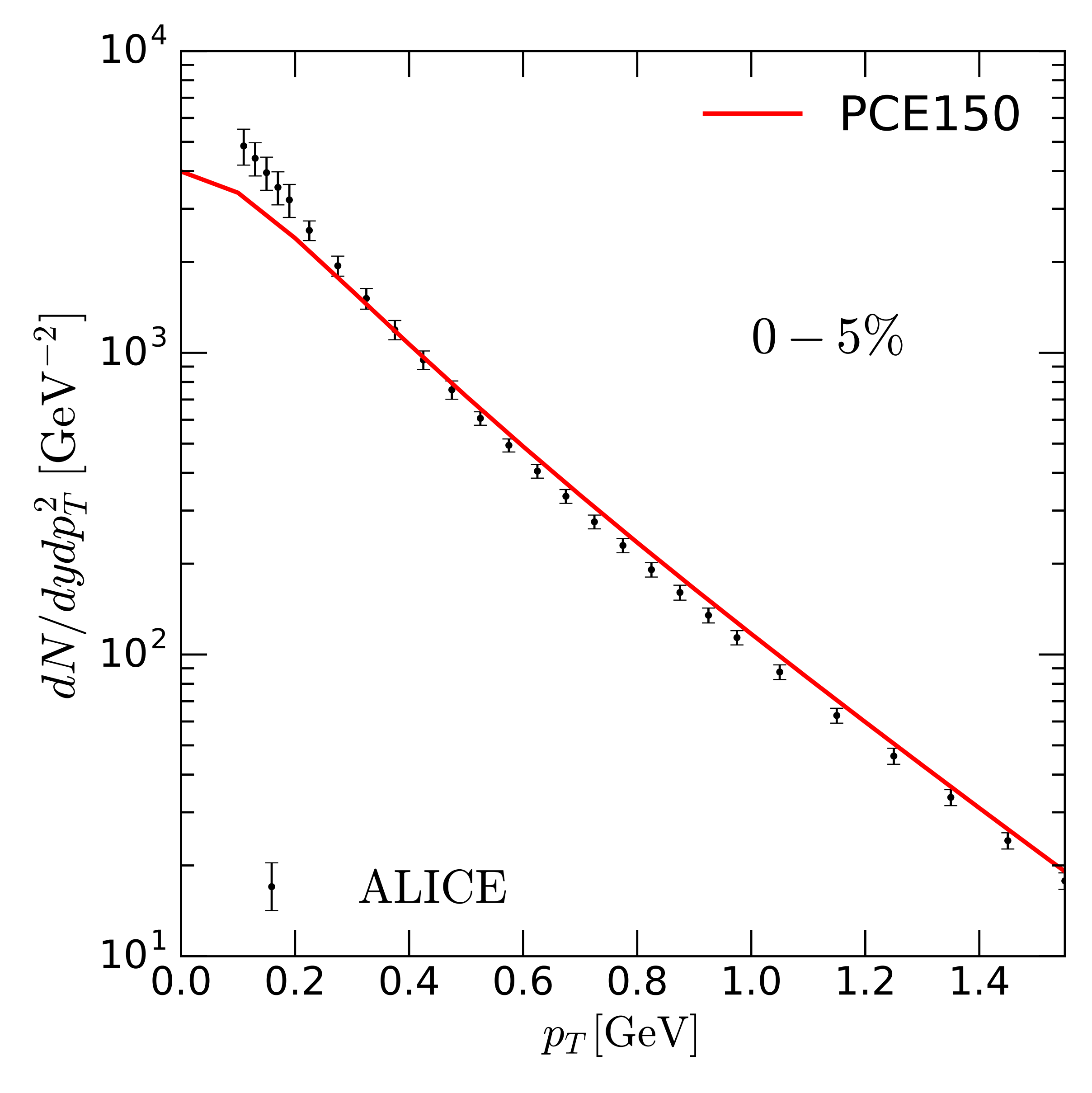
Pasi Huovinen
Uniwersytet Wrocławski

Phase diagram of strongly interacting matter:
From Lattice QCD to Heavy-Ion Collision Experiments
November 28, 2017, ECT*, Trento

in collaboration with
Pok Man Lo
and M. Marczenko, K. Redlich, C. Sasaki

The speaker has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 665778 via the National Science Center, Poland, under grant Polonez DEC-2015/19/P/ST2/03333

Pion p_T spectrum at LHC (Pb+Pb at $\sqrt{s_{NN}} = 2.76\,\mathrm{TeV}$)

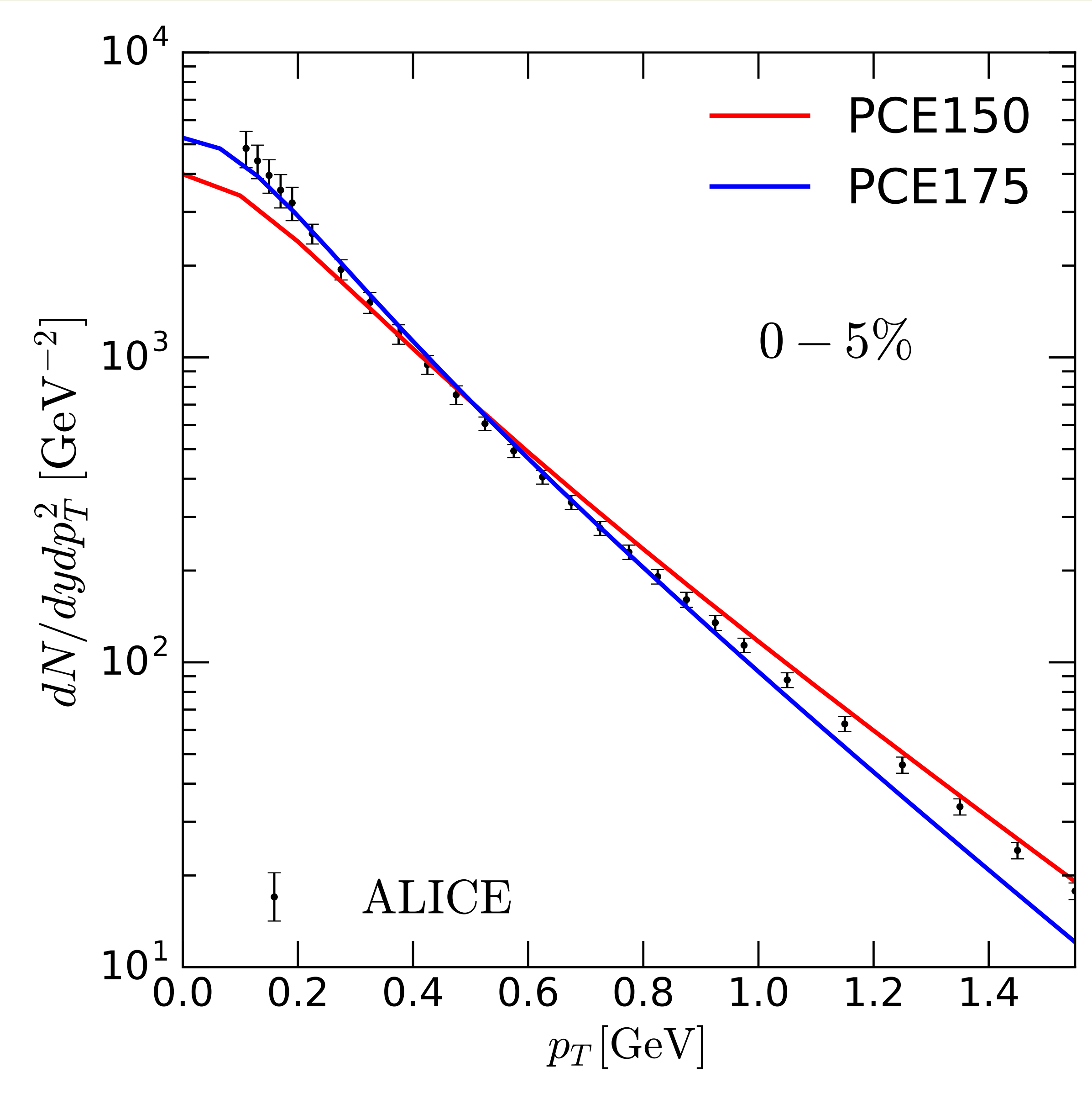


- **viscous hydro**
- **initial state:** **pQCD+saturation**
- $\tau_0 \approx 0.2\mathrm{fm}/c$

PCE150:
fit to π, K, p yields
no fit to spectrum

©H. Niemi

Pion p_T spectrum at LHC (Pb+Pb at $\sqrt{s_{NN}} = 2.76\,\mathrm{TeV}$)



- **viscous hydro**
- **initial state: pQCD+saturation**
- $\tau_0 \approx 0.2\mathrm{fm}/c$

PCE150:
fit to π, K, p yields
no fit to spectrum

PCE175:
no fit to yields
fits the spectrum

©H. Niemi

- **need more resonances**
- **yield proportional to Boltzmann factor**

$$N \propto \exp\left(-\frac{m}{T}\right)$$

- **need more resonances**

- **yield proportional to Boltzmann factor**

$$N \propto \exp\left(-\frac{m}{T}\right)$$

- **resonance mass?**

- **need more resonances**

- **yield proportional to Boltzmann factor**

$$N \propto \exp\left(-\frac{m}{T}\right)$$

- **resonance mass?**

- **usually no width, i.e. resonances have their pole mass**

effect of Breit-Wigner width on number density:

$$n = \int d^3\mathbf{p}\, f(p)$$

$$\Rightarrow n = \int d^3\mathbf{p} \int dm^2 \frac{d\rho}{dm^2}\, f(p,m)$$

where

$$\frac{d\rho}{dm^2} = \frac{1}{N}\frac{m_R\,\Gamma}{(m^2 - m_R^2)^2 + m_R^2\Gamma^2},$$

with normalisation

$$N = \int_{m_0}^{\infty} dm^2 \frac{m_R\,\Gamma}{(m^2 - m_R^2)^2 + m_R^2\Gamma^2}$$

For ρ^0 $m_R = 775.26\,\mathbf{MeV}$ **and** $\Gamma = 147.8\,\mathbf{MeV}$

effect of Breit-Wigner width on number density:

$$n = \int \mathrm{d}^3\mathbf{p}\, f(p)$$

$$\Rightarrow n = \int \mathrm{d}^3\mathbf{p} \int \mathrm{d}m^2 \frac{\mathrm{d}\rho}{\mathrm{d}m^2}\, f(p,m)$$

where

$$\frac{\mathrm{d}\rho}{\mathrm{d}m^2} = \frac{1}{N} \frac{m_R \Gamma}{(m^2 - m_R^2)^2 + m_R^2 \Gamma^2},$$

with normalisation

$$N = \int_{m_0}^{\infty} \mathrm{d}m^2 \frac{m_R \Gamma}{(m^2 - m_R^2)^2 + m_R^2 \Gamma^2}$$

For ρ^0 $m_R = 775.26\,\mathbf{MeV}$ **and** $\Gamma = 147.8\,\mathbf{MeV}$

effect of Breit-Wigner width on number density:

$$n = \int d^3\mathbf{p}\, f(p)$$

$$\Rightarrow n = \int d^3\mathbf{p} \int dm^2 \frac{d\rho}{dm^2} f(p,m)$$

where

$$\frac{d\rho}{dm^2} = \frac{1}{N}\frac{m_R\Gamma}{(m^2 - m_R^2)^2 + m_R^2\Gamma^2},$$

with normalisation

$$N = \int_{m_0}^{\infty} dm^2 \frac{m_R\Gamma}{(m^2 - m_R^2)^2 + m_R^2\Gamma^2}$$

For ρ^0 $m_R = 775.26\,\mathbf{MeV}$ **and** $\Gamma = 147.8\,\mathbf{MeV}$

effect of Breit-Wigner width on number density:

$$\begin{aligned} n &= \int \mathrm{d}^3\mathbf{p}\, f(p) \\ \Rightarrow n &= \int \mathrm{d}^3\mathbf{p} \int \mathrm{d}m^2 \frac{\mathrm{d}\rho}{\mathrm{d}m^2} f(p,m) \end{aligned}$$

where

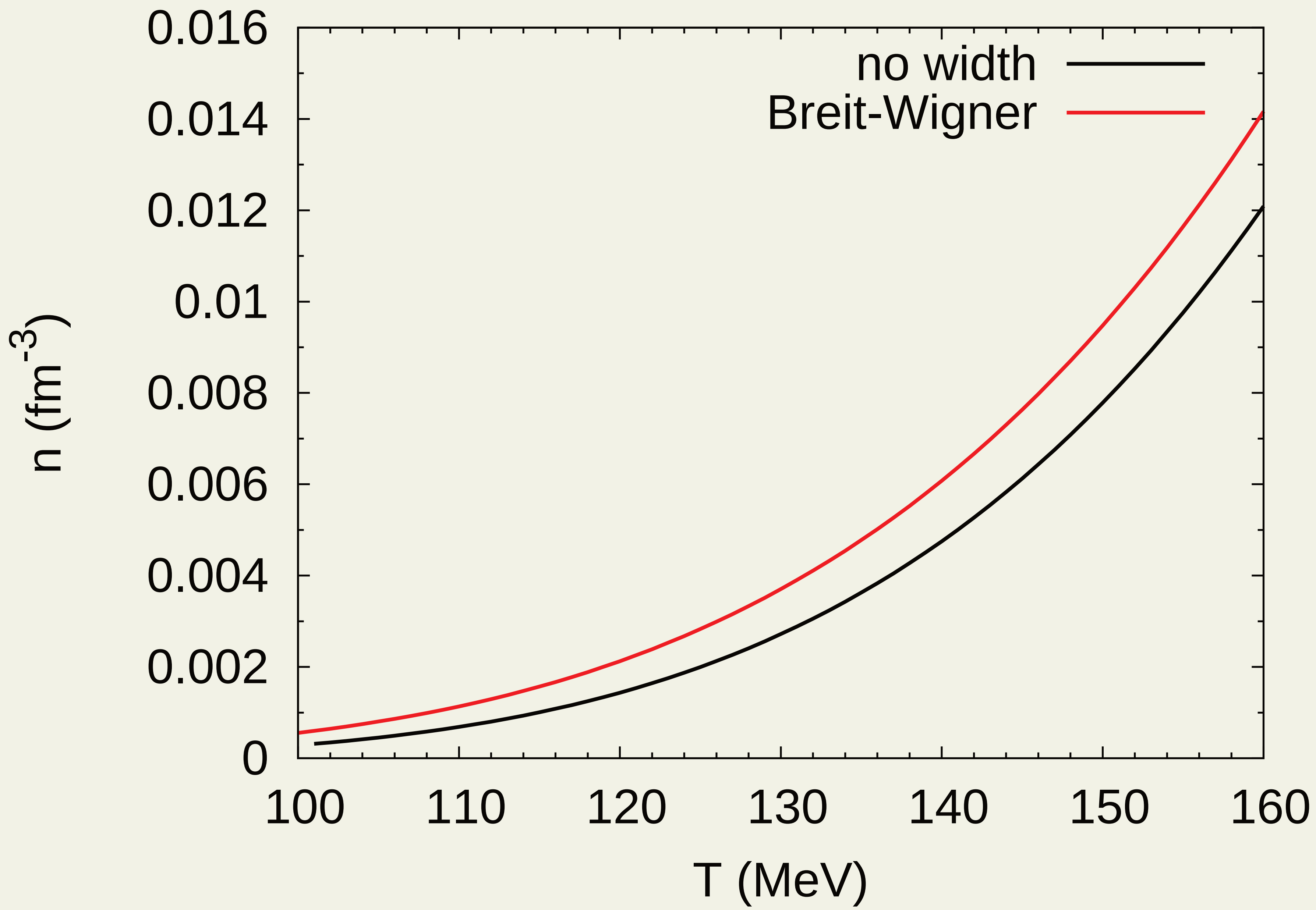
$$\frac{\mathrm{d}\rho}{\mathrm{d}m^2} = \frac{1}{N} \frac{m_R \Gamma}{(m^2 - m_R^2)^2 + m_R^2 \Gamma^2},$$

with normalisation

$$N = \int_{m_0}^{\infty} \mathrm{d}m^2 \frac{m_R \Gamma}{(m^2 - m_R^2)^2 + m_R^2 \Gamma^2}$$

