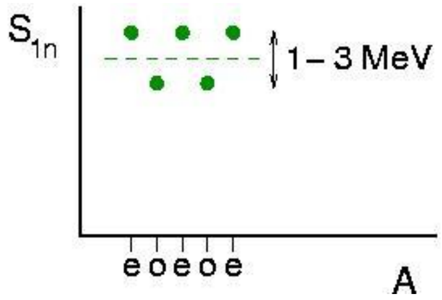
INTRODUCTION

First historical evidence for pairing interactions: Nuclear Masses

	$M(A,Z)$ [u]	$B(A,Z)$ [MeV]	B/A [MeV]	S_{1n} [MeV]
$^{131}_{54}$Xe	130.9051	1103.5	8.42	6.6
$^{132}_{54}$Xe	131.9042	1112.5	8.43	9.0
$^{133}_{54}$Xe	132.9058	1119.0	8.41	6.5
$^{134}_{54}$Xe	133.9054	1127.4	8.41	8.4
$^{135}_{54}$Xe	134.9070	1134.0	8.40	6.6
$^{136}_{54}$Xe	135.9072	1141.9	8.40	8.0



Semiempirical Binding Energy Formula (Von Weiszacker, 1935)

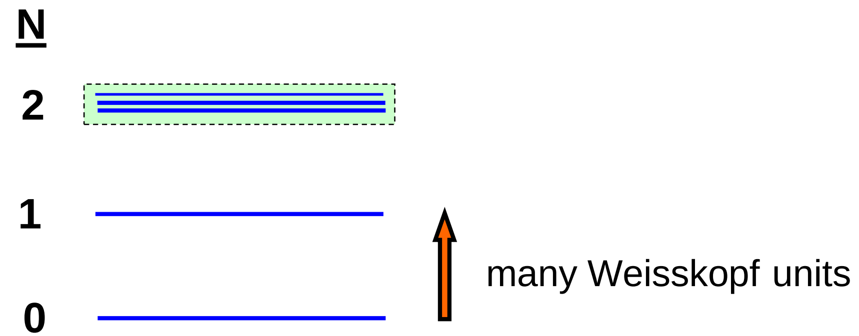
$$B(A,Z) = a_v\ A - a_s\ A^{2/3} - a_c\ \frac{Z^2}{A^{1/3}} - a_a\ \frac{(A-2Z)^2}{A} + \delta_p(A)$$

where,

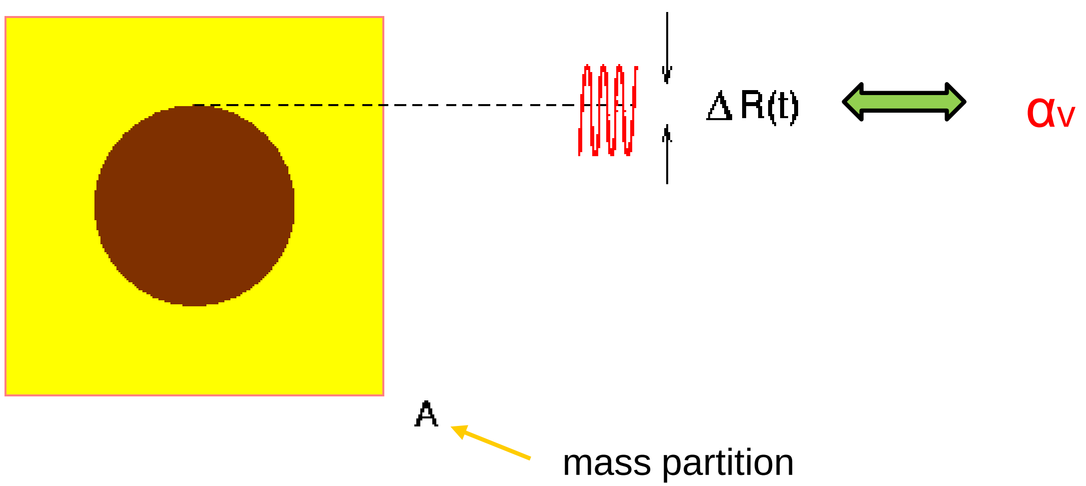
$$\delta_p(A) = \begin{cases} +a_p\ A^{-1/2} & \text{even-even} \quad (a_p\ A^{-3/4}) \\ 0 & \text{even-odd / odd-even} \\ -a_p\ A^{-1/2} & \text{odd-odd} \quad (a_p\ A^{-3/4}) \end{cases}$$

we will try to understand the value (and A-dependence) of this constant …

Experimental evidence:

