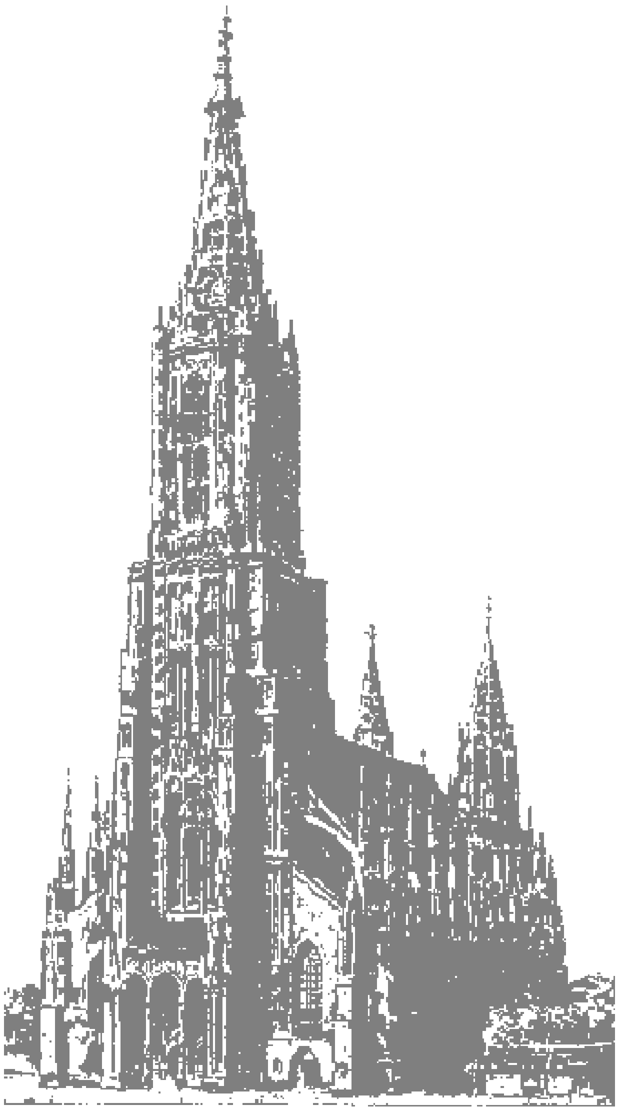


Few-body physics induced by p-wave resonance

Maxim A. Efremov

Institut für Quantenphysik, Universität Ulm

in collaboration with

Wolfgang P. Schleich (Universität Ulm)

Lev Plimak & Misha Ivanov (Max-Born-Institut, Berlin)

ECT*-Workshop: "Resonances and non-hermitian quantum mechanics in nuclear and atomic physics"

Trento, June 23-27, 2014

Outline

- Two-body resonant state and forces
- 3 dimensions and s-wave resonance:

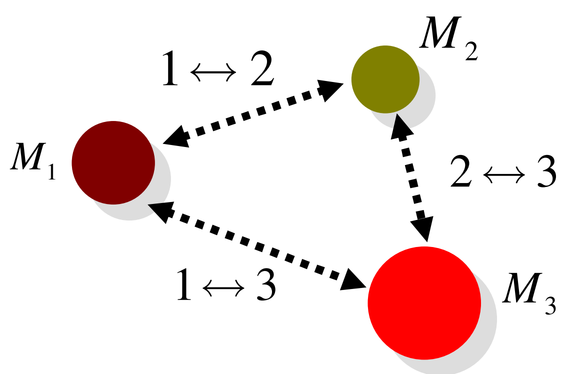
  Efimov effect

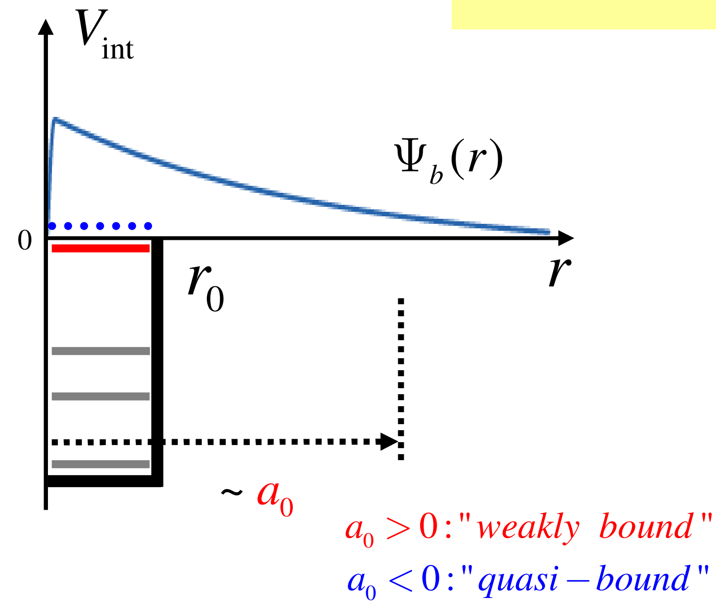
  - theory and experiments

  - three messages
- 3 dimensions and p-wave resonance
- 2 dimensions and p-wave resonance
- Conclusion and outlook