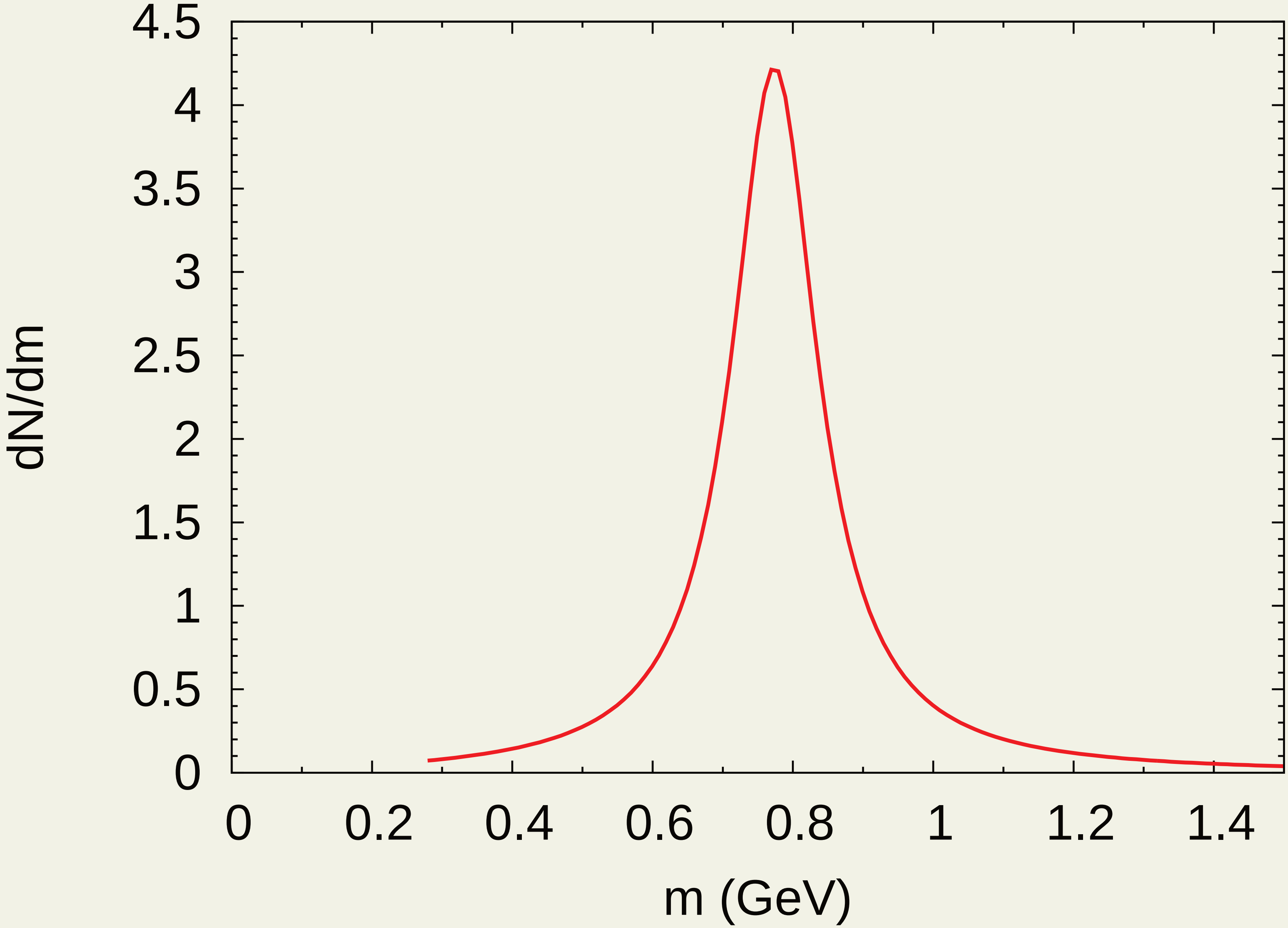
For ρ^0 $m_R = 775.26\,\mathbf{MeV}$ **and** $\Gamma = 147.8\,\mathbf{MeV}$

ρ-density

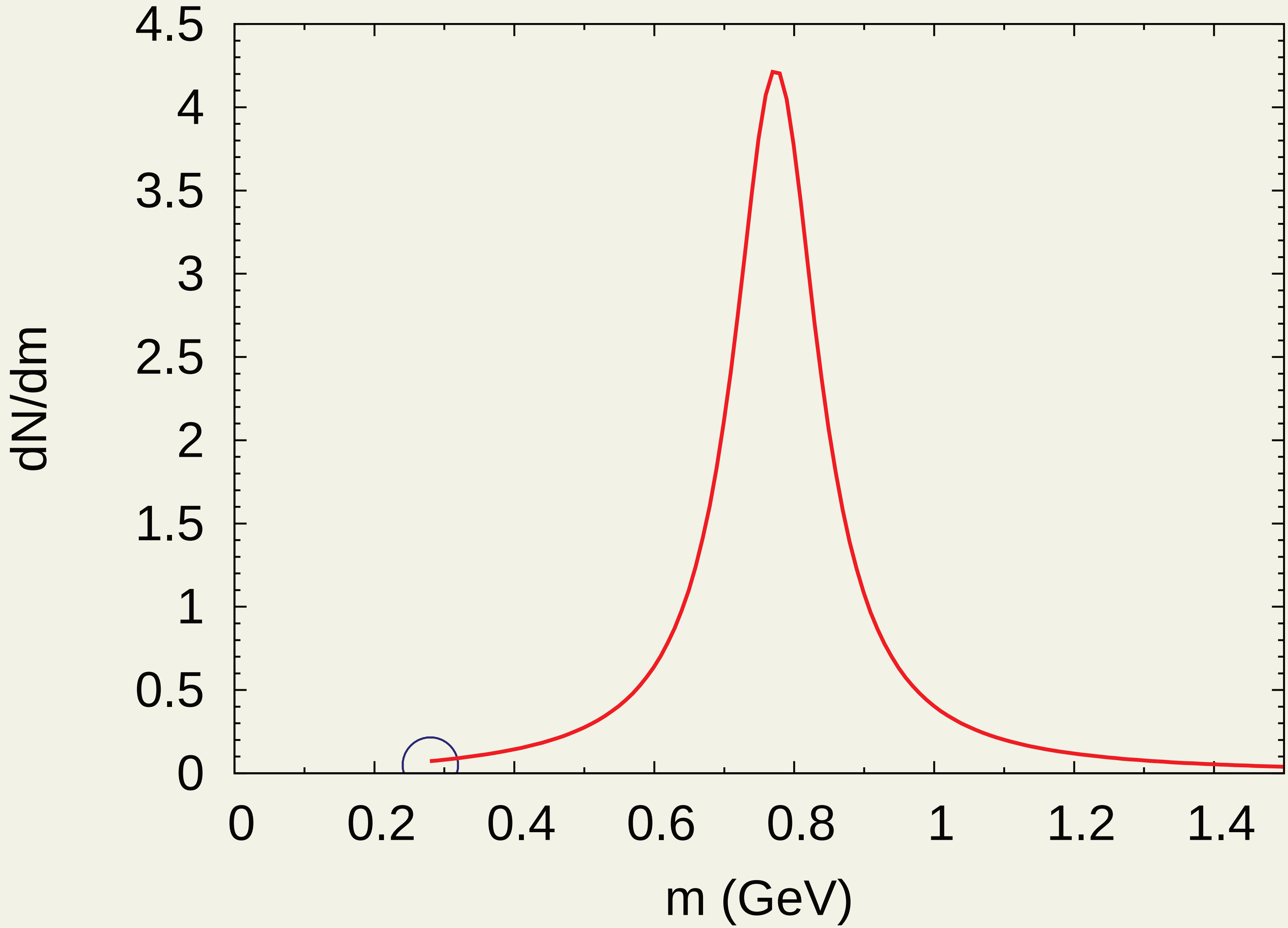


0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
n (fm^{-3})
100
110
120
130
140
150
160
T (MeV)
no width
Breit-Wigner

Breit-Wigner



Breit-Wigner



Mass dependent width

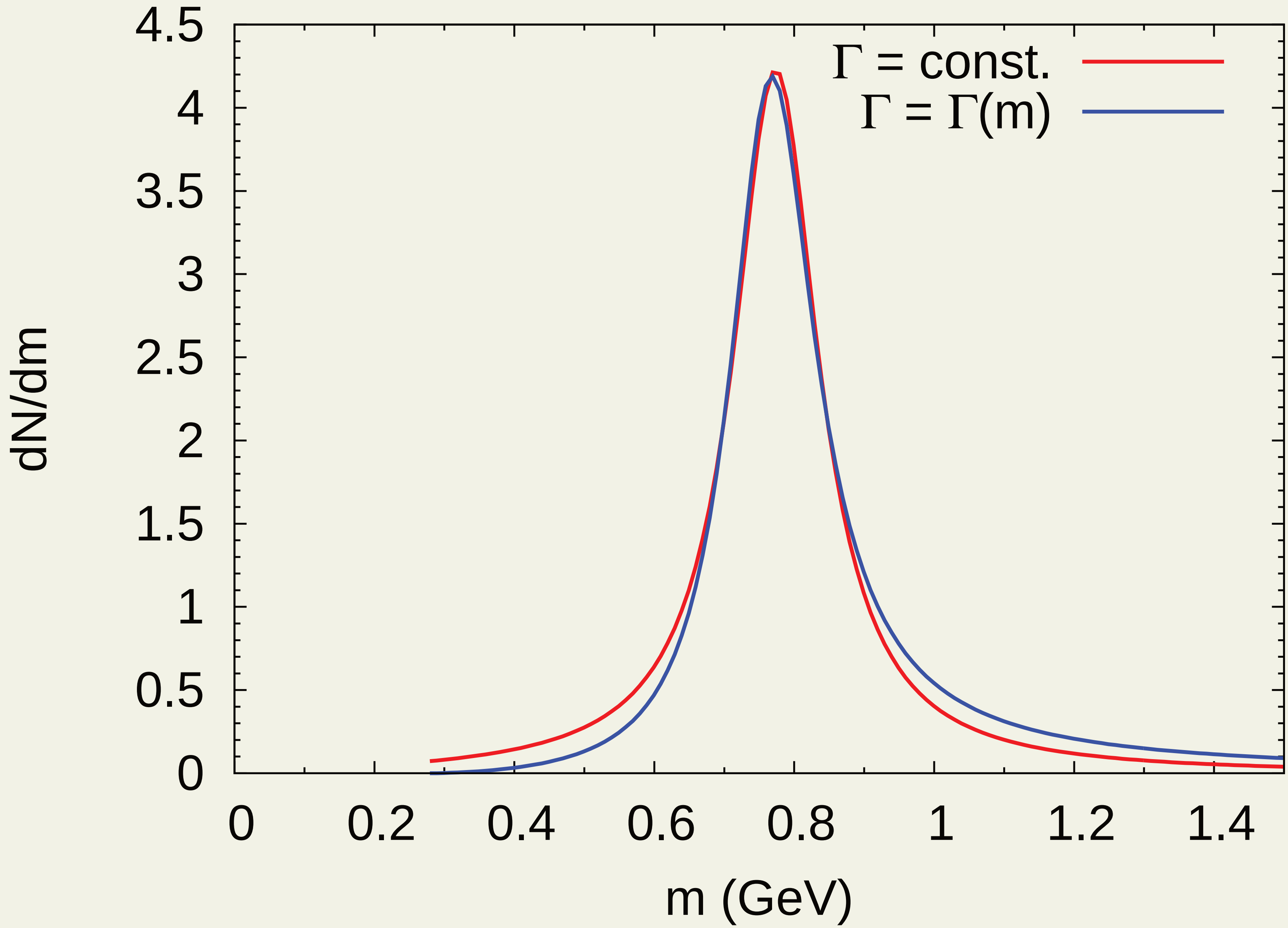
$$\frac{\mathrm{d}\rho}{\mathrm{d}m^2} = \frac{1}{N}\frac{m_R\,\Gamma(m)}{(m^2-m_R^2)^2+m_R^2\Gamma(m)^2},$$

with width

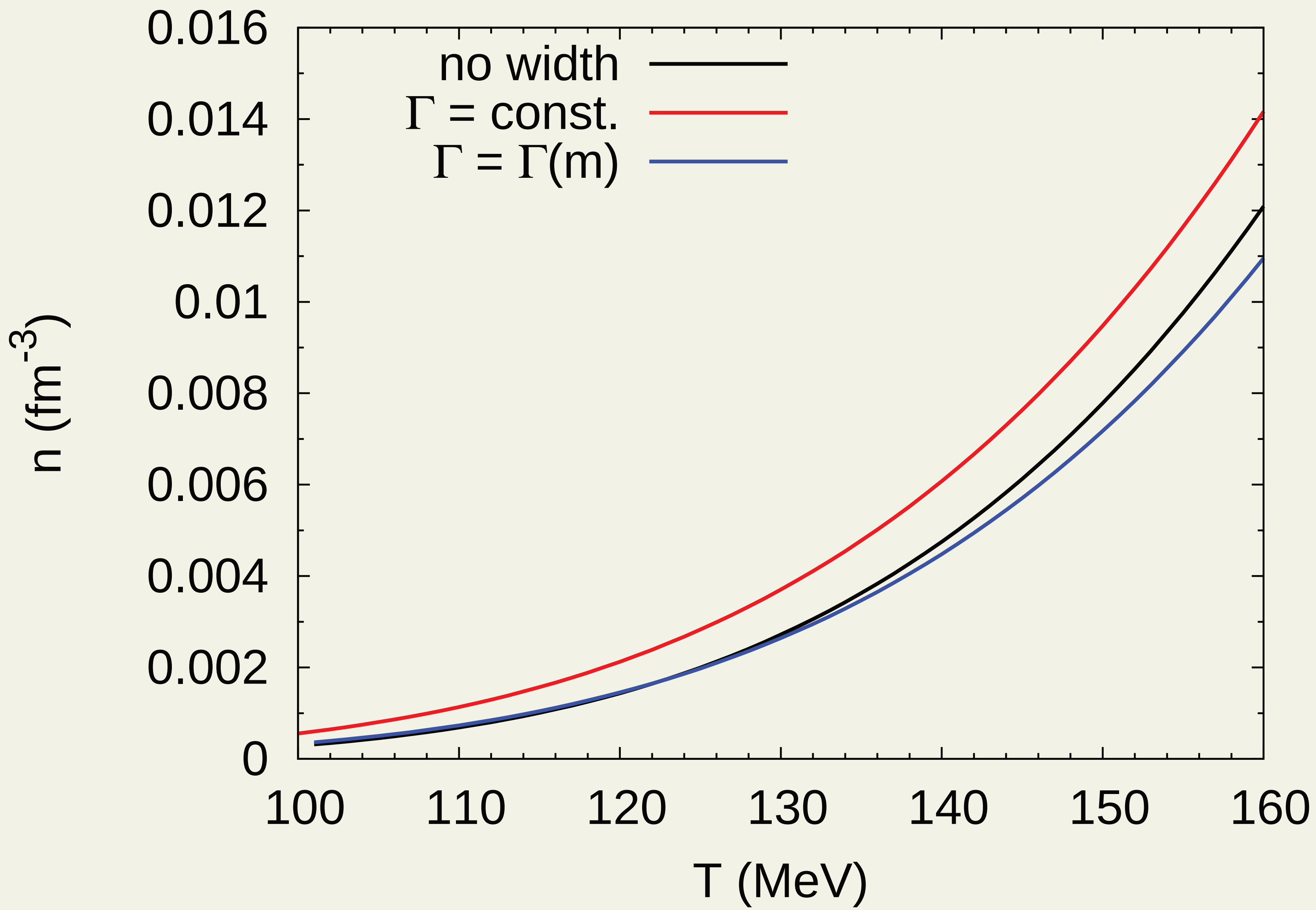
$$\Gamma(m) = \frac{1}{2}\,\frac{p_{\mathrm{CMS}}^3 r_0^2}{1+p_{\mathrm{CMS}}^2 r_0^2}$$

where $r_0 = 6.3$ **GeV**$^{-1}$

Breit-Wigner



ρ-density



0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
100
110
120
130
140
150
160
n (fm-3)
T (MeV)
no width
Γ = const.
Γ = Γ(m)

relativistic Breit-Wigner

$$\frac{\mathrm{d}\rho}{\mathrm{d}m^2} = \frac{1}{N}\frac{m_R\,\Gamma(m)}{(m^2 - m_R^2)^2 + m_R^2\Gamma(m)^2}$$

or:

$$\frac{\mathrm{d}\rho}{\mathrm{d}m^2} = \frac{1}{N}\frac{m\,\Gamma(m)}{(m^2 - m_R^2)^2 + m^2\Gamma(m)^2}$$

relativistic Breit-Wigner

$$\frac{\mathrm{d}\rho}{\mathrm{d}m^2} = \frac{1}{N}\frac{m_R\,\Gamma(m)}{(m^2-m_R^2)^2+m_R^2\Gamma(m)^2}$$

or:

$$\frac{\mathrm{d}\rho}{\mathrm{d}m^2} = \frac{1}{N}\frac{m\,\Gamma(m)}{(m^2-m_R^2)^2+m^2\Gamma(m)^2}$$

But if $\Gamma(m) \propto m$ at large m,

$$N = \int_{m_0}^{\infty} \mathrm{d}m^2 \frac{m\,\Gamma(m)}{(m^2-m_R^2)^2+m^2\Gamma(m)^2} = \infty$$

Particle Data Group about ρ:

`...the line shape does not correspond to`
`a relativistic Breit-Wigner function...but`
`requires some additional shape parameter`

Garbage in, garbage out

Dashen-Ma-Bernstein: Phys. Rev. 187, 345 (1969)

If interactions mediated by $narrow$ **resonances, properties of interacting hadron gas are those of noninteracting hadron-resonance gas**

Dashen-Ma-Bernstein: Phys. Rev. 187, 345 (1969)

If interactions mediated by *narrow* resonances, properties of interacting hadron gas are those of noninteracting hadron-resonance gas

Dashen-Ma-Bernstein: S-matrix formulation of statistical mechanics:

⇒ Second virial coefficient can be evaluated in terms of scattering phase shift (as far as interaction is manifested in elastic scattering)