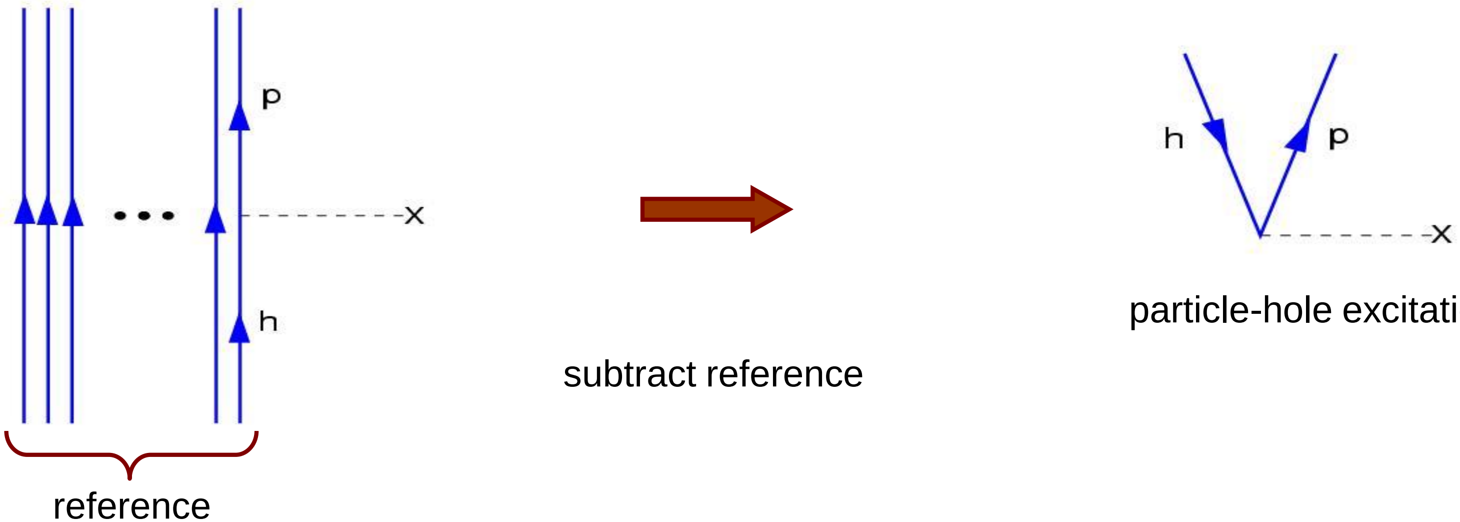


<u>Collective</u> excitation of the nuclear density surface

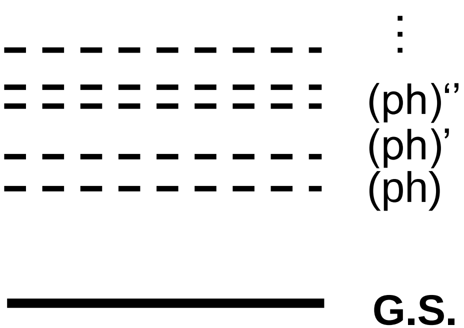


$$R = R_{\circ}\left[1 + \sum_{\mu} \alpha_{\lambda\mu} Y^{*}_{\lambda\mu}\right] \rightarrow R_{\circ}\left[1 + \alpha^{v}\right]$$

Basic inelastic excitation by a one-body field



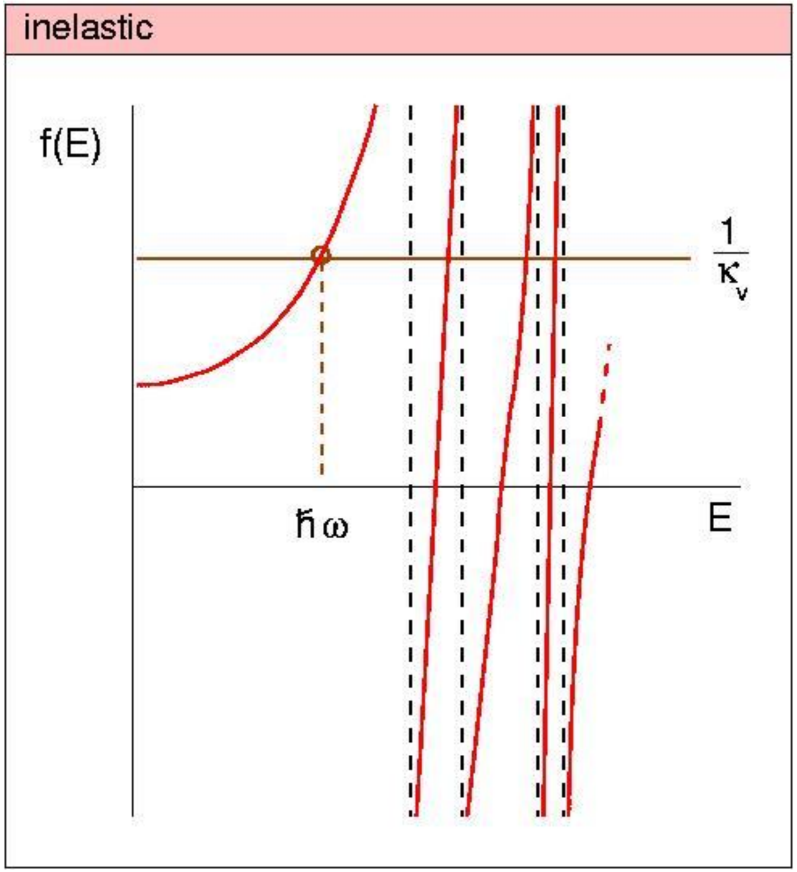
but then, the excitation spectrum would be