Resonant two-body forces and state

$$|a_0| >> r_0$$





universal properties
for s-wave resonance

binding energy: $E_b \sim \dfrac{1}{m a_0^2}$

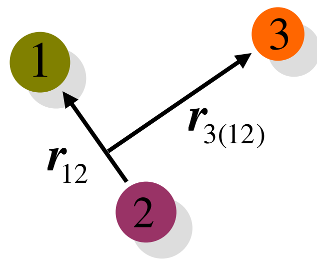
bound length: $\langle r \rangle \sim a_0$

Three-body system: Efimov effect

$V_{ij} : |(a_0)_{ij}| >> (r_0)_{ij}$

$ij = \{12, 13, 23\}$

$R = \sqrt{r_{12}^2 + r_{3(12)}^2}$

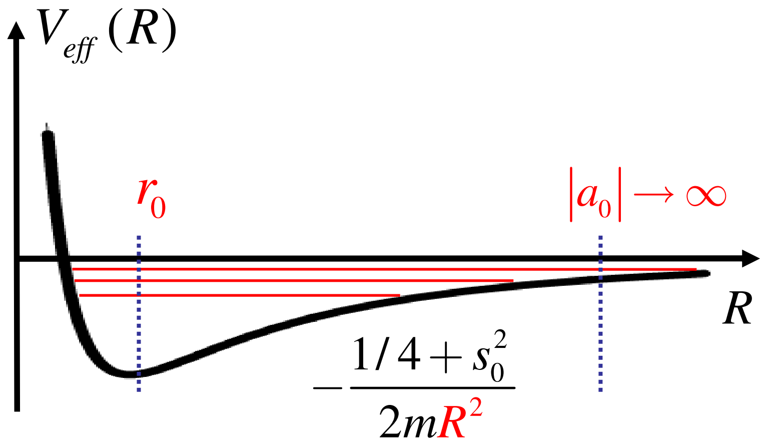


The effective Schrödinger equation

$(r_0 << R << |a_0|)$ *Vitaly Efimov (1970)*

$$\frac{1}{2m}\left(-\frac{d^2}{dR^2} - \frac{s_0^2 + 1/4}{R^2}\right)\Psi_0(R) = E\Psi_0$$

$$s_0 = s_0(m_1, m_2, m_3, a_{12}, a_{13}, a_{23})$$



$$E_n = E_0 \exp\left(-\frac{2\pi}{s_0} n\right), \quad n \gg 1, \quad E_0 \sim \frac{1}{m r_0^2}$$

Number of Efimov states:

$$N = \frac{s_0}{\pi} \ln\left(\frac{|a_0|}{r_0}\right) >> 1$$

$r_0 \to 0$: *Thomas effect (1935)*

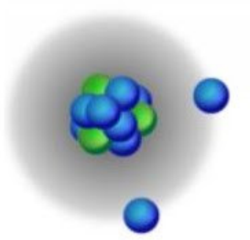
$|a_0| \to \infty$: *Efimov effect (1970)*

Efimov states in nuclear physics

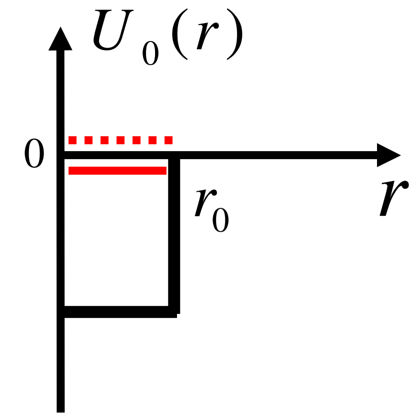
- potential ("shape") resonance – not tunable

Nuclear physics: $^{14}\mathrm{Be}, ^{18}\mathrm{C}, ^{20}\mathrm{C}, ..$

A.S. Jensen et al., RMP 76, 215 (2004)

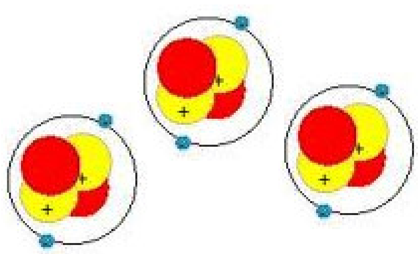


$a_0 \gg r_0$



Molecular physics: $^{4}\mathrm{He} - atoms$

J.P. Toennies et al., PRL 95, 063002 (2005)