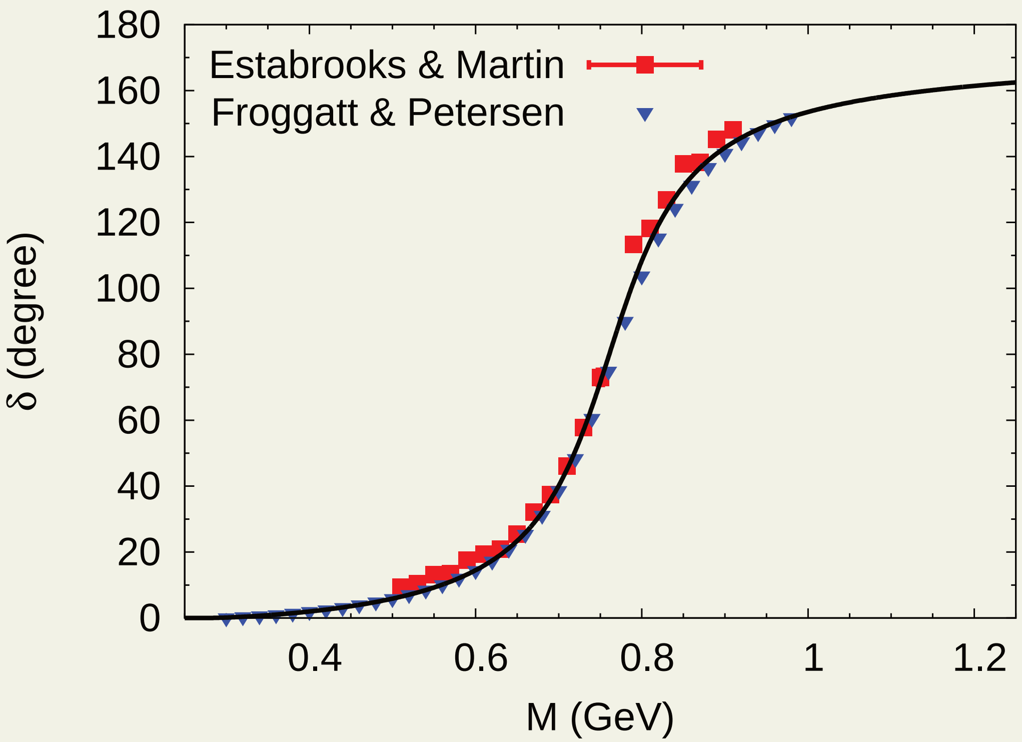
⇒ relativistic Beth-Uhlenbeck form

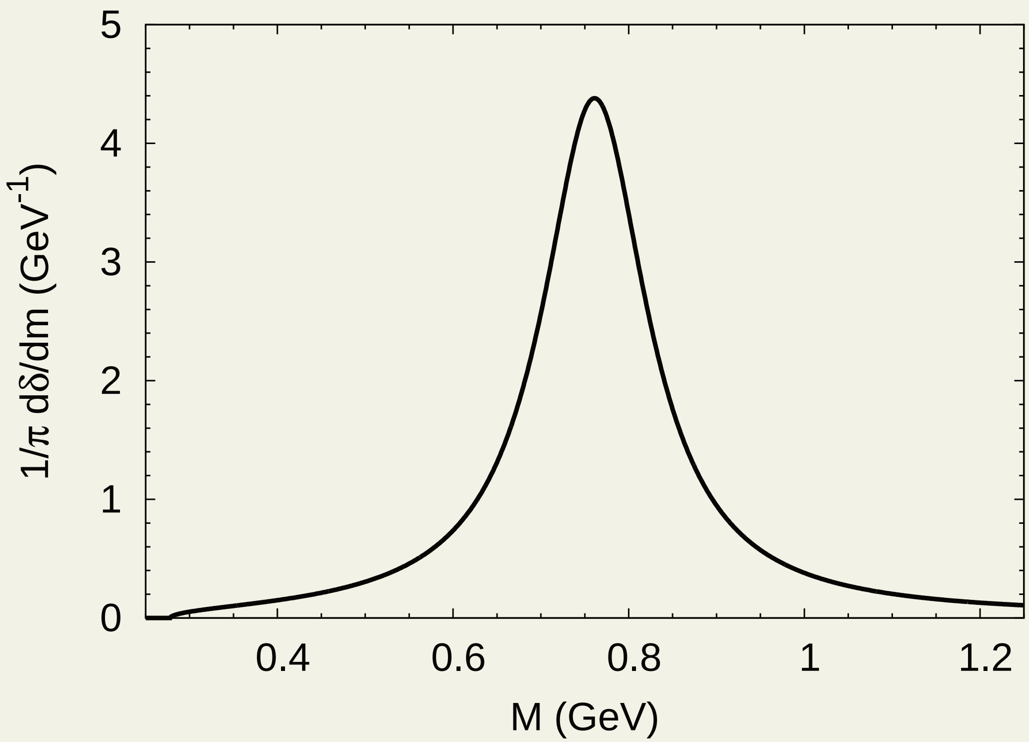
S-matrix

- **effects of interactions expressed in terms of scattering phase shifts**

$$n = \int d^3\mathbf{p} \int dm \frac{d\rho}{dm} f(p,m) \qquad \text{with} \qquad \frac{d\rho}{dm} = \frac{1}{\pi}\frac{d\delta}{dm}$$

- $\pi\pi$ **scattering, P-wave, i.e.** ρ **resonance**



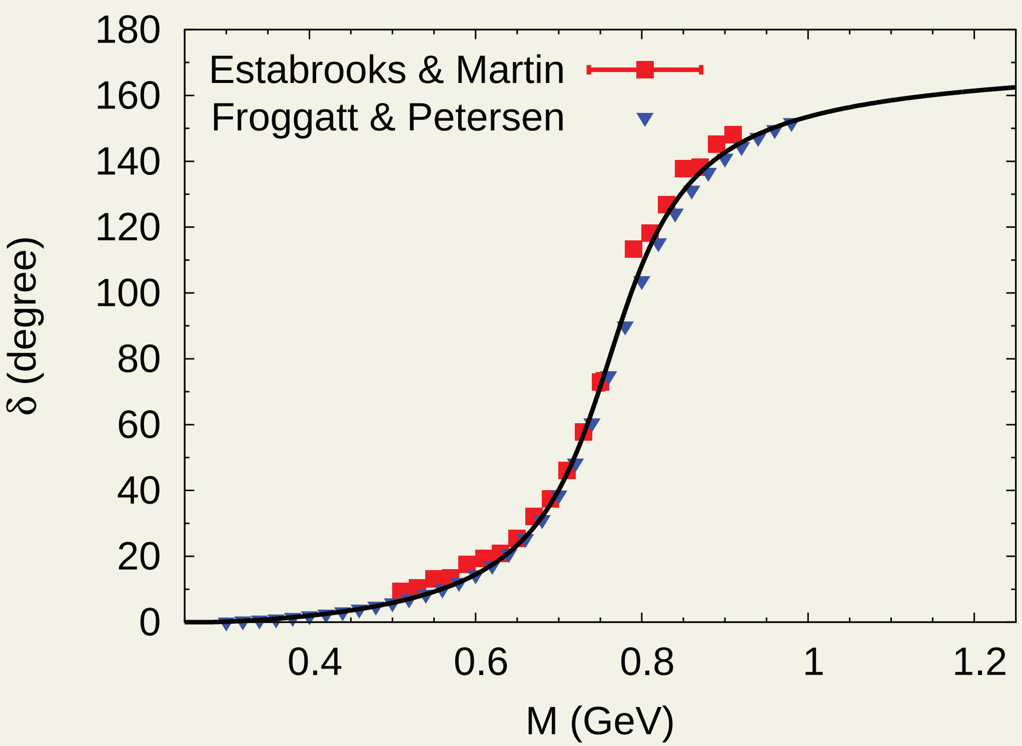


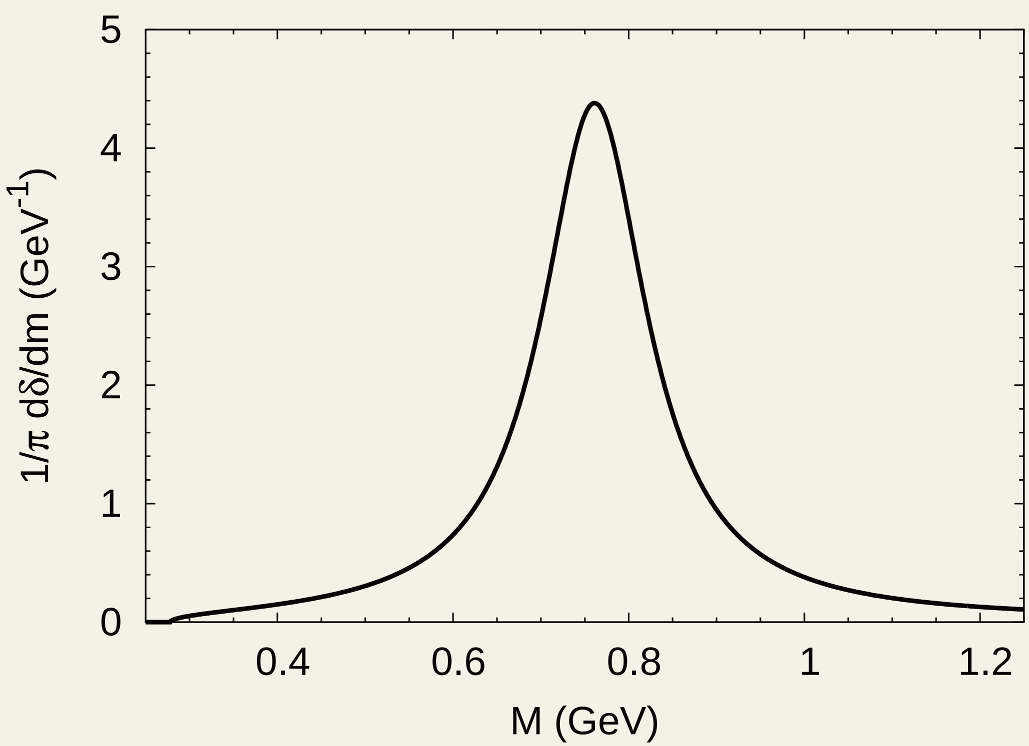
S-matrix

- **effects of interactions expressed in terms of scattering phase shifts**

$$n = \int \mathrm{d}^3\mathbf{p} \int \mathrm{d}m \frac{\mathrm{d}\rho}{\mathrm{d}m} f(p,m) \qquad \text{with} \qquad \frac{\mathrm{d}\rho}{\mathrm{d}m} = \frac{1}{\pi}\frac{\mathrm{d}\delta}{\mathrm{d}m}$$

- $\pi\pi$ **scattering, P-wave, i.e.** ρ **resonance**



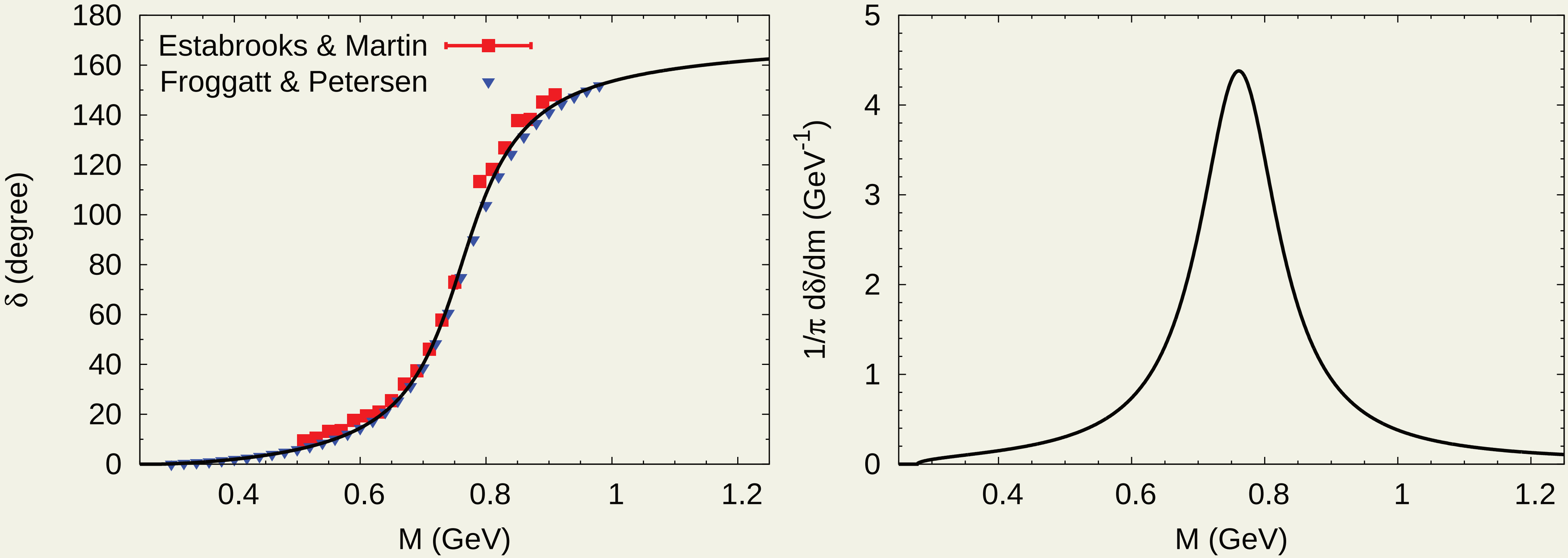


S-matrix

- **effects of interactions expressed in terms of scattering phase shifts**

$$n = \int \mathrm{d}^3\mathbf{p} \int \mathrm{d}m \frac{\mathrm{d}\rho}{\mathrm{d}m} f(p,m) \qquad \text{with} \qquad \frac{\mathrm{d}\rho}{\mathrm{d}m} = \frac{1}{\pi}\frac{\mathrm{d}\delta}{\mathrm{d}m}$$

- **$\pi\pi$ scattering, P-wave, i.e. ρ resonance**

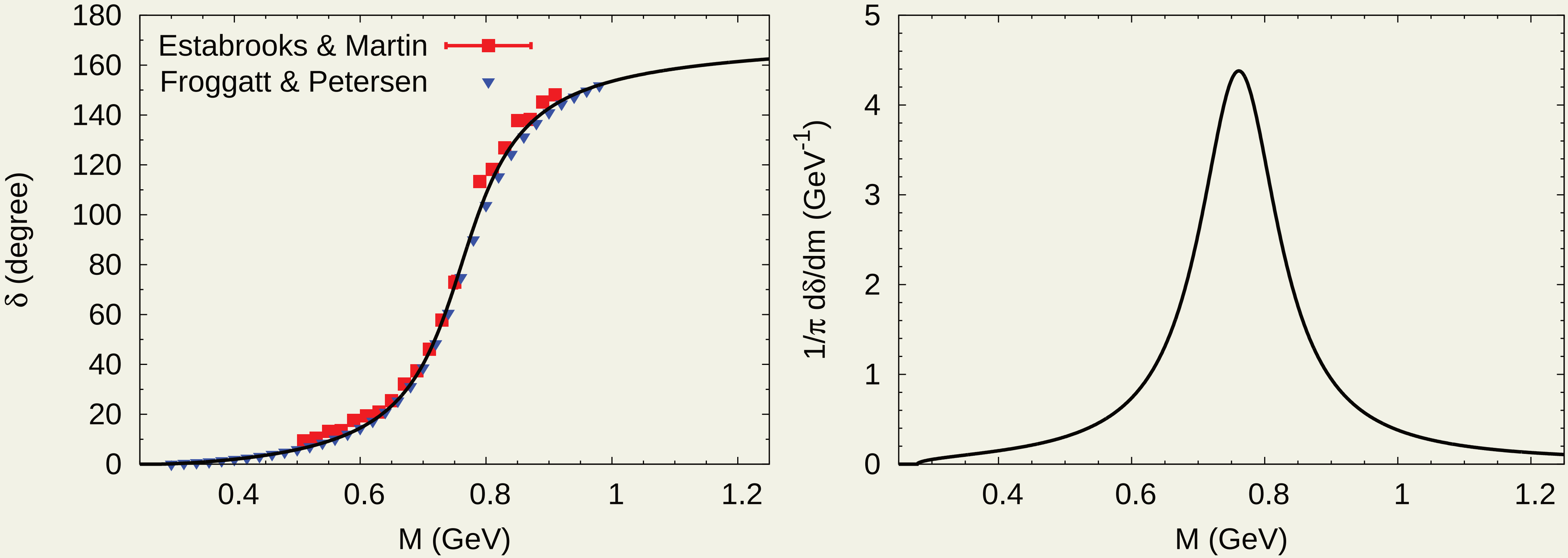


S-matrix

- **effects of interactions expressed in terms of scattering phase shifts**

$$n = \int d^3\mathbf{p} \int dm \frac{d\rho}{dm} f(p,m) \qquad \text{with} \qquad \frac{d\rho}{dm} = \frac{1}{\pi}\frac{d\delta}{dm}$$

- $\pi\pi$ **scattering, P-wave, i.e.** ρ **resonance**

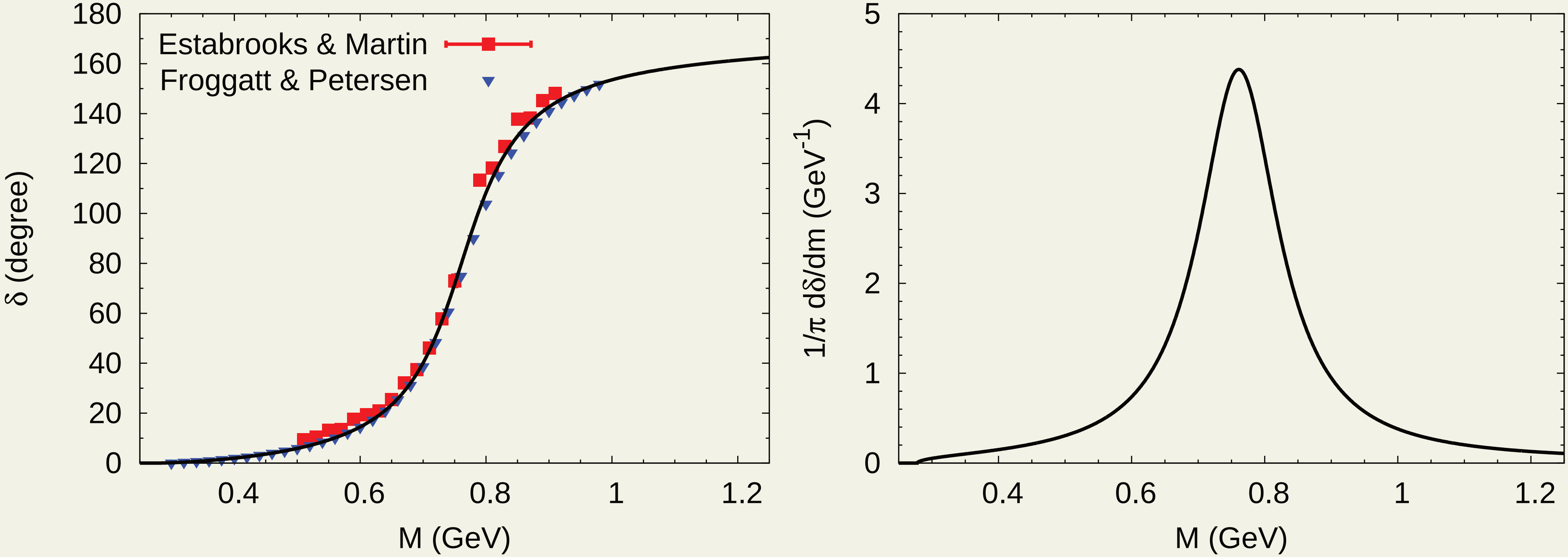


S-matrix

- **effects of interactions expressed in terms of scattering phase shifts**

$$n = \int \mathrm{d}^3\mathbf{p} \int \mathrm{d}m \frac{\mathrm{d}\rho}{\mathrm{d}m} f(p,m) \qquad \text{with} \qquad \frac{\mathrm{d}\rho}{\mathrm{d}m} = \frac{1}{\pi}\frac{\mathrm{d}\delta}{\mathrm{d}m}$$

