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The University of Nottingham

Robustness of Entanglement in Analogue Gravity Systems

Quantum Mechanics Tests in Particle, Atomic, Nuclear and Complex Systems: 50 years after Bell's renowned theorem

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Nicolai Friis - IQOQI Innsbruck

work in collaboration with

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Outline

Introduction & Motivation

- Why Analogue Gravity? Why care about quantumness?
- Typical situations in AG — Theory & Practice

Entanglement in Analogue Gravity[1]

- Covariance Matrix Formalism
- State Transformation — Nonzero Temperatures
- Generation of entanglement in AG

Example: Quench of a Bose-Einstein Condensate (BEC)

- Illustration of Temperature & Dispersion Effects
- Entanglement Resonances in a BEC

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Motivation to Study Analogue Gravity/Relativistic Effects

- Interesting effects of QFT in curved spacetimes
  e.g., Hawking-, Unruh-, dynamical Casimir effect, Firewall problem
  Bonus: possibly gain insights into quantum gravity
- Problem: black holes, large accelerations, expanding universes not directly accessible $\Rightarrow$ Tests in analogue gravity systems[2,3]

Quantumness of simulated effects?

- If quantum features are to be tested
  $\Rightarrow$ quantumness of simulation must be asserted[4]
- Identify quantumness via entanglement generation—Bell inequalities

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Typical Situations in AG - Theory

Analogue Spacetime

Medium (e.g., BEC, water tank) mimics curved spacetime structure

Phonons & collective excitations represent excitations of quantum field

Bogoliubov transformation relates modes in two regions

"Inside" ⇔ "Outside"

"Before" ⇔ "After"

In homogeneous media, free modes ⇒ conservation of momentum

only Bogoliubov coefficients α_{kk} and $\beta_{k(-k)}$

Signature of Quantumness?

Theoretical settings predict entanglement between "Inside" and "Outside" modes, e.g., in an expanding universe[5]

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Typical Situations in AG - Practical Limitations

Homogeneity & Boundaries

Problem: Media usually not completely homogeneous, modes not really plane waves

No elementary way to fix this, but entanglement generation limited by temperature even in idealized case.

Nonlinear Dispersion

Typically effects of nonlinear dispersion— intrinsic feature of the system

Nonzero Temperatures

Experiments conducted at nonzero temperature, e.g., water tanks, even BECs in nK regime.

What is the the threshold temperature for entanglement generation?

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Gaussian States - covariance matrix formalism

$$\Gamma_{ij} = \langle\, \mathbb{X}_i \mathbb{X}_j + \mathbb{X}_j \mathbb{X}_i \,\rangle - 2\,\langle\, \mathbb{X}_i \,\rangle \langle\, \mathbb{X}_j \,\rangle$$

quadratures: $\mathbb{X}_{(2n-1)} = \frac{1}{\sqrt{2}}(a_n + a_n^\dagger)$ and $\mathbb{X}_{(2n)} = \frac{-i}{\sqrt{2}}(a_n - a_n^\dagger)$

Bogoliubov transformation $\Rightarrow$ symplectic transformation[6] S

$$\tilde{\Gamma} = S\,\Gamma\,S^T \quad \text{where} \quad S = \begin{pmatrix} \mathcal{M}_{kk} & \mathcal{M}_{k(-k)} \\ \mathcal{M}_{(-k)k} & \mathcal{M}_{(-k)(-k)} \end{pmatrix}$$

$$\mathcal{M}_{nn} = \begin{pmatrix} \mathrm{Re}\,\alpha_{nn} & \mathrm{Im}\,\alpha_{nn} \\ -\mathrm{Im}\,\alpha_{nn} & \mathrm{Re}\,\alpha_{nn} \end{pmatrix}, \quad \mathcal{M}_{n(-n)} = \begin{pmatrix} -\mathrm{Re}\,\beta_{n(-n)} & \mathrm{Im}\,\beta_{n(-n)} \\ \mathrm{Im}\,\beta_{n(-n)} & \mathrm{Re}\,\beta_{n(-n)} \end{pmatrix}$$

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$$\mathcal{M}_{nn} = \begin{pmatrix} \mathrm{Re}\,\alpha_{nn} & \mathrm{Im}\,\alpha_{nn} \\ -\mathrm{Im}\,\alpha_{nn} & \mathrm{Re}\,\alpha_{nn} \end{pmatrix}, \quad \mathcal{M}_{n(-n)} = \begin{pmatrix} -\mathrm{Re}\,\beta_{n(-n)} & \mathrm{Im}\,\beta_{n(-n)} \\ \mathrm{Im}\,\beta_{n(-n)} & \mathrm{Re}\,\beta_{n(-n)} \end{pmatrix}$$

$\alpha_{kk} = e^{i\Theta}\alpha_{(-k)(-k)}$, and $\beta_{k(-k)} = e^{i\Theta}\beta_{(-k)k}$,

[6] see, e.g., recent work: N. Friis and I. Fuentes, J. Mod. Opt. **60**, 22 (2013) [arXiv:1204.0617].