Efimov states in atom physics

- Feshbach resonance - tunable

F. Ferlaino and R. Grimm, Physics 3, 9 (2010)

$$a_0 = a_{Bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$$

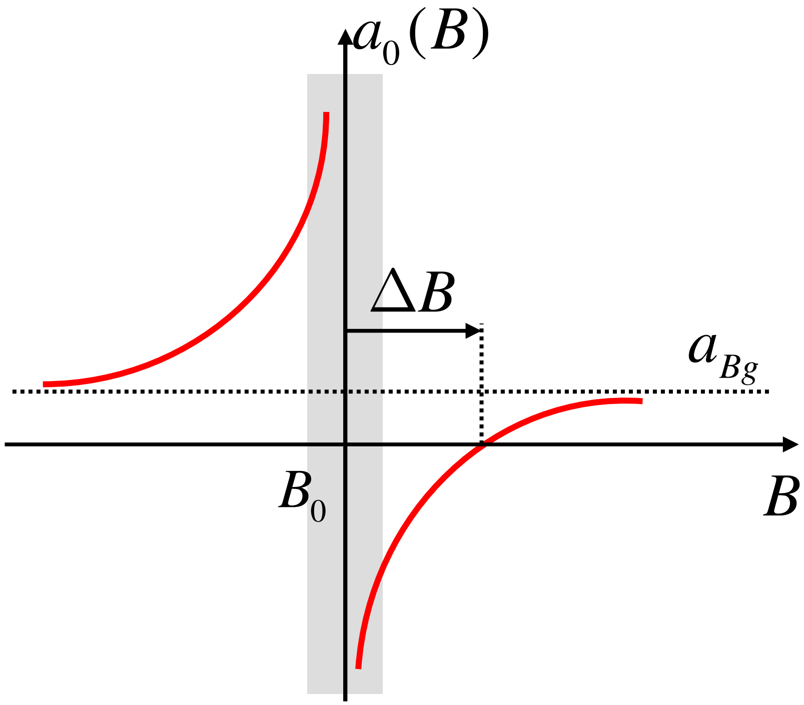
Cs *T. Kraemer et al., Nature 404, 315 (2006)*
S. Knoop et al., Nature Physics 5, 227 (2009)
B. Huang et al., PRL 112, 190401 (2014)

K *M. Zaccanti et al., Nature Physics 5, 586 (2009)*

Li *N. Gross et al., PRL 103, 163202 (2009)*
S.E. Pollack et al., Science 18, 1683 (2009)
N. Gross et al., PRL 105, 103203 (2010)
T.B. Ottenstein et al., PRL 101, 203202 (2008)
A.N.Wenz et al., PRA 80, 040702(R) (2009)
J.H. Huckans et al., PRL 102, 165302 (2009)
J.R. Williams et al., PRL 103, 130404 (2009)
T. Lompe et al., PRL 105, 103201 (2010)
S. Nakajima et al., PRL 105, 023201 (2010)
T. Lompe et al., Science 330, 940 (2010)
S. Nakajima et al., PRL 106, 143201 (2011)

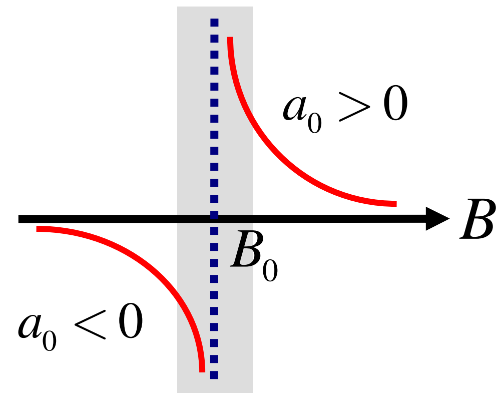
K-Rb *G. Barontini et al., PRL 103, 043201 (2009)*