- **$\pi\pi$ scattering, P-wave, i.e. ρ resonance**

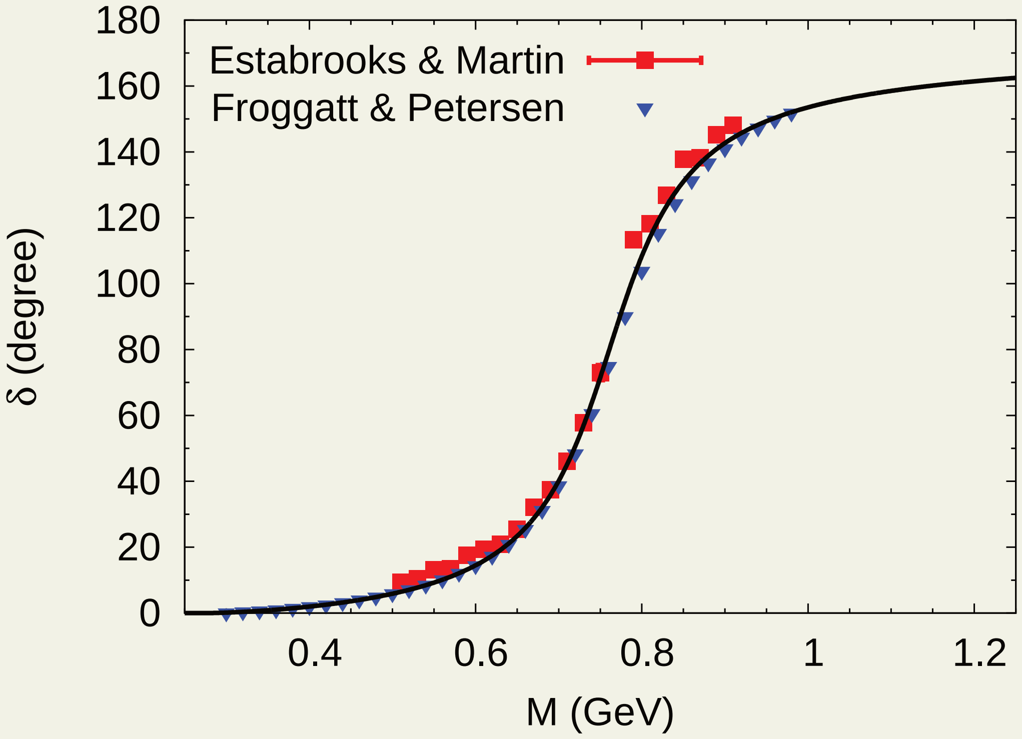


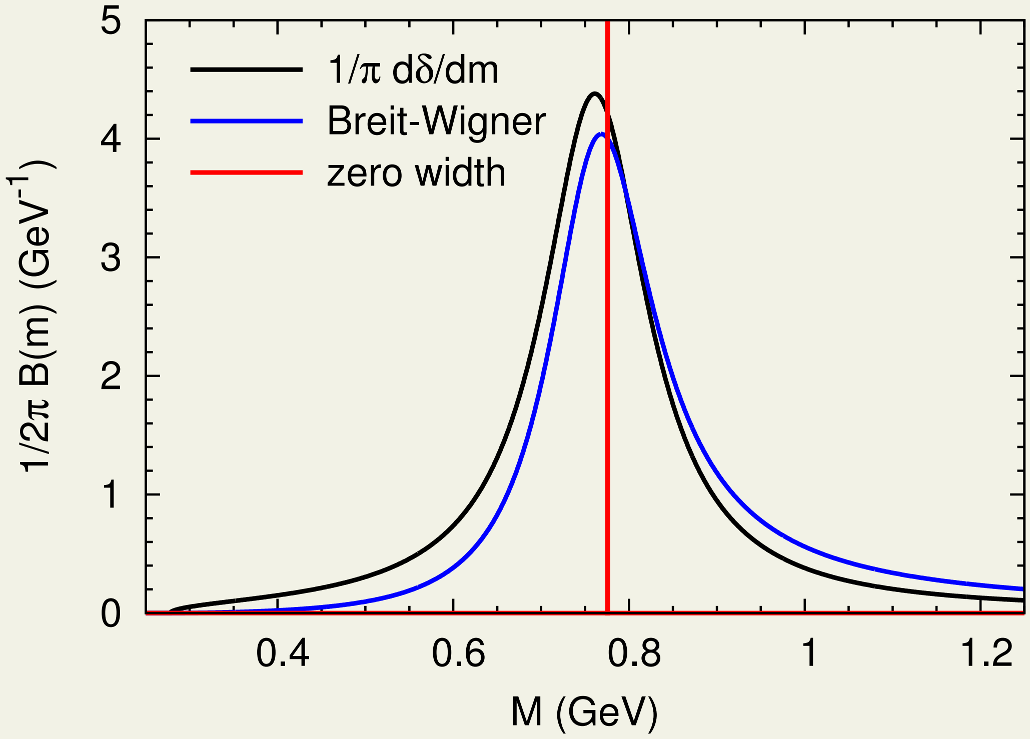
S-matrix

- **effects of interactions expressed in terms of scattering phase shifts**

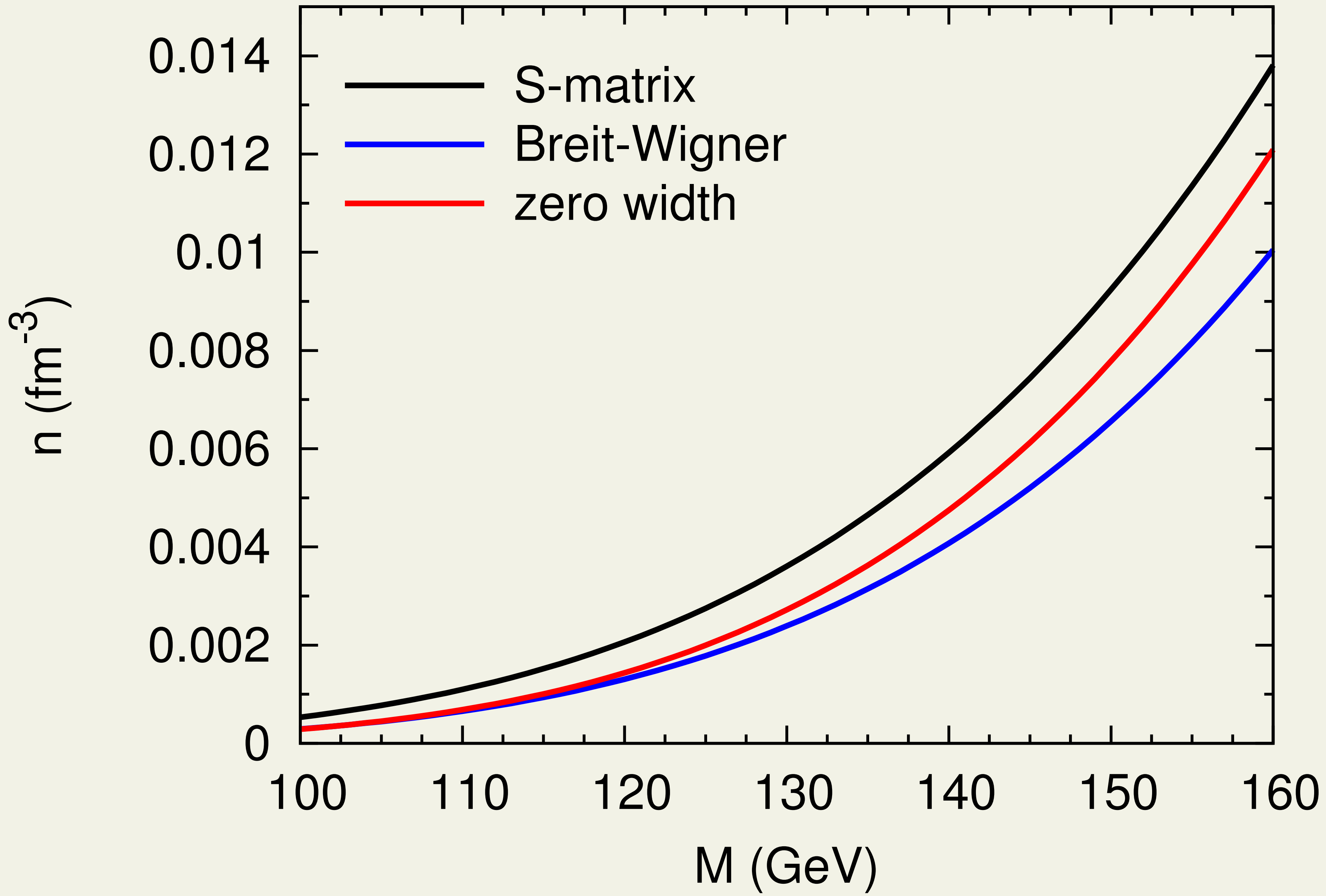
$$n = \int \mathrm{d}^3\mathbf{p} \int \mathrm{d}m \frac{\mathrm{d}\rho}{\mathrm{d}m} f(p,m) \qquad \text{with} \qquad \frac{\mathrm{d}\rho}{\mathrm{d}m} = \frac{1}{\pi}\frac{\mathrm{d}\delta}{\mathrm{d}m}$$

- **$\pi\pi$ scattering, P-wave, i.e. ρ resonance**



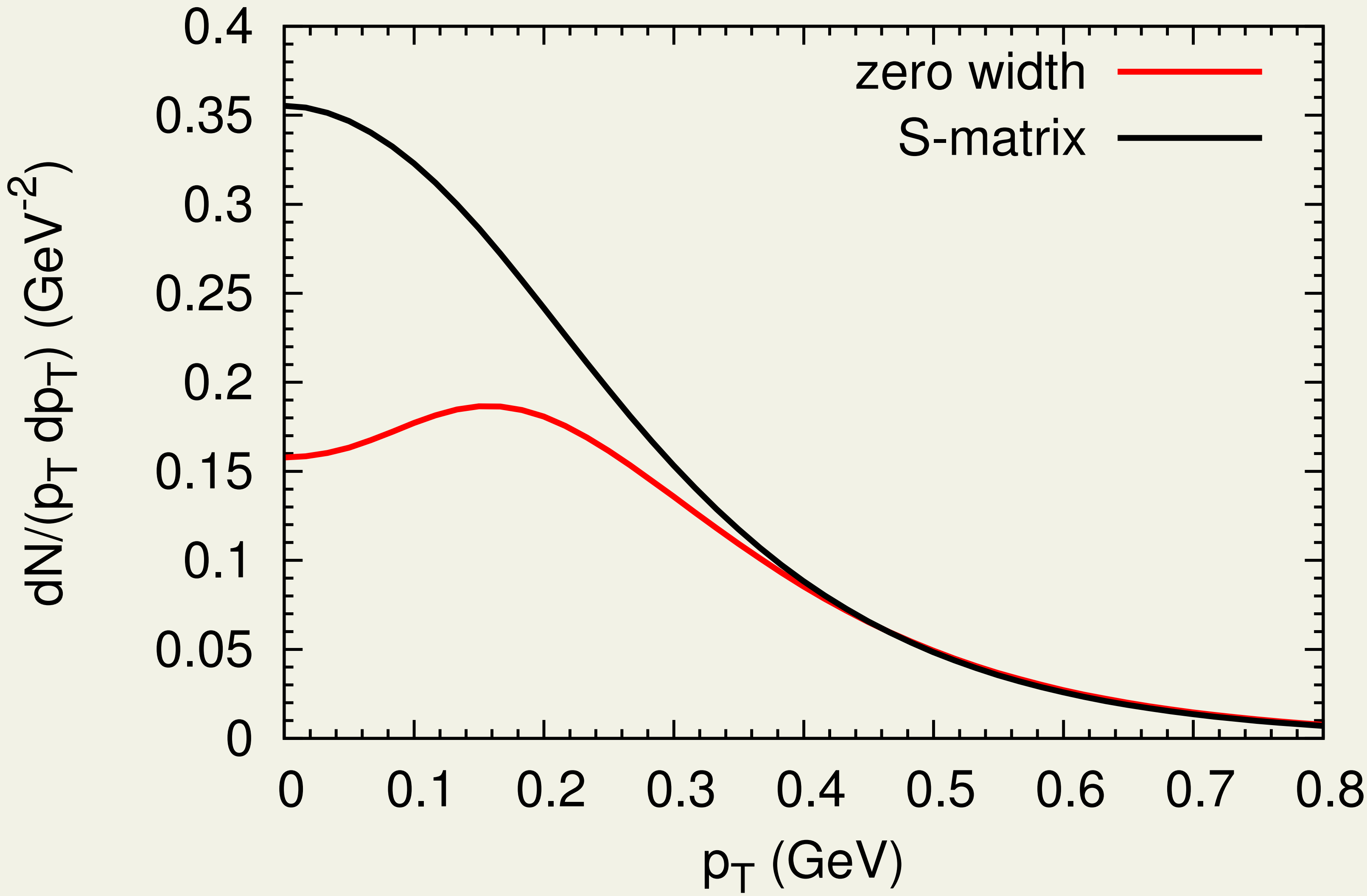


ρ-density



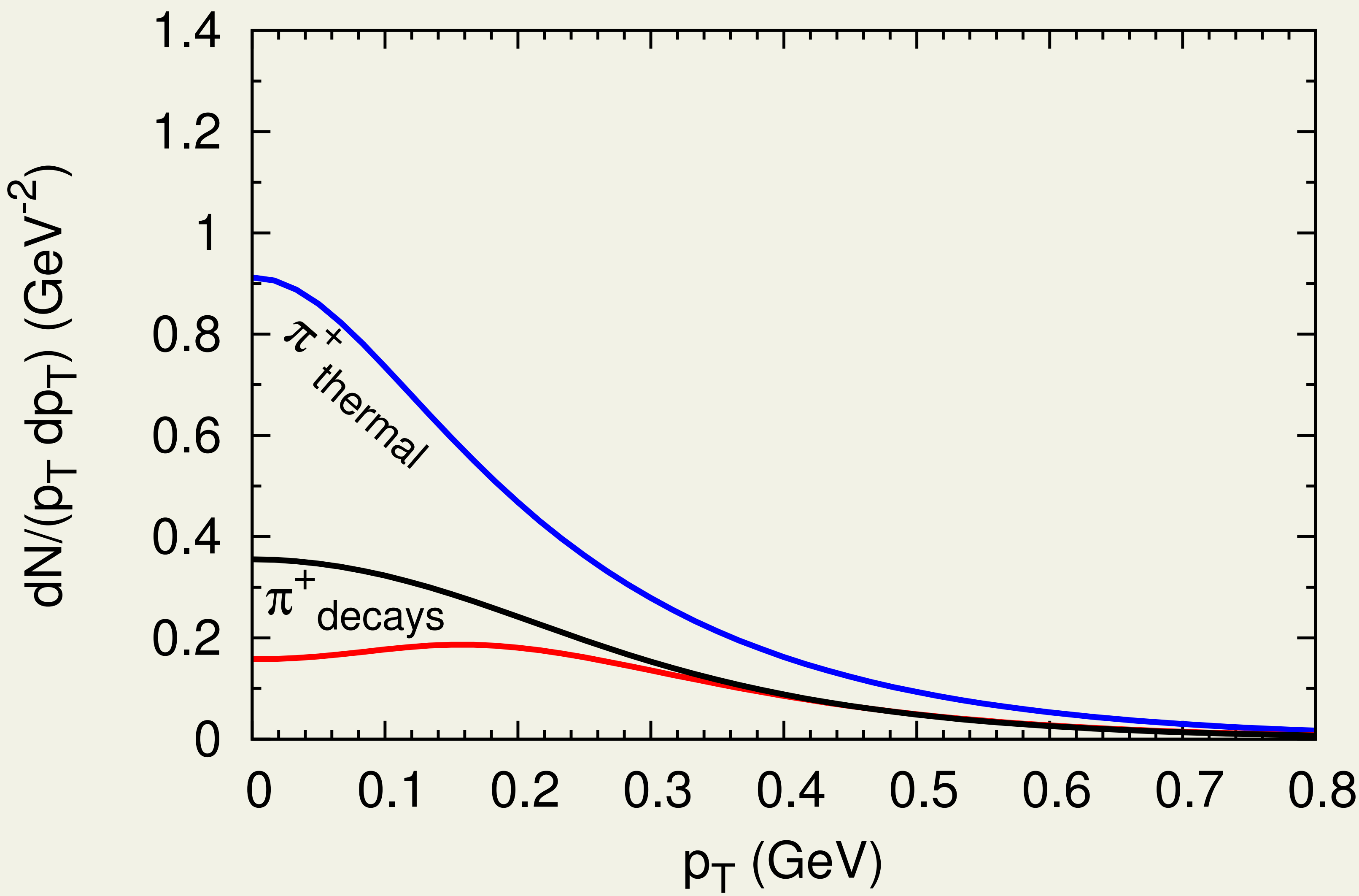
S-matrix
Breit-Wigner
zero width
n (fm^{-3})
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
100
110
120
130
140
150
160
M (GeV)