State Transformation — Nonzero Temperatures

Initial separable state

modes k and $-k$ have the same initial frequency $\omega_{\text{in}} = \omega_{\text{in}}(|k|)$

thermal state at temperature T: $\Gamma_{\text{th}}(T) = \coth\left(\frac{\hbar\,\omega_{\text{in}}}{2\,k_{\text{B}}\,T}\right)\mathbb{1}$

Transformed state: $\omega_{\text{in}} \to \omega_{\text{out}}$

$$\tilde{\Gamma} = S\,\Gamma_{\text{th}}(T)\,S^{\text{T}} = \begin{pmatrix} \tilde{\Gamma}_k & C \\ C^{\text{T}} & \tilde{\Gamma}_{-k} \end{pmatrix}$$

reduced state covariance matrices of the individual modes

$$\tilde{\Gamma}_k = \tilde{\Gamma}_{-k} = \coth\left(\frac{\hbar\,\omega_{\text{in}}}{2\,k_{\text{B}}\,T}\right)(2|\beta_{k(-k)}|^2 + 1)\,\mathbb{1}$$

define characteristic *entanglement temperature* T_{E} via

$$\coth\left(\frac{\hbar\,\omega_{\text{out}}}{2\,k_{\text{B}}\,T_{\text{E}}}\right) = (2|\beta_{k(-k)}|^2 + 1)$$

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Generation of Entanglement in AG

Entanglement of Formation

$$E_{oF}(\rho_{\mathrm{AB}}) \,=\, \inf_{\{(p_i,\psi_i^{\mathrm{AB}})\}} \sum_i p_i\, \mathcal{E}(|\,\psi_i^{\mathrm{AB}}\,\rangle)$$

E_{oF} computable for symmetric two-mode Gaussian states[7]

$$E_{oF} \,=\, \begin{cases} h(\nu_-) & \text{if } \; 0 \leq \nu_- < 1 \\ 0 & \text{if } \; \nu_- \geq 1 \end{cases}$$

where $h(x) \,=\, \frac{(1+x)^2}{4\,x} \ln \frac{(1+x)^2}{4\,x} \,-\, \frac{(1-x)^2}{4\,x} \ln \frac{(1-x)^2}{4\,x}$

ν_- ... smallest symplectic eigenvalue of the partial transpose

Here:[1] $\nu_-(T) \,=\, \coth\!\left(\frac{\hbar\,\omega_{\mathrm{in}}}{2\,k_{\mathrm{B}}\,T}\right)\left(|\alpha_{kk}| - |\beta_{k(-k)}|\right)^2$

$\nu_-(T_{\mathrm{SD}}) = 1$ *sudden death temperature* $T_{\mathrm{SD}} \,=\, 2\,\frac{\omega_{\mathrm{in}}}{\omega_{\mathrm{out}}}\,T_{\mathrm{E}}$

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Single Quench Bogoliubov Transformation

BEC Setting

Homogeneous BEC → single, sudden change in density: "quench" at t_0

Nonlinear dispersion relation: $\omega^2 = c^2\,k^2 + \epsilon^2\,k^4\,, \quad \epsilon = \hbar/(2m)$

(i) affects k-dependence of temperature distribution

(ii) enters directly in the Bogoliubov coefficients

Bogoliubov coefficients[8]

$$\alpha_{kk} = \frac{1}{2}\left(\sqrt{\frac{\omega_{\mathrm{out}}}{\omega_{\mathrm{in}}}} + \sqrt{\frac{\omega_{\mathrm{in}}}{\omega_{\mathrm{out}}}}\right) e^{i\,(\omega_{\mathrm{out}} - \omega_{\mathrm{in}})t_0}\,,$$

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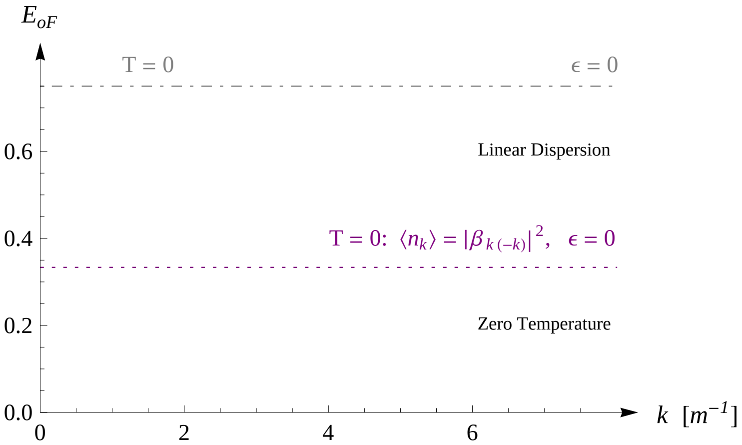