Li-Cs *S.-K. Tung et al., arXiv:1402.5943*
R. Pires et al., arXiv:1403.7246



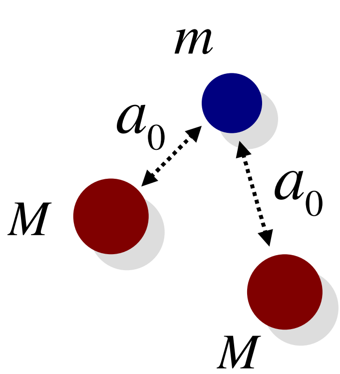
Efimov effect: summary

- **adjustable** and **large** scattering length $a_0(B)$

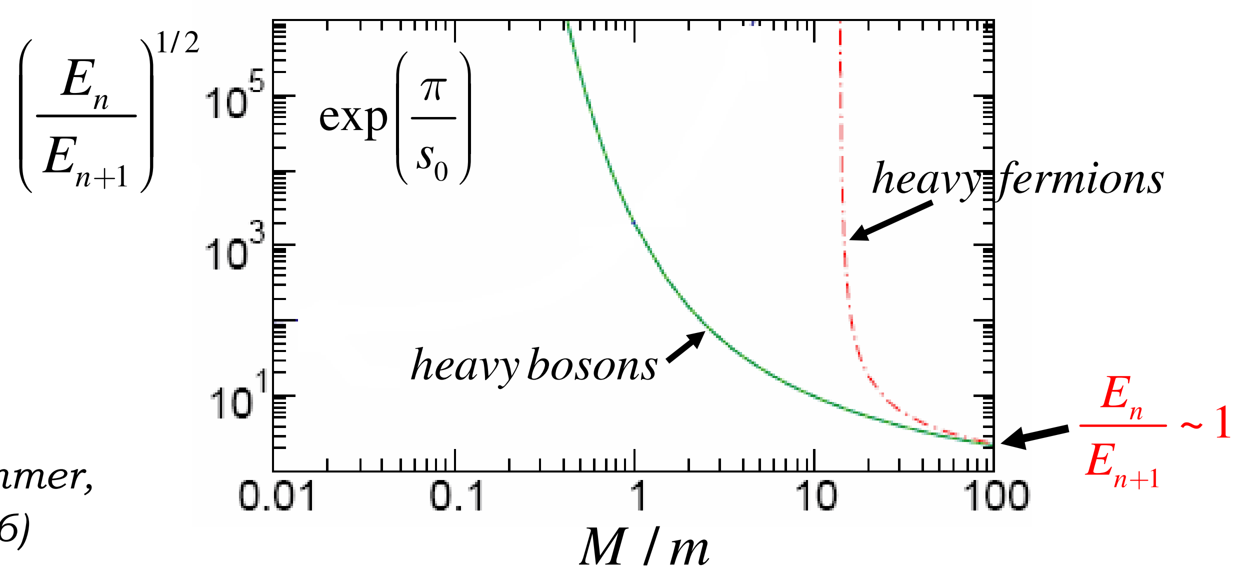


- two species with **different masses** $M \gg m$

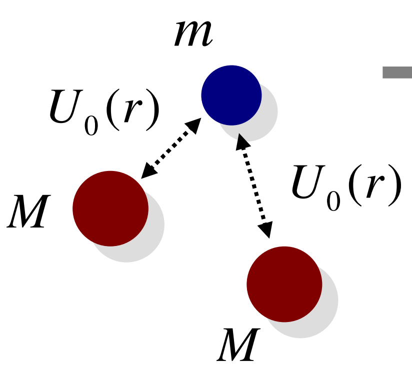
- **statistics** of atoms



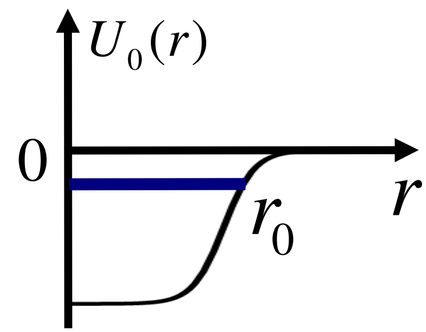
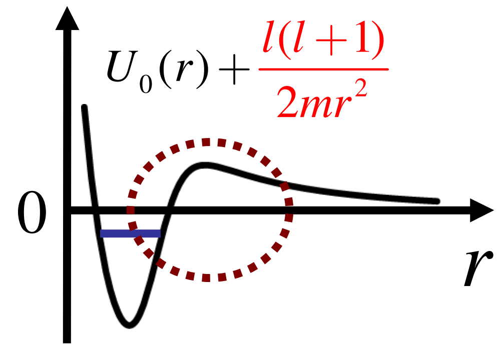
E. Braaten and H.W. Hammer, Phys. Rep. 428, 259 (2006)



Dimensionality and resonance symmetry

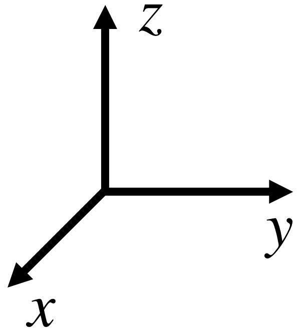


symmetry of two − body resonant state → l

d	$l=0$	$l=1$
	 a_0	 a_1, R_1
3	*Efimov effect* $V_{eff}(R) \sim -\frac{1}{mR^2}$ $E_n \sim \exp(\alpha n);\ N \sim \frac{M}{m}\ln\left(\frac{a_0}{r_0}\right)$	?
2	*finite number of trimers* $V_{eff}(\rho) \sim \exp\left(-\frac{\rho}{a_0^{(2)}}\right)$	?