Pions from ρ decays



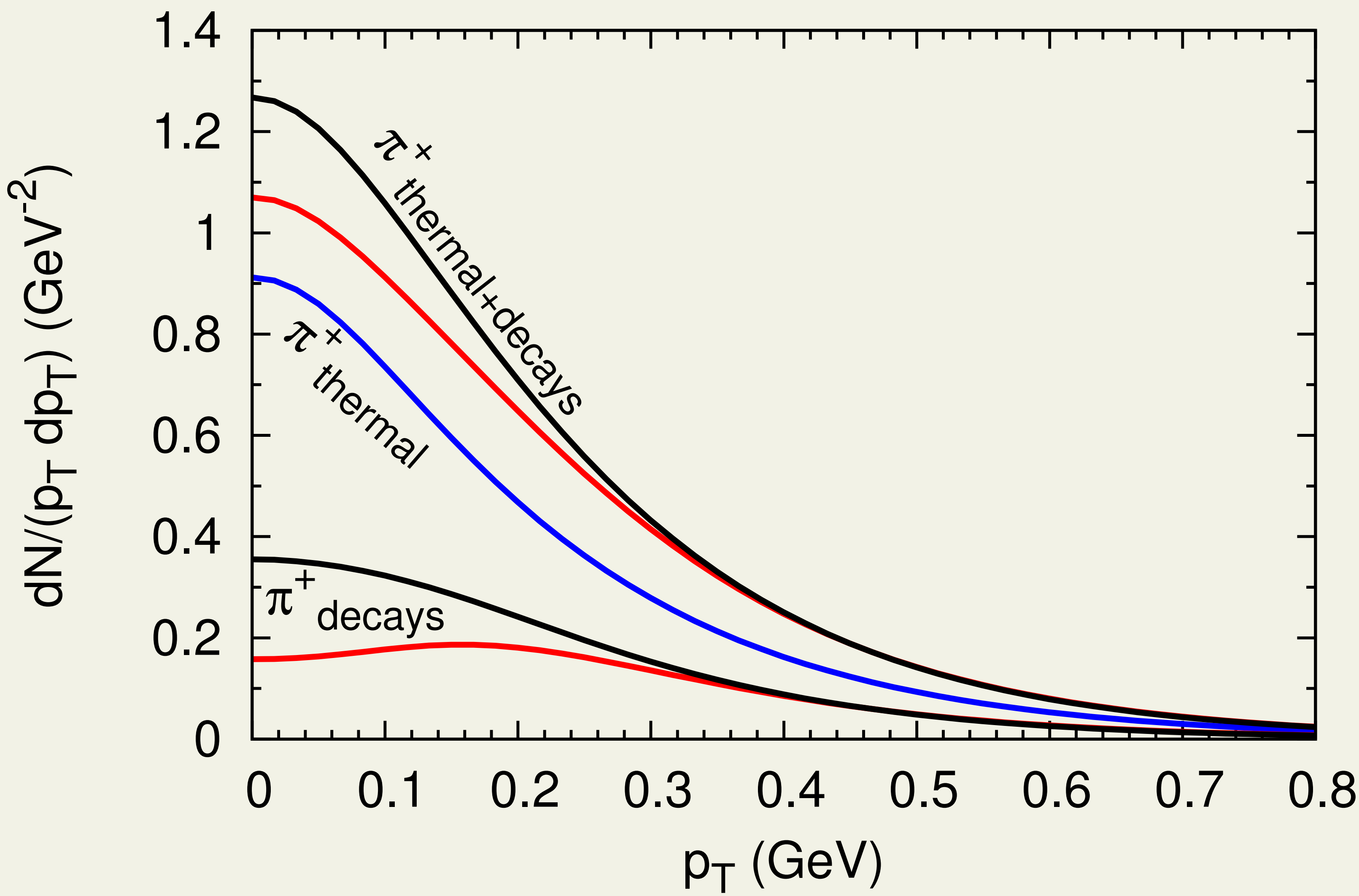
- **static source,** $T = 155\,\mathrm{MeV}$

Thermal pions + pions from ρ decays



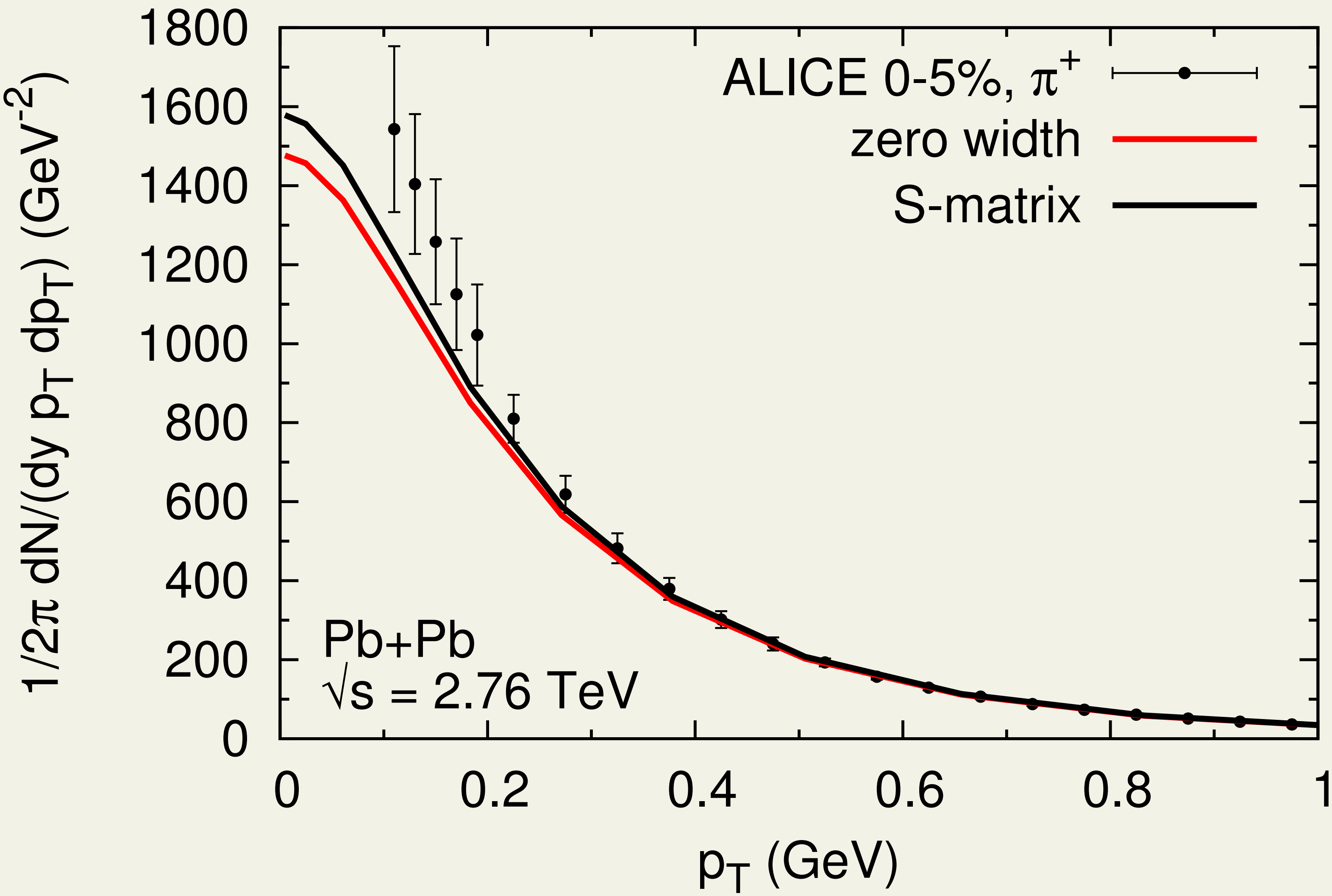
- **static source,** $T = 155\,\mathrm{MeV}$

Thermal pions + pions from ρ decays



- **static source,** $T = 155\,\mathrm{MeV}$

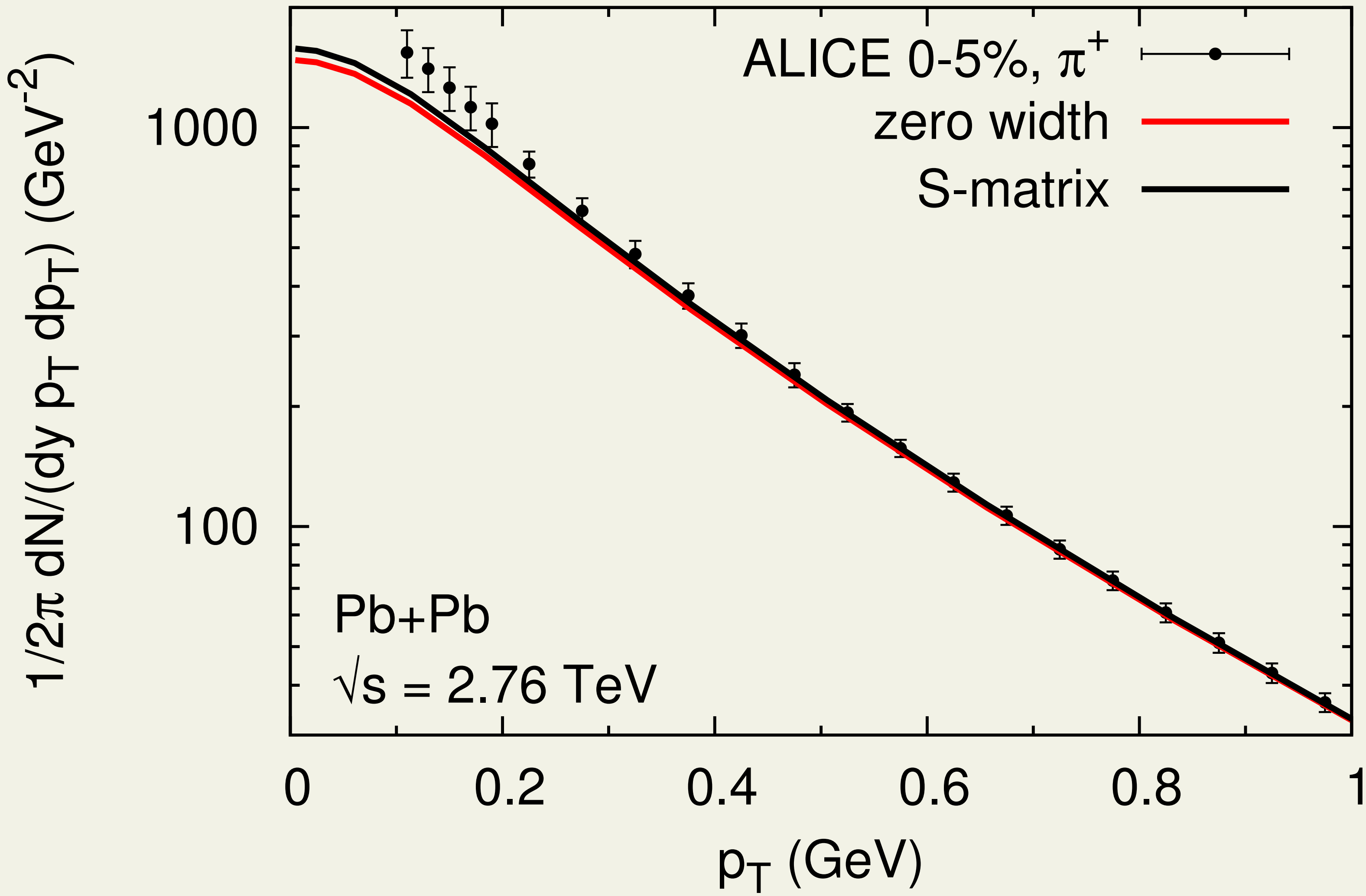
Pions from blast wave



- $\tau = 14.1\,\text{fm}$
- $R = 10\,\text{fm}$
- $v_{max} = 0.8$

- **all resonances up to 2 GeV**
- **Beth-Uhlenbeck for rhos**
- **zero width for everything else**

Pions from blast wave



- $\tau = 14.1\,\mathrm{fm}$
- $R = 10\,\mathrm{fm}$
- $v_{max} = 0.8$

- **all resonances up to 2 GeV**
- **Beth-Uhlenbeck for rhos**
- **zero width for everything else**

caveats

- **so far only rho mesons**

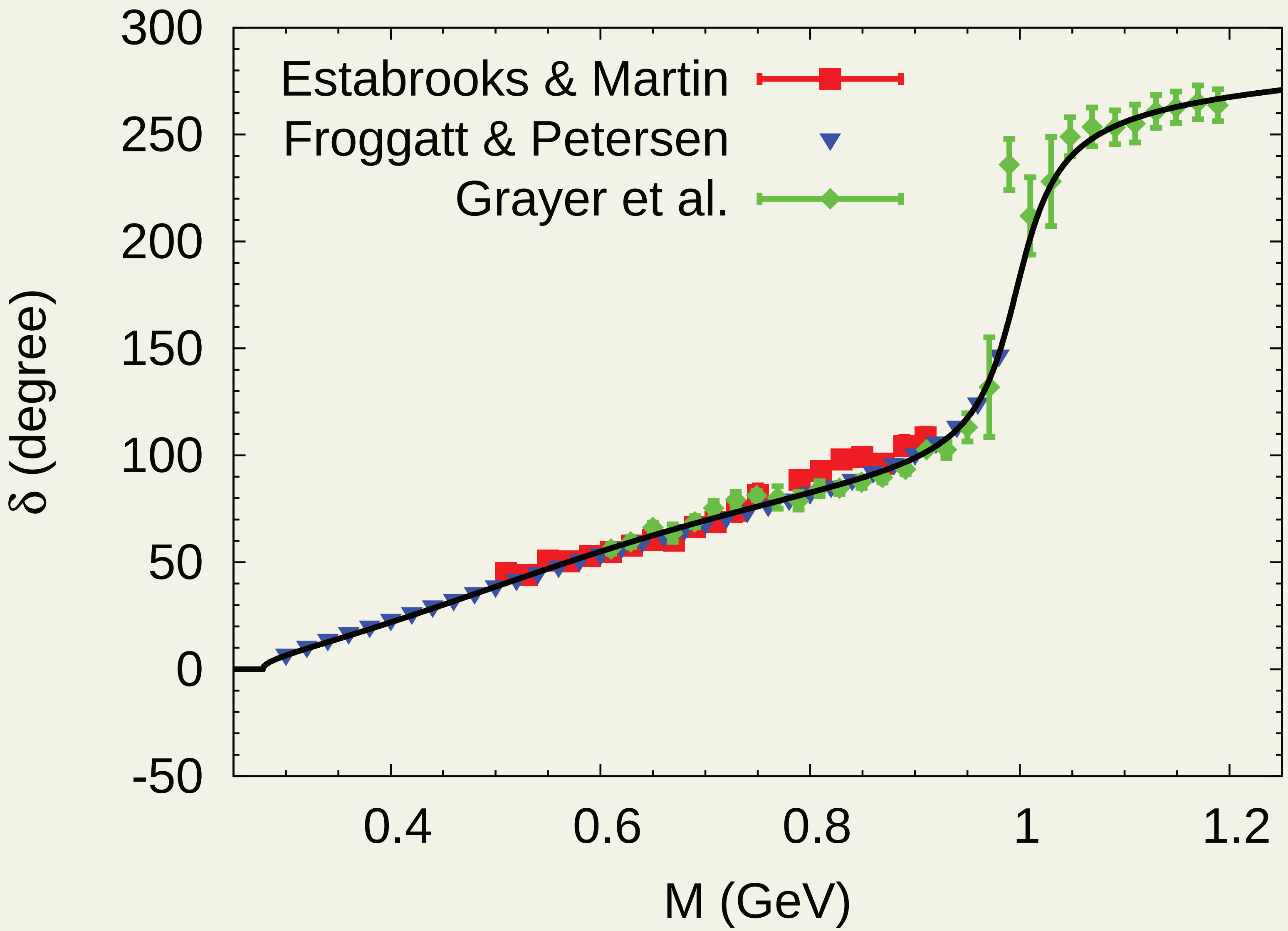
caveats

- so far only rho mesons
- Beth-Uhlenbeck applicable to elastic scatterings only!