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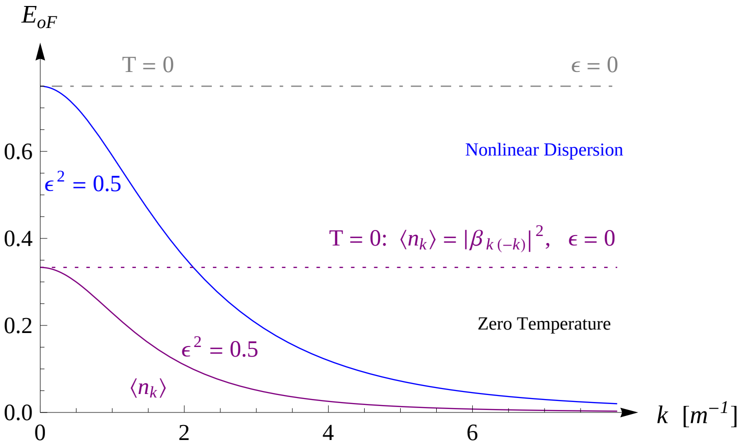

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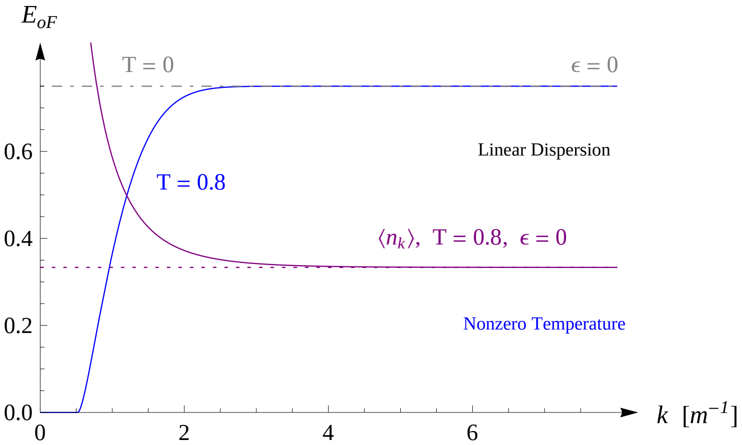

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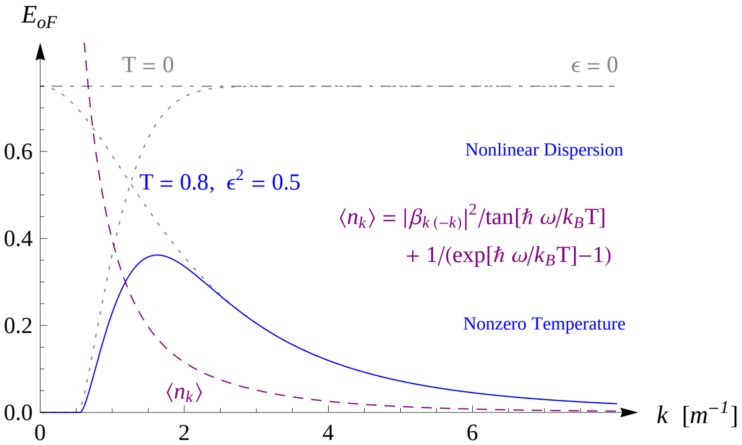

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Entanglement Resonances in a BEC

General entanglement resonances for 2 modes

symplectic transformation decomposed as $S = S_{\mathsf{P}} S_{\mathsf{A}}$

(i) passive: rotations and beam splitting $S_{\mathsf{P}}^T S_{\mathsf{P}} = \mathbb{1}$

(ii) active: single- and two-mode squeezing $S_{\mathsf{A}} = S_{\mathsf{A}}^T$

Here: $S_{\mathsf{A}} = S_{\mathsf{TMS}}(r)$ pure 2-mode squeezing

Resonance Condition: if $[S\,, S^T] = 0 \;\Rightarrow\; S(r_1)S(r_2) = S(r_1 + r_2)$

Entanglement resonances for BEC

Combine quenches $\omega_{\text{in}} \rightarrow \omega_{\text{out}}$ and $\omega_{\text{out}} \rightarrow \omega_{\text{in}}$

Tune transformation to enhance entanglement at particular frequencies

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Summary & Conclusion

Robustness of Entanglement in Analogue Gravity

- Homogeneous/Idealized system $\Rightarrow$ analytic results for generated entanglement
- Fixed transformation & frequency $\Rightarrow$ sudden death temperature T_{SD}
- Inhomogeneous/Realistic system $\Rightarrow$ T_{SD} provides upper bound
- Resonances can enhance entanglement generation
- Note also the recent related results[9–11].

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