Three dimensions and P-wave resonance

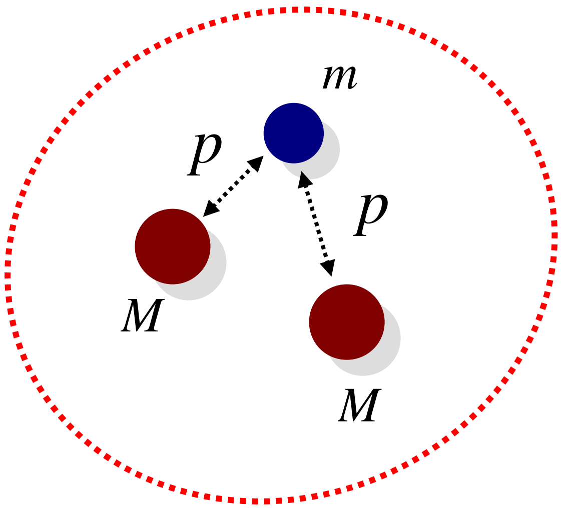
3d



$$S_0(k) - 1 = -\frac{2ik}{a_0^{-1} + ik}$$

$$S_1(k) - 1 = -\frac{2ik^3}{\left[a_1^{-1} + \frac{1}{2}R_1 k^2 - ik^3\right]}$$

p - wave resonance



$M \gg m$

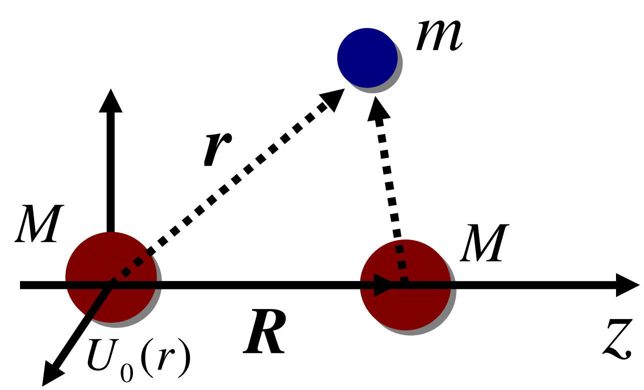
- Skornyakov-Ter-Martirosyan (Petrov, Fedorov, etc.)
- Effective field theory (Braaten & Hammer)
- Adiabatic hyperspherical functions (Green, Macek, Esry, Nielsen, etc.)
- Born-Oppenheimer approximation, $M \gg m$ (Fonseca)

Born-Oppenheimer approximation

$$\Phi(\boldsymbol{r},\boldsymbol{R}) = \sum_{m_l=-1,0,1} \chi_+^{(m_l)}(\boldsymbol{r};\boldsymbol{R})F_+^{(m_l)}(\boldsymbol{R}) + \chi_-^{(m_l)}(\boldsymbol{r};\boldsymbol{R})F_-^{(m_l)}(\boldsymbol{R})$$

I step: Light particle in two-well potential

$$\left(-\frac{1}{2m}\frac{\partial^2}{\partial \boldsymbol{r}^2} + U_0(\boldsymbol{r}) + U_0(\boldsymbol{r}-\boldsymbol{R})\right)\chi_\pm(\boldsymbol{r}) = E_\pm(\boldsymbol{R})\chi_\pm(\boldsymbol{r})$$



II step: Dynamics of two heavy particles

$$\left(-\frac{1}{M}\frac{\partial^2}{\partial \boldsymbol{R}^2} + E_\pm(\boldsymbol{R})\right)F(\boldsymbol{R}) = \varepsilon F(\boldsymbol{R})$$

ε : three-body binding energy

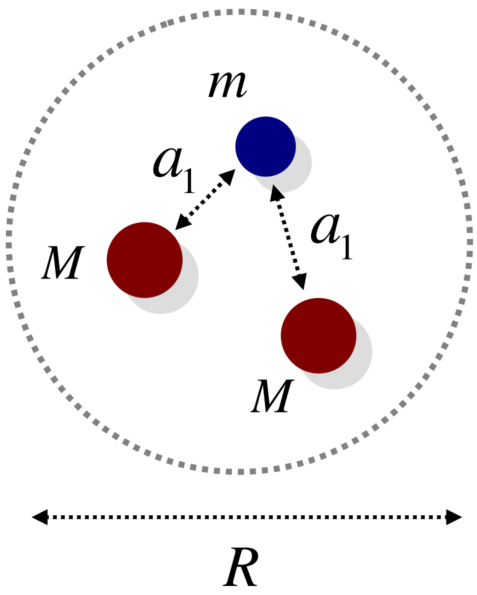
Boundary condition: $F_\pm(\boldsymbol{R})|_{R\to\infty} = 0$