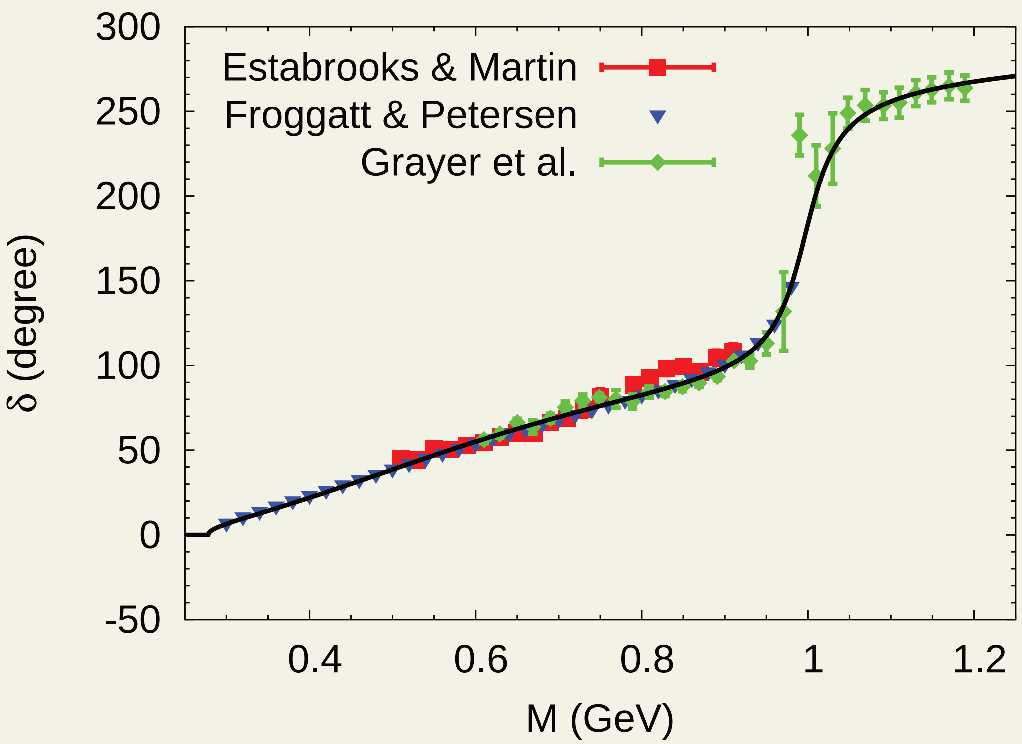
caveats

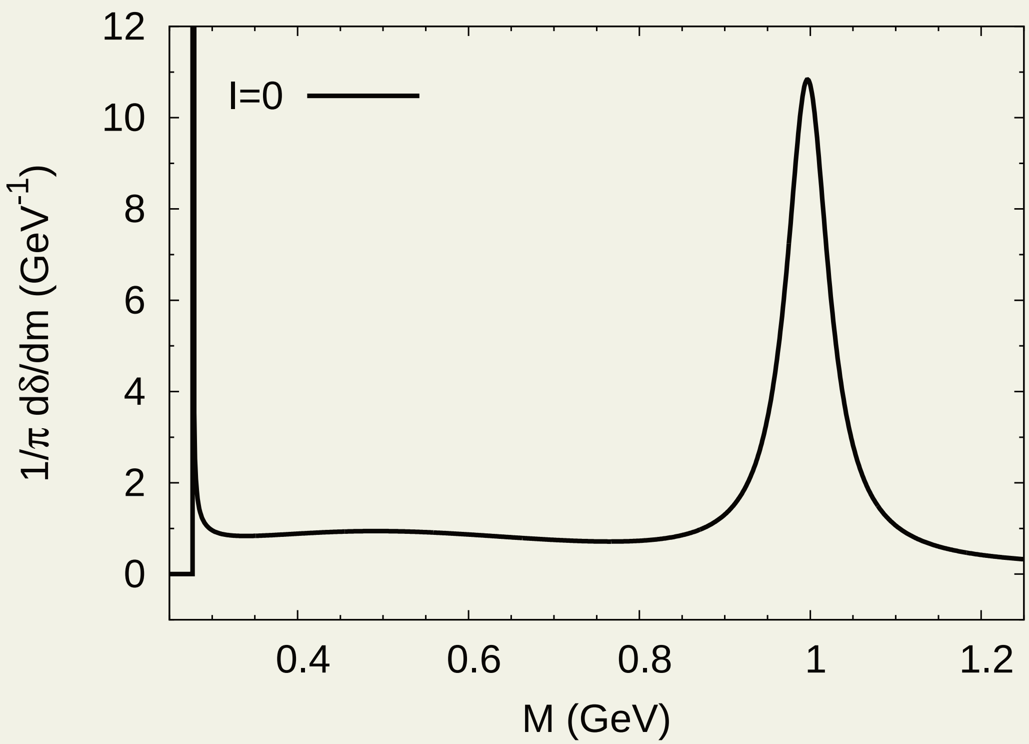
- **so far only rho mesons**
- **Beth-Uhlenbeck applicable to elastic scatterings only!**
- **how about σ a.k.a. $f_0(500)$?**

S-wave $\pi\pi$ scattering

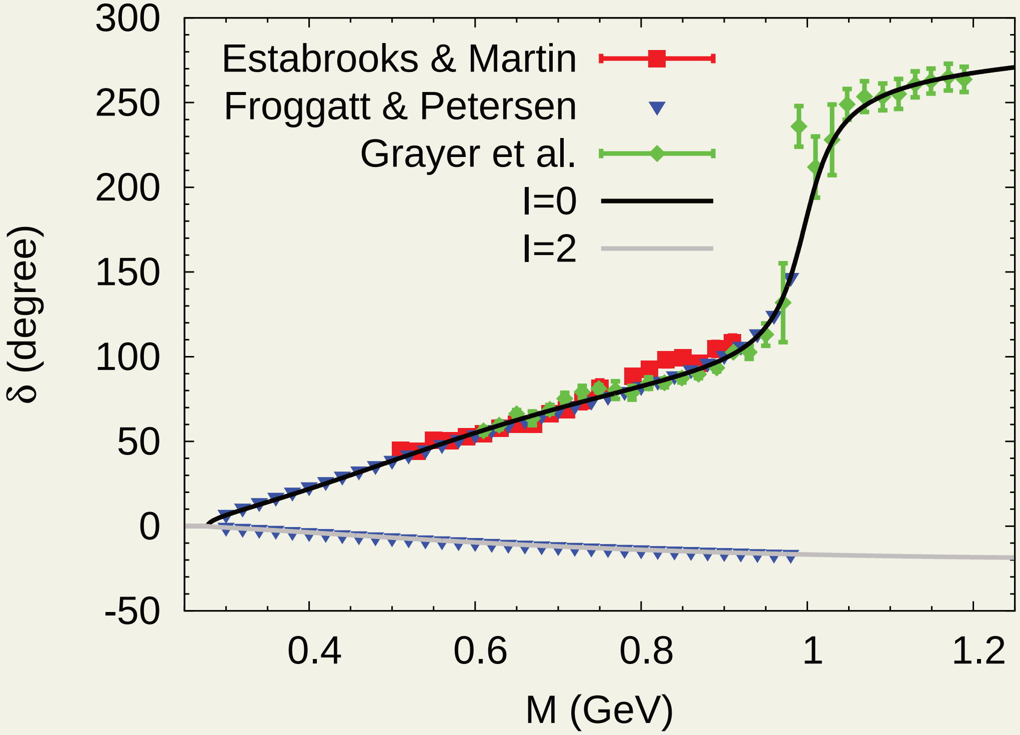


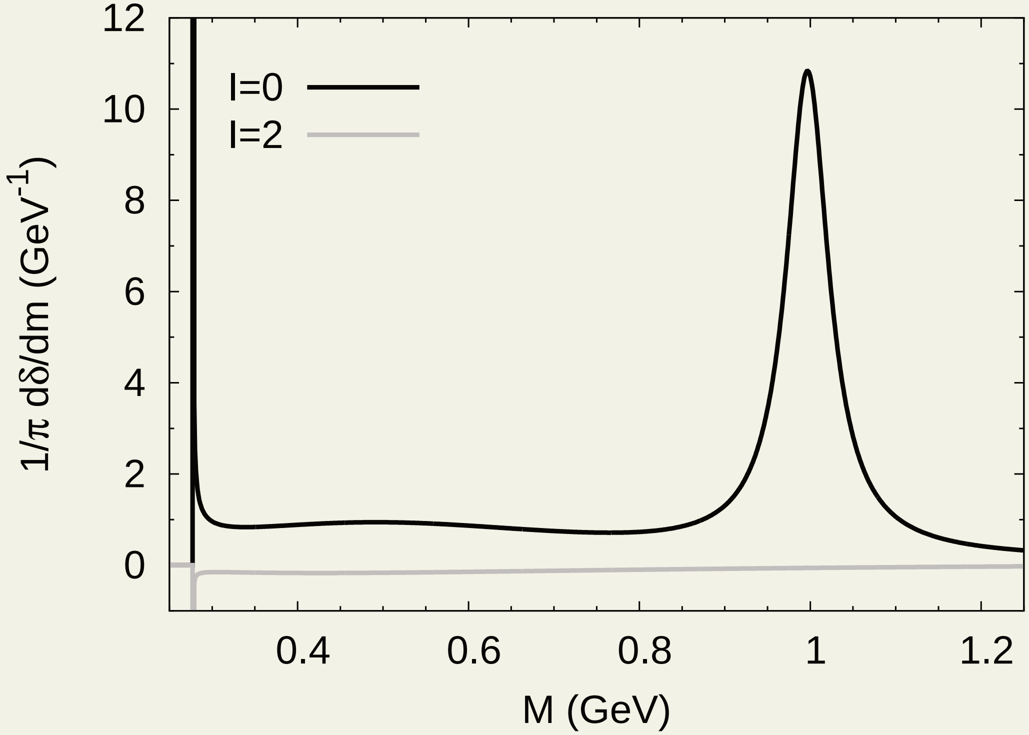
S-wave $\pi\pi$ scattering



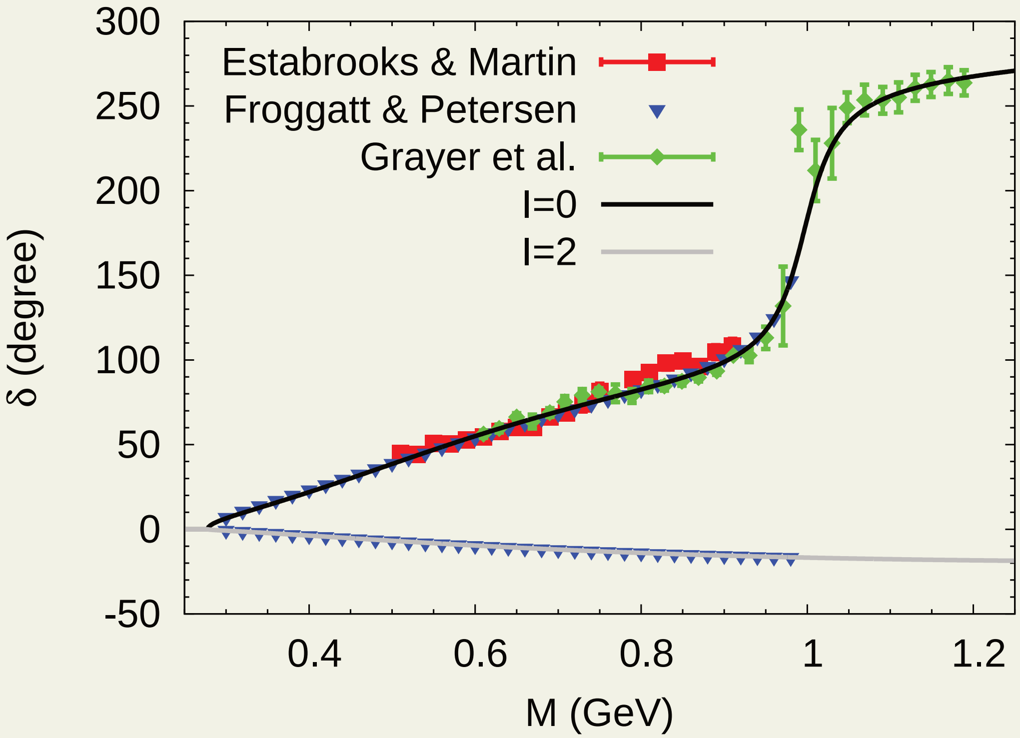


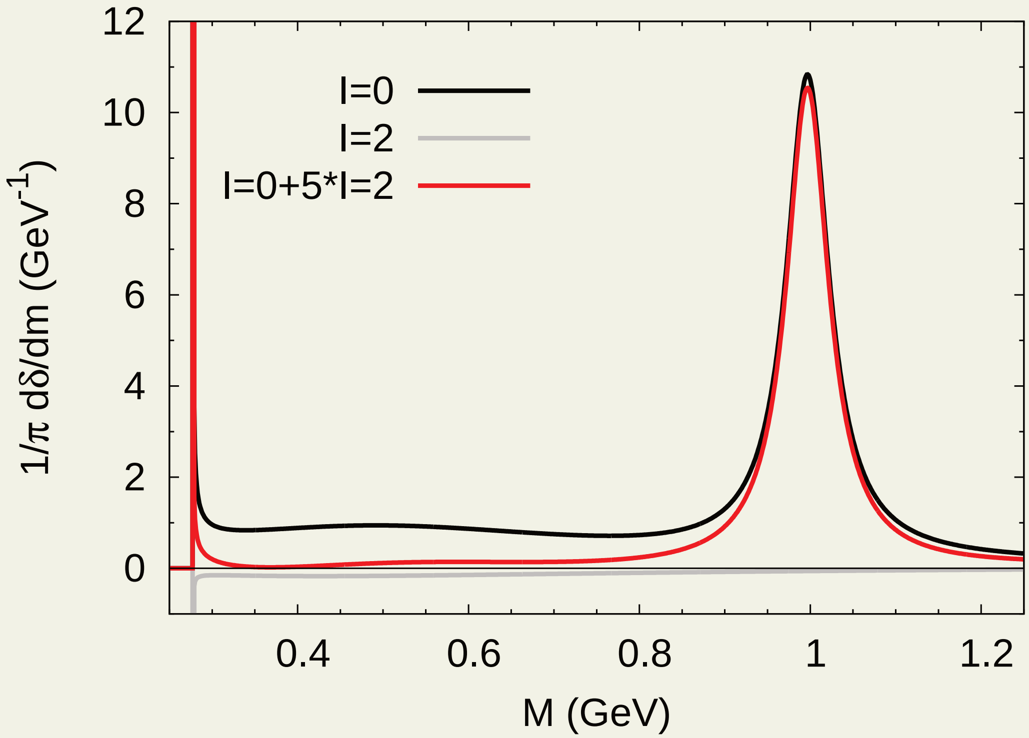
S-wave $\pi\pi$ scattering





S-wave $\pi\pi$ scattering

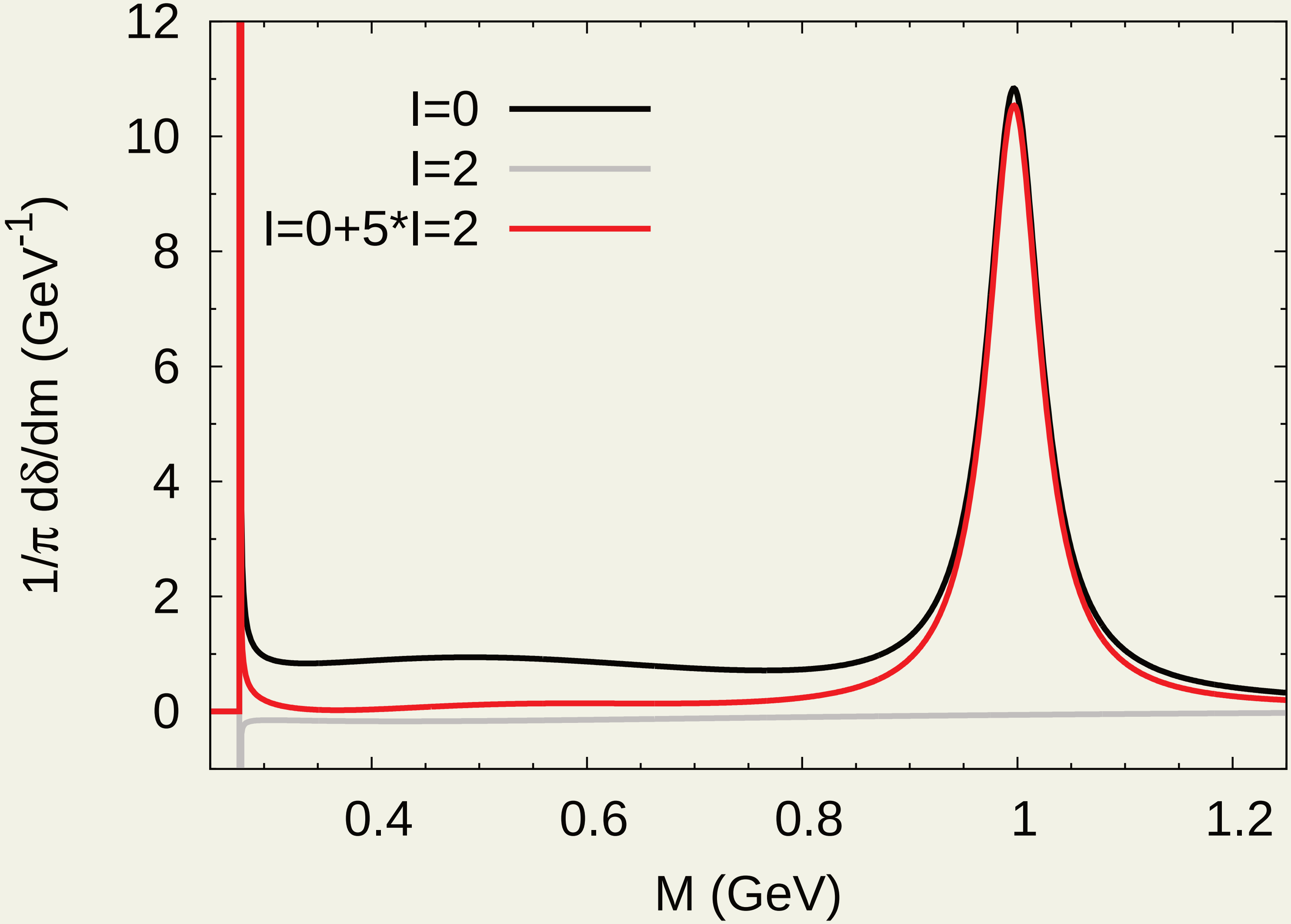




as advocated by

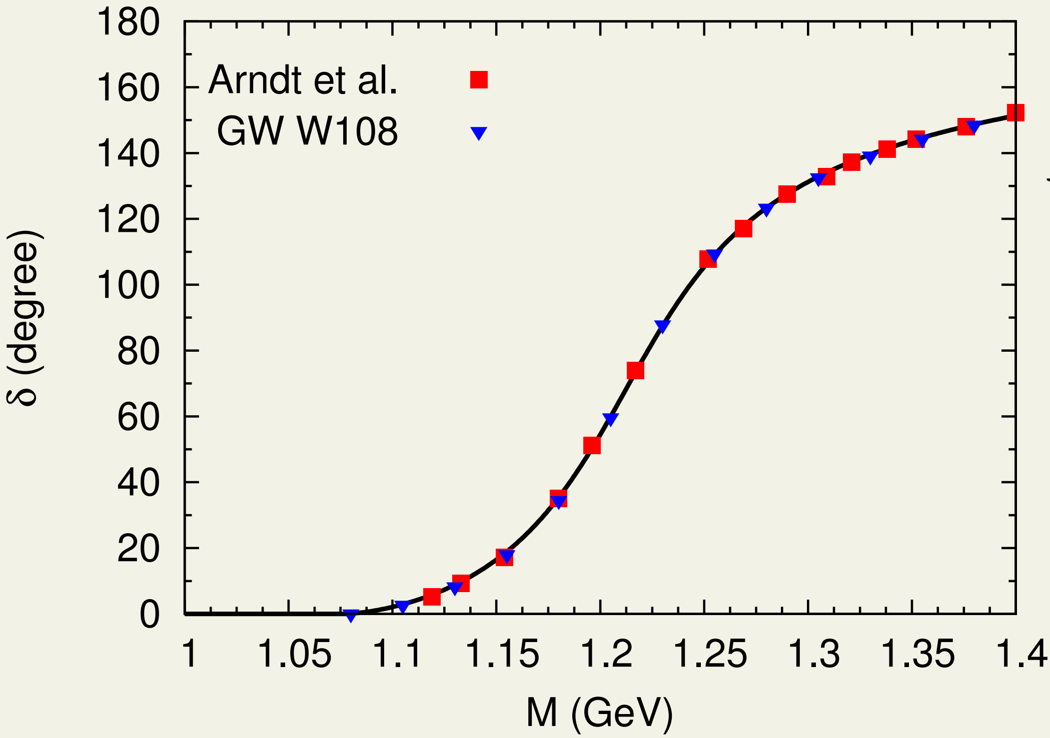
- **Broniowski, Giacosa & Begun, PRC92, 034905 (2015)**
- **Prakash & Venugopalan, NPA546, 718 (1992)**

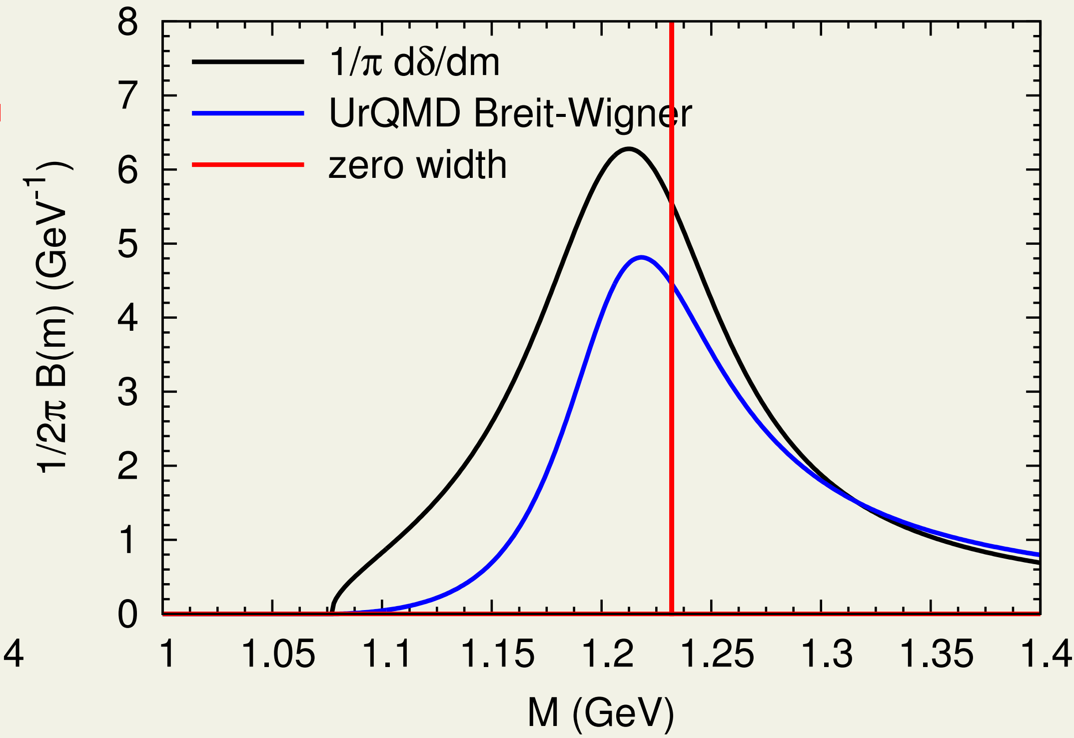
S-wave $\pi\pi$ scattering



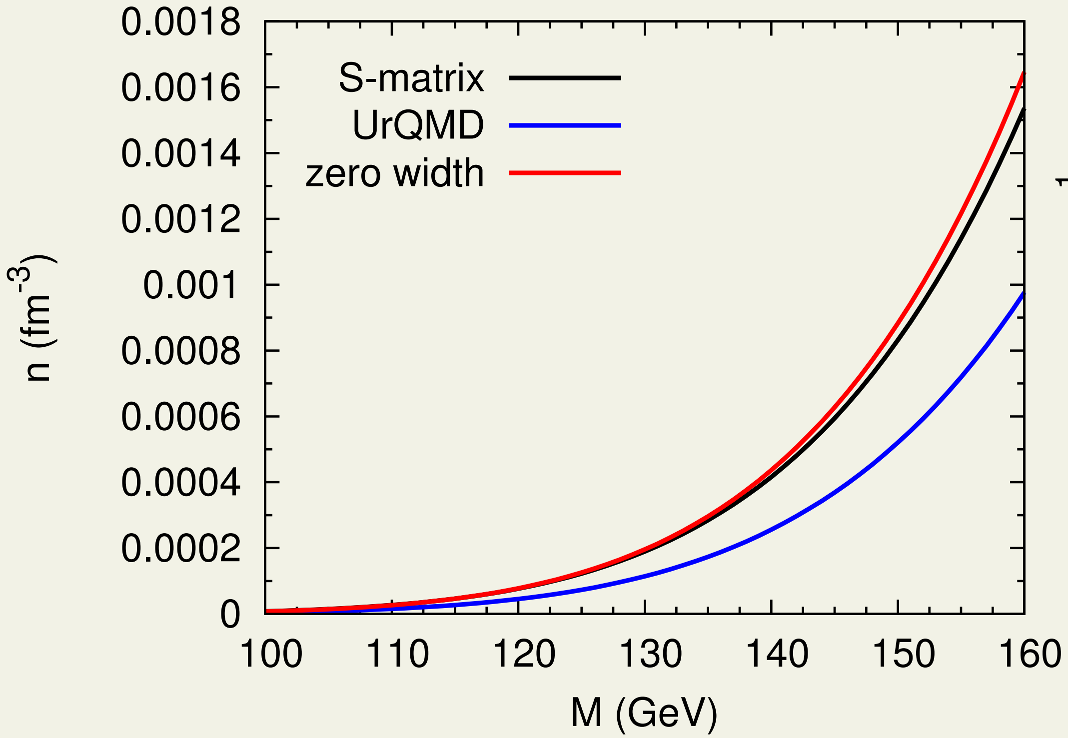
- $f_0(980)$ **nicely described**

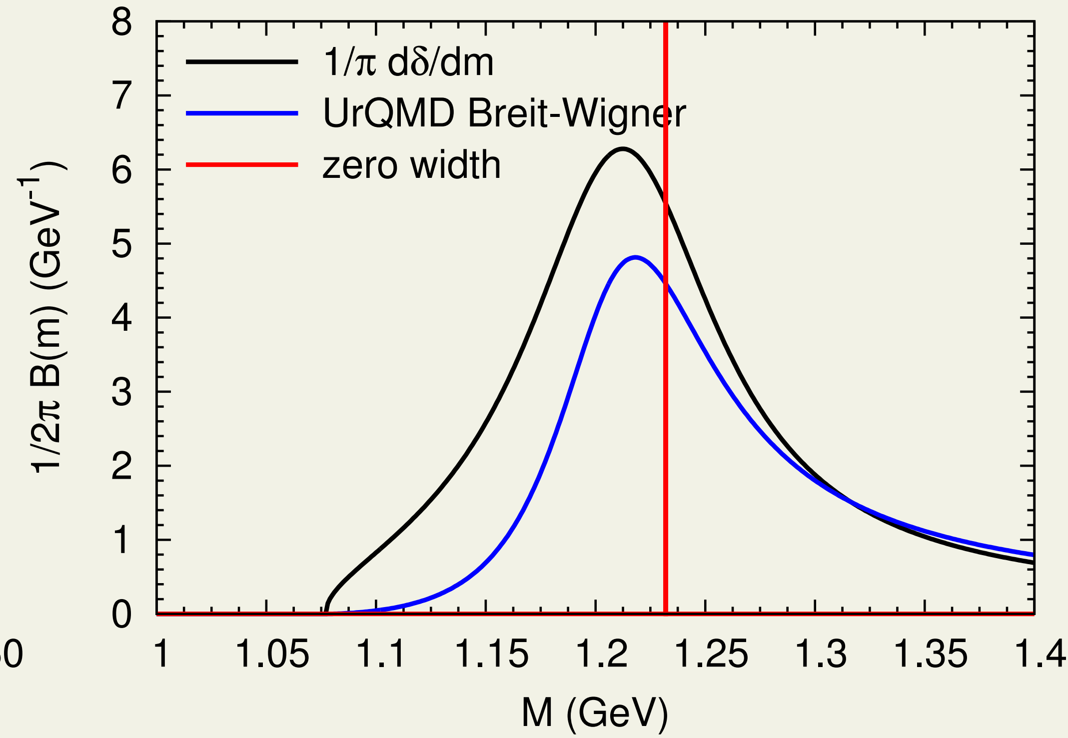
P_{33} πN scattering, a.k.a. Δ



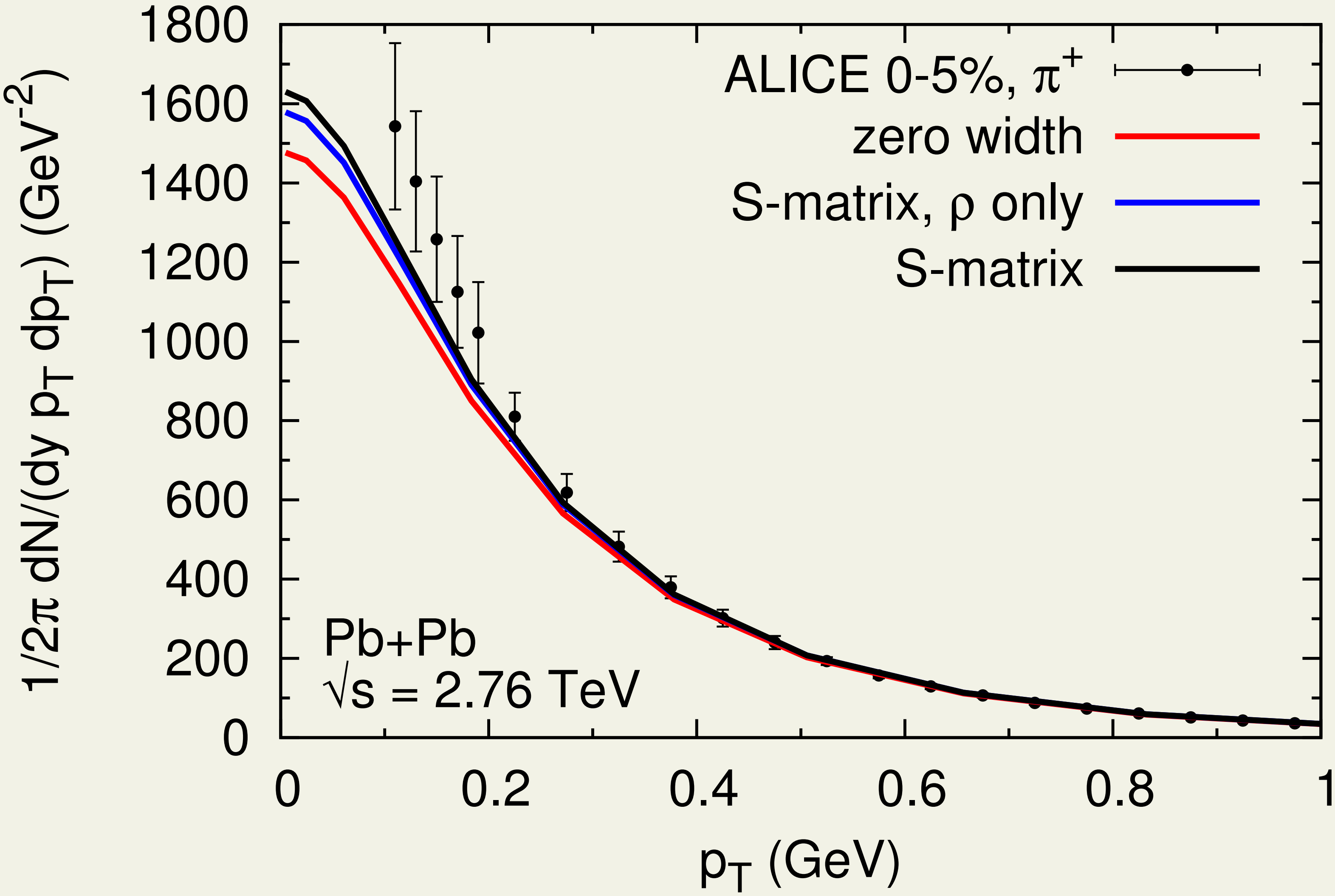


P_{33} πN scattering, a.k.a. Δ





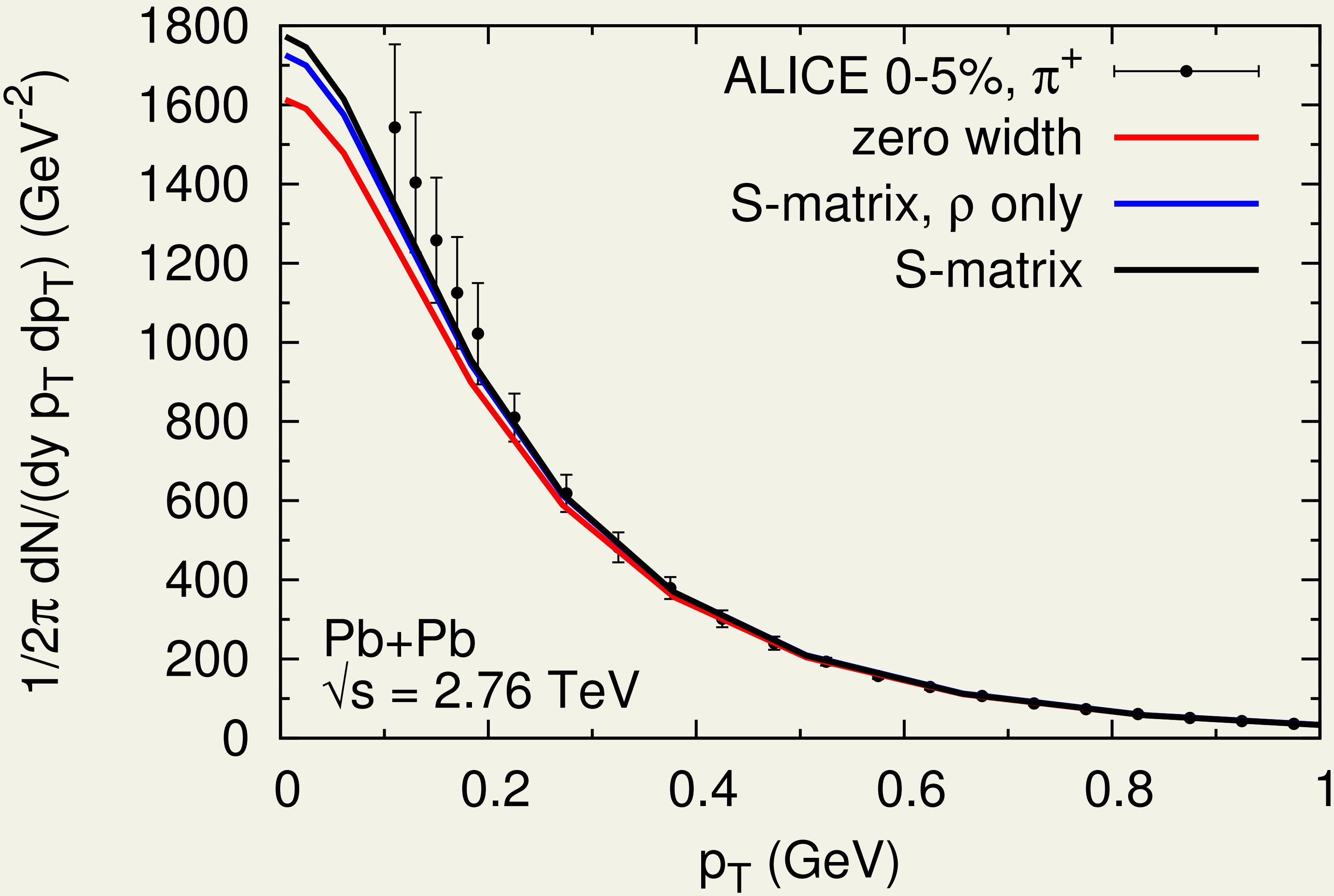
Pions from blast wave, $T = 150$ MeV



- $\tau = 14.1\,\mathrm{fm}$
- $R = 10\,\mathrm{fm}$
- $v_{max} = 0.8$

- **all resonances up to 2 GeV**
- **Beth-Uhlenbeck for** ρ, Δ, $f_0(980)$, $K^*(892)$, $K_0^*(1430)$
- **zero width for everything else**