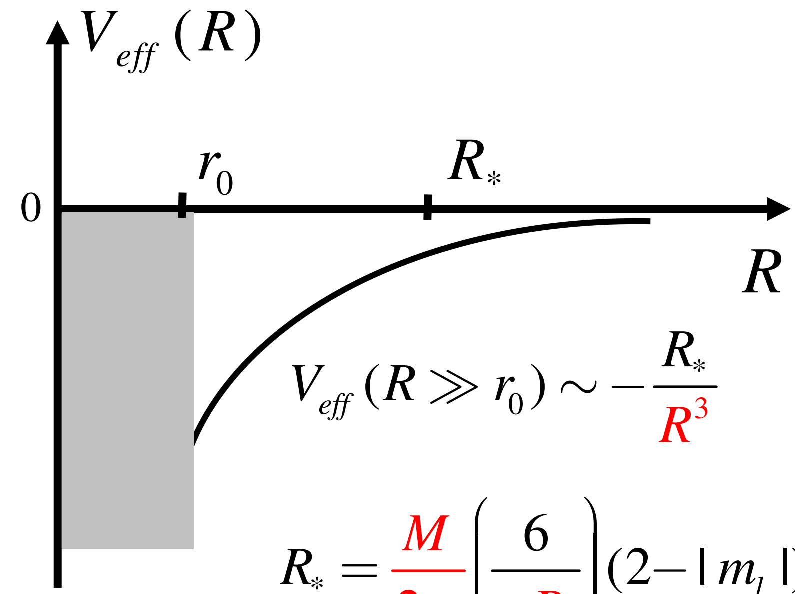
Universal three-body states: 3d and P-wave

p − wave resonance

$$a_1^{-1} = 0$$

$$R_1 > 0$$



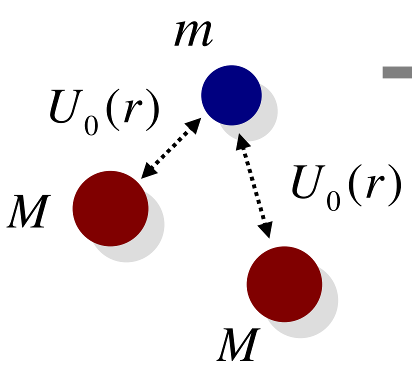


$$R_* = \frac{M}{2m}\left(\frac{6}{r_0 R_1}\right)(2-|m_l|)r_0$$

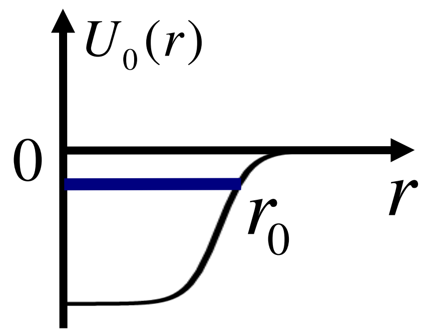
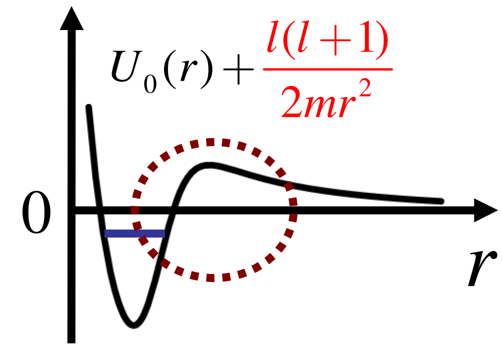
- effective range: $R_* \gg r_0$
- finite number of bound states: $N \approx \frac{R_*}{8r_0} \sim \frac{M}{m}\frac{1}{r_0 R_1}$
- unique energy spectrum: $E_n = -\frac{\alpha}{MR_*^2}(n-n_*)^6$
- scattering amplitudes: $\tan\delta_0(kR_* \ll 1) = -kR_* \ln(kR_*), \tan\delta_{L>0}(kR_* \ll 1) \sim kR_*$

*M.A.E, L. Plimak, M. Ivanov, W.P. Schleich, PRL **111**, 113201 (2013)*

Dimensionality and resonance symmetry



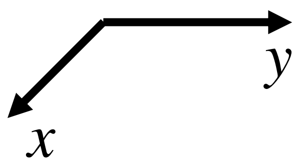
$symmetry\ of\ two-body\ resonant\ state$ → l