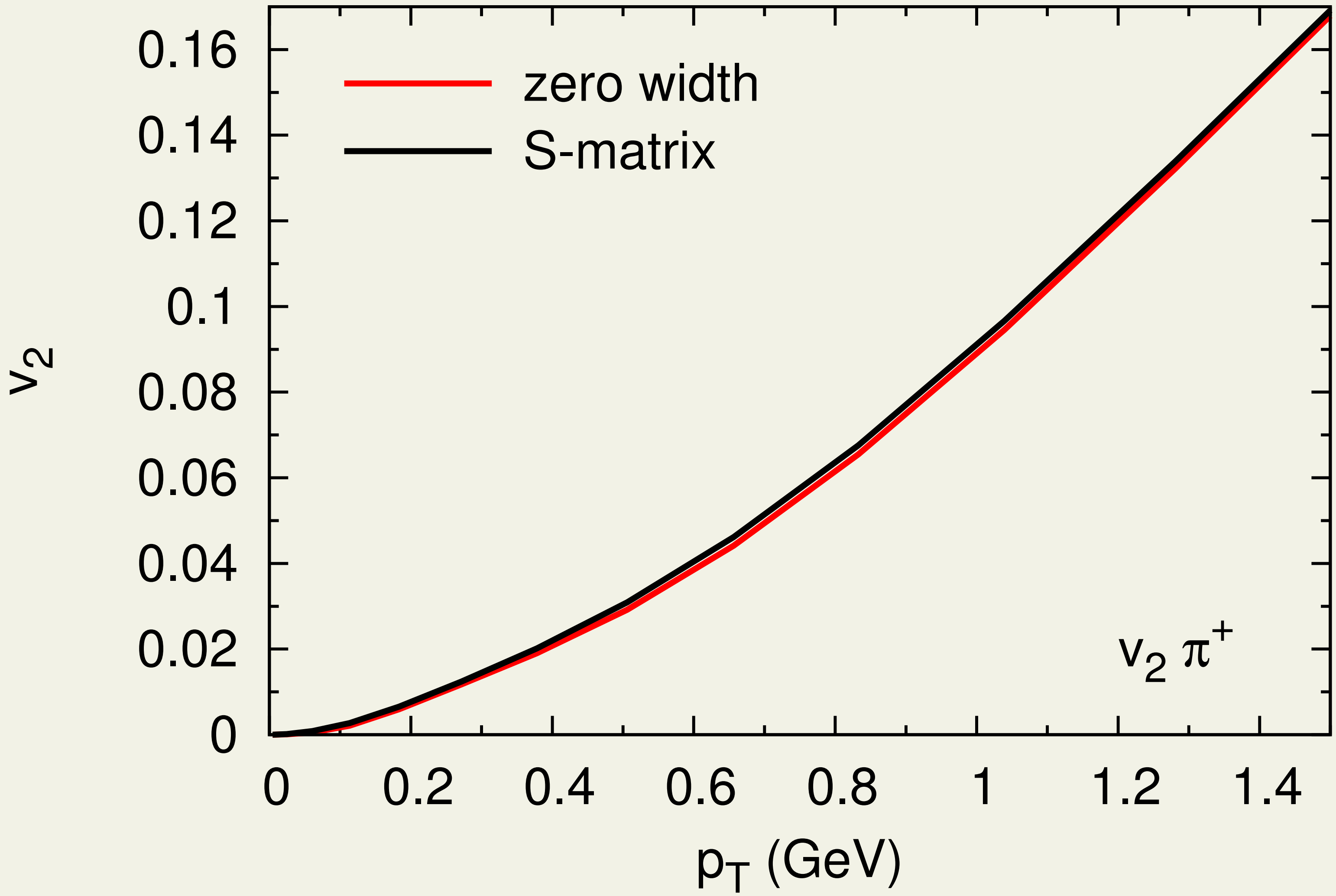
Pions from blast wave, $T = 120$, $T_{\text{chem}} = 150$ MeV



- $\tau = 31.0\,\text{fm}$
- $R = 10\,\text{fm}$
- $v_{max} = 0.87$

- **all resonances up to 2 GeV**
- **Beth-Uhlenbeck for** ρ, Δ, $f_0(980)$, $K^*(892)$, $K_0^*(1430)$
- **zero width for everything else**

v_2 of pions from blast wave, $T = 120$, $T_{\text{chem}} = 150$ MeV

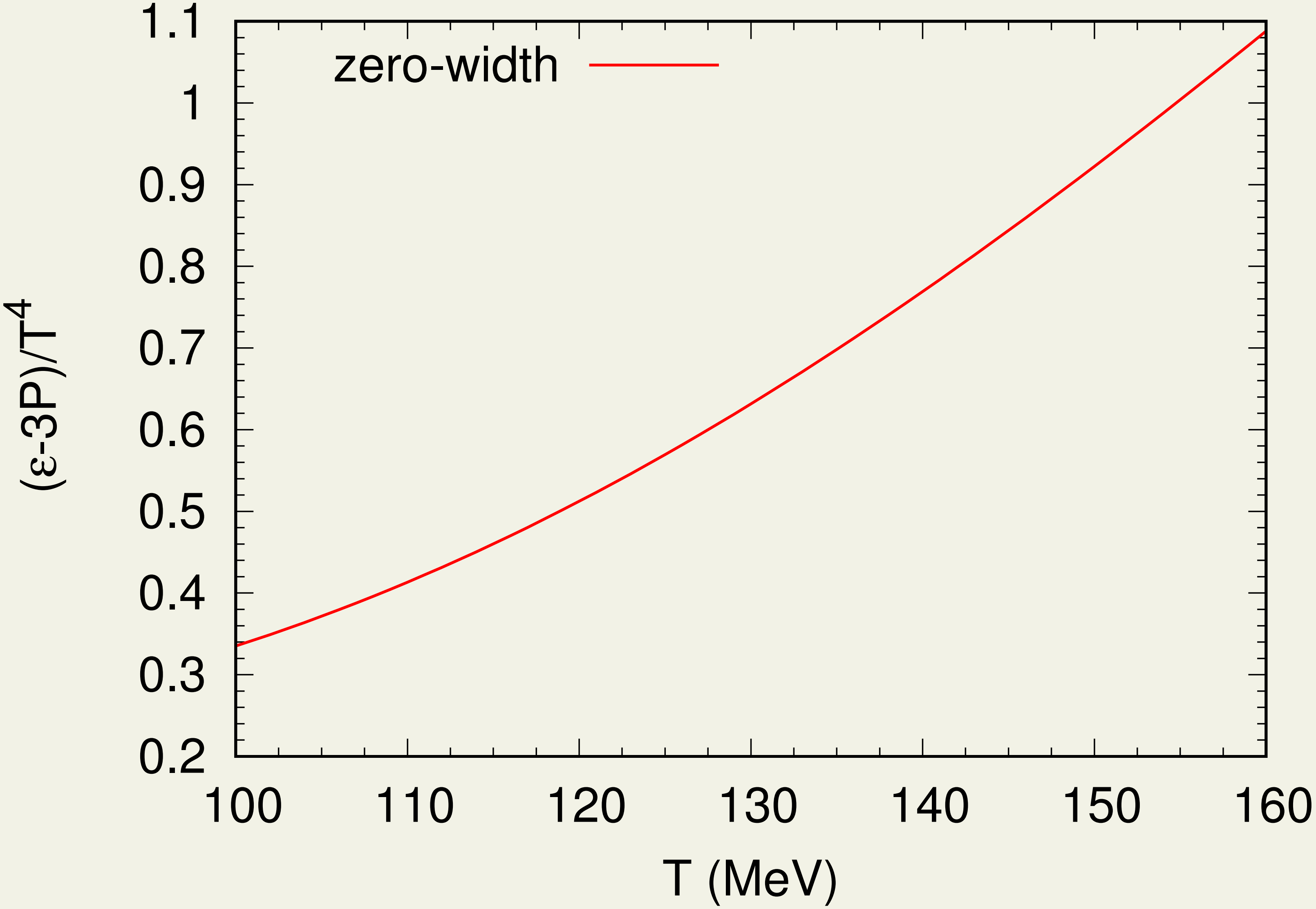


- $\tau = 31.0\,\text{fm}$
- $R = 10\,\text{fm}$
- $v_{max} = 0.87$

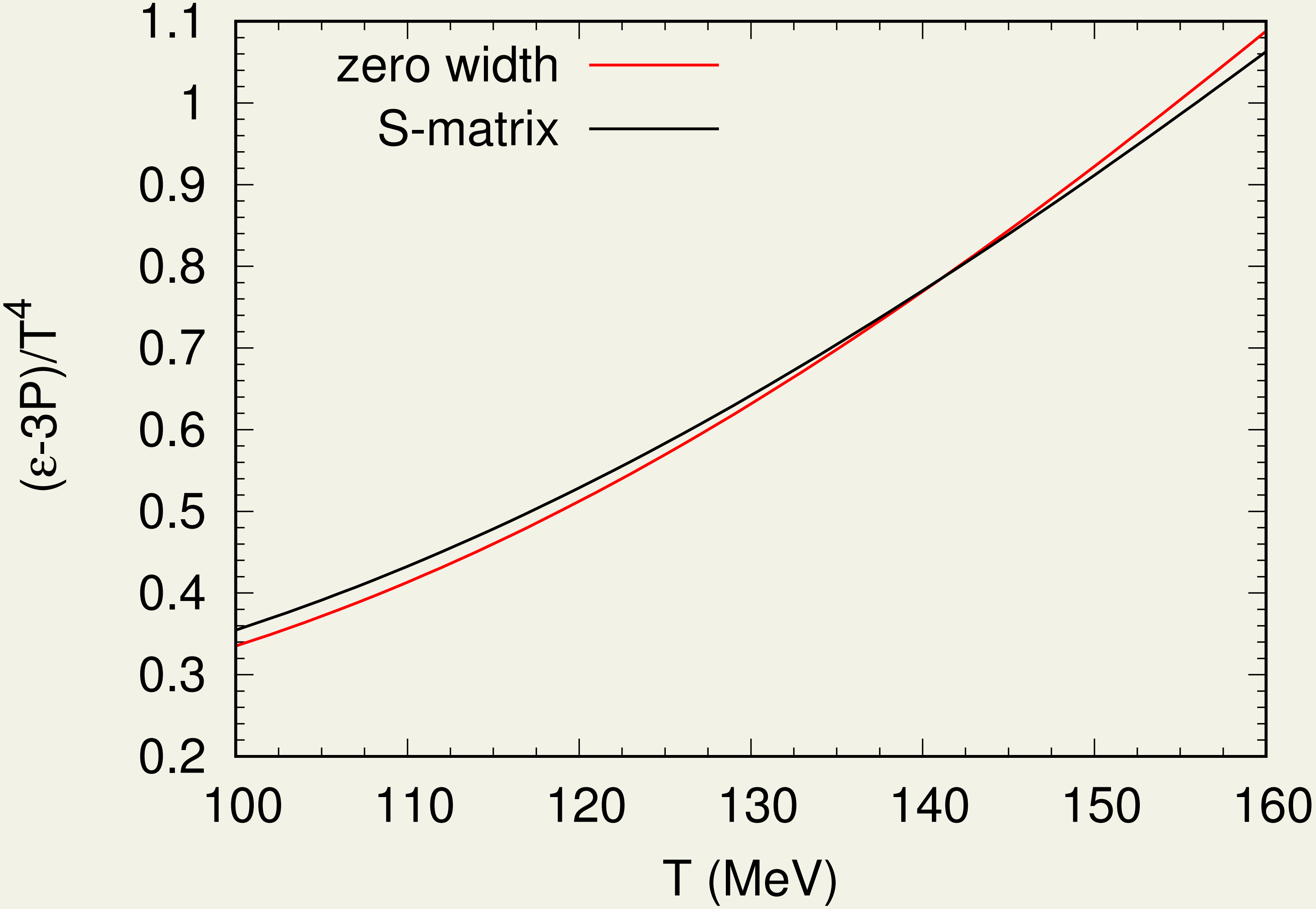
- **all resonances up to 2 GeV**
- **Beth-Uhlenbeck for** ρ, Δ, $f_0(980)$, $K^*(892)$, $K_0^*(1430)$
- **zero width for everything else**

and EoS?

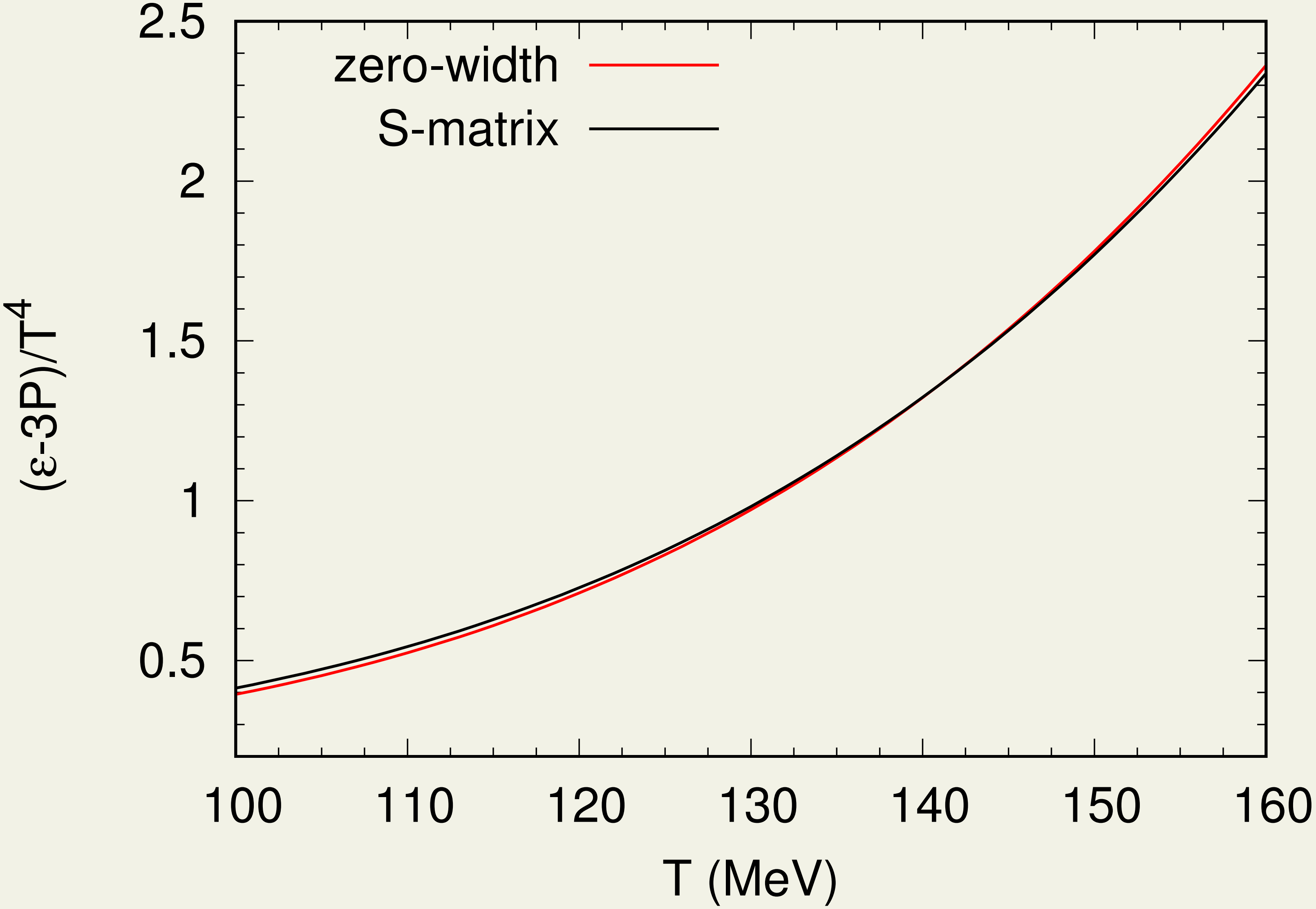
π, K, N, ρ, $f_0(980)$, K^*, $K_0(1430)$, Δ



π,K,N,ρ,$f_0(980)$,K^*,$K_0(1430)$,Δ



the whole zoo



Summary

- **Resonance widths change the low-p_T distribution of pions**

Summary

- **Resonance widths change the low-p_T distribution of pions**
  - **Fortunately $v_2(p_T)$ is not affected**

Summary

- **Resonance widths change the low-p_T distribution of pions**
  - **Fortunately $v_2(p_T)$ is not affected**
- **Effect on EoS uncertain**

Summary

- Resonance widths change the low-p_T distribution of pions
  - Fortunately $v_2(p_T)$ is not affected
- Effect on EoS uncertain
- Better treatment of resonances needed

Summary

- Resonance widths change the low-p_T distribution of pions
  - Fortunately $v_2(p_T)$ is not affected
- Effect on EoS uncertain
- Better treatment of resonances needed - we are working on it. . .

♻ This talk consisted of 100% recycled electrons