$l=0$	$l=1$
	
a_0	a_1, R_1

d	$l=0$	$l=1$
3	*Efimov effect* $V_{eff}(R) \sim -\frac{1}{mR^2}$ $E_n \sim \exp(\alpha n);\ N \sim \frac{M}{m}\ln\left(\frac{a_0}{r_0}\right)$	$V_{eff}(R) \sim -\frac{R_*}{R^3};\quad R_* \sim \frac{M}{m} r_0$ $E_n \sim (n-n_*)^6;\quad N \sim \frac{M}{m}\frac{1}{r_0 R_1}$
2	*finite number of trimers* $V_{eff}(\rho) \sim \exp\left(-\frac{\rho}{a_0^{(2)}}\right)$	?

Two dimensions and P-wave resonance

$2d$



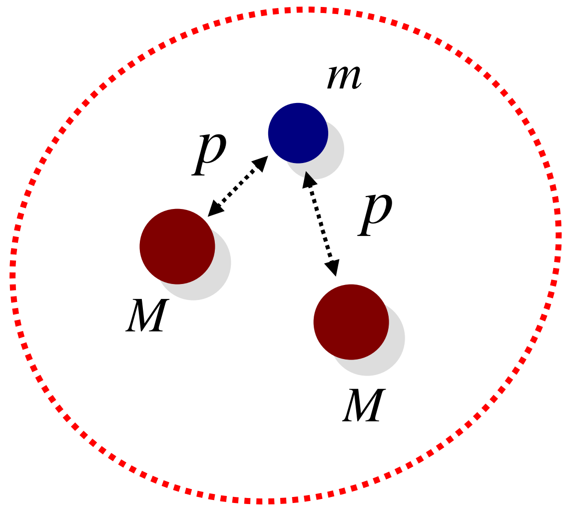
$$S_m(k) - 1 = \frac{2i}{\cot\delta_m - i}$$

$$m = 0, \pm 1, \pm 2, \ldots$$

$$\cot\delta_0(k) \cong \frac{2}{\pi}\left(\gamma + \ln\frac{k a_0^{(2)}}{2}\right)$$

$$\cot\delta_1(k) \cong \frac{2}{\pi}\left(-\frac{1}{a_1^{(2)} k^2} + \ln(k r_1)\right)$$

p - *wave resonance*



$$M \gg m$$

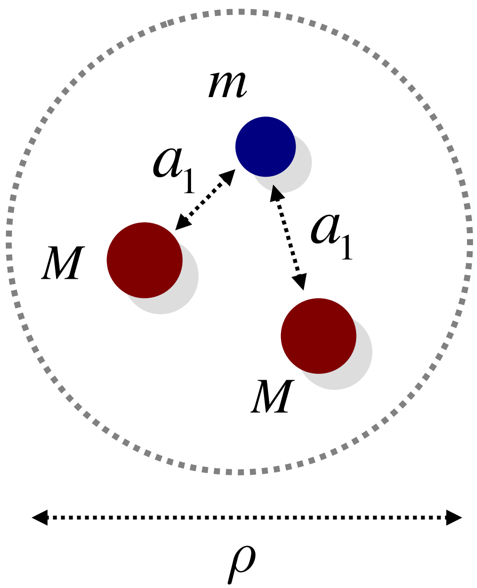
$$a_0^{(2)} \geq 0,\ a_1^{(2)} \geq 0 \ \ and \ \ 0 < r_1\big|_{\substack{p-wave \\ resonance}} \leq \frac{1}{2} e^{\gamma} r_0$$

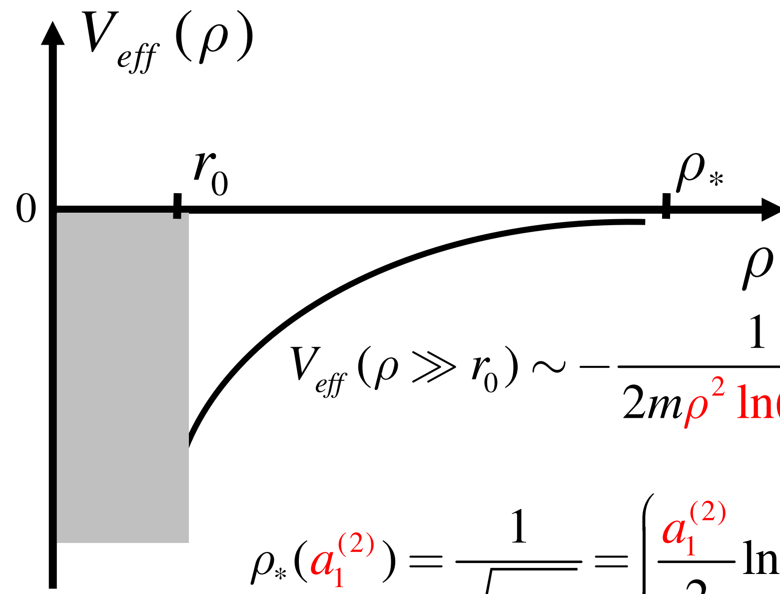
H.-W. Hammer and D. Lee, Ann. Phys. 325, 2212 (2010)
M. Randeriaet al., Phys. Rev. B 41, 327 (1990)
S.A. Rakityansky and N. Elander, J. Phys. A 45, 135209 (2012)

Universal three-body states: 2d and P-wave

$p - wave\ resonance$

$a_1^{(2)} \gg r_0^2$





$$V_{eff}(\rho \gg r_0) \sim -\frac{1}{2m\rho^2 \ln(\rho / r_1)}$$

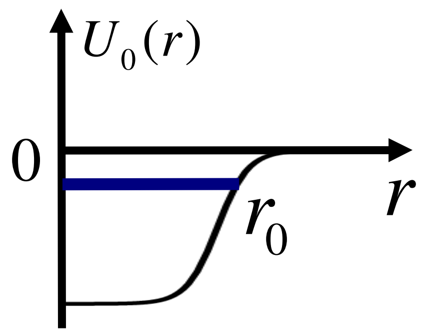
$$\rho_*(a_1^{(2)}) = \frac{1}{\sqrt{m\varepsilon_p}} = \left(\frac{a_1^{(2)}}{2} \ln \frac{a_1^{(2)}}{2r_1^2}\right)^{1/2}$$

- unique energy spectrum: $E_n = -\frac{E_0}{n^2} \exp\left(-\frac{\pi^2}{2} \frac{m}{M} n^2\right)$; $n \gg 1$; $E_0 \sim \frac{1}{mr_0^2}$
- infinite number of bound states: $N = \frac{1}{\pi}\left(\frac{2M}{m} \ln \frac{a_1^{(2)}}{2r_1^2}\right)^{1/2}$
- atom-molecule scattering: $\ln \frac{A_{at-mol}}{\rho_*(a_1^{(2)})} = -\frac{m}{2M} \frac{\pi N(a_1^{(2)})}{\cot\left[\pi N(a_1^{(2)})\right]}$

Summary and outlook

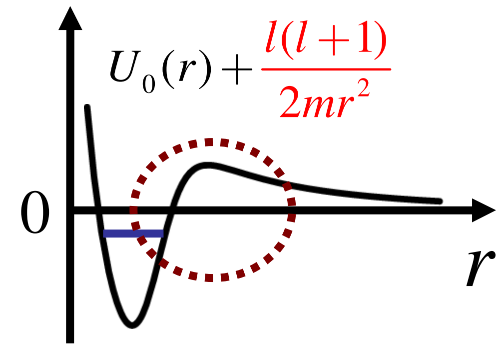
symmetry of two − body resonant state →

$l=0$



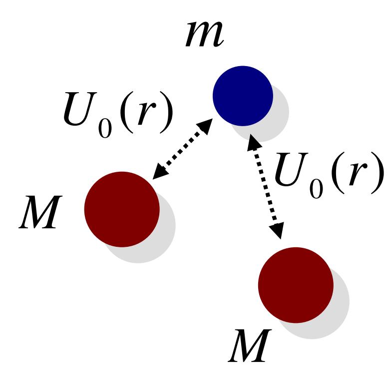
a_0

$l=1$



a_1, R_1

$l>1$



d	$l=0$	$l=1$	$l>1$
3	*Efimov effect* $V_{eff}(R)\sim-\frac{1}{mR^2}$ $E_n\sim\exp(\alpha n);\ N\sim\frac{M}{m}\ln\left(\frac{a_0}{r_0}\right)$	$V_{eff}(R)\sim-\frac{R_*}{R^3};\quad R_*\sim\frac{M}{m}r_0$ $E_n\sim(n-n_*)^6;\quad N\sim\frac{M}{m}\frac{1}{r_0R_1}$	?
2	*finite number of trimers* $V_{eff}(\rho)\sim\exp\left(-\frac{\rho}{a_0^{(2)}}\right)$	$V_{eff}(\rho)\sim-\frac{1}{2m\rho^2\ln(\rho/r_1)}$ $E_n\sim n^{-2}\exp\left[-(\pi^2m/2M)n^2\right]$	?