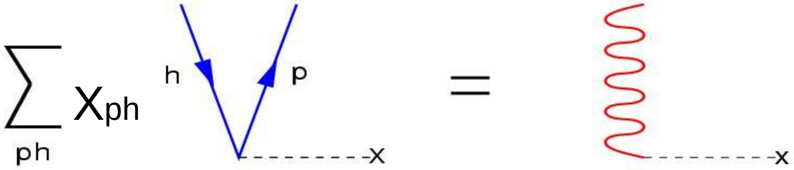
Add residual interactions: $H = H_o + \kappa^v Q^+ Q$, where $Q = \sum_{\alpha\beta} \langle\alpha|f|\beta\rangle a_\beta^+ a_\alpha$

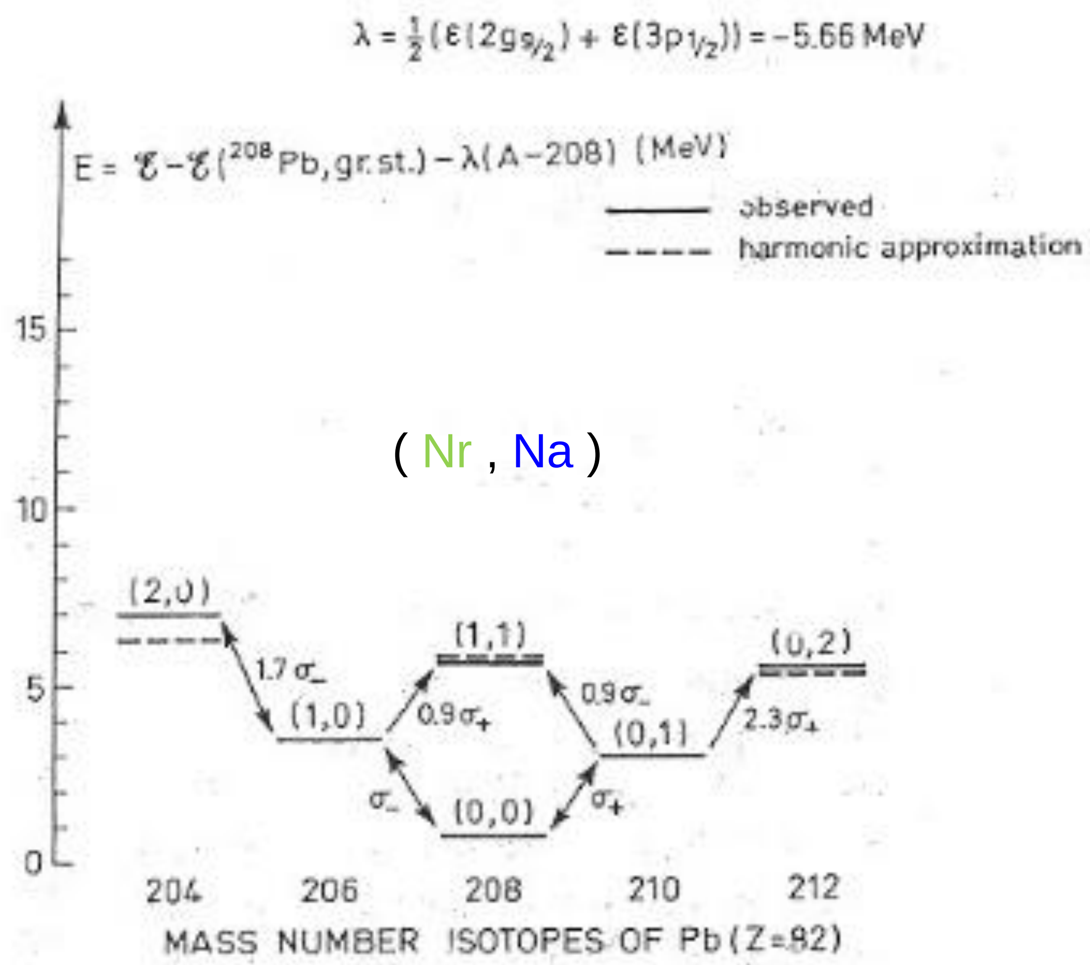
SURFACE VIBRATIONS — Microscopic Interpretation (II)

Dispersion relation: $f(E) = \frac{1}{\kappa_v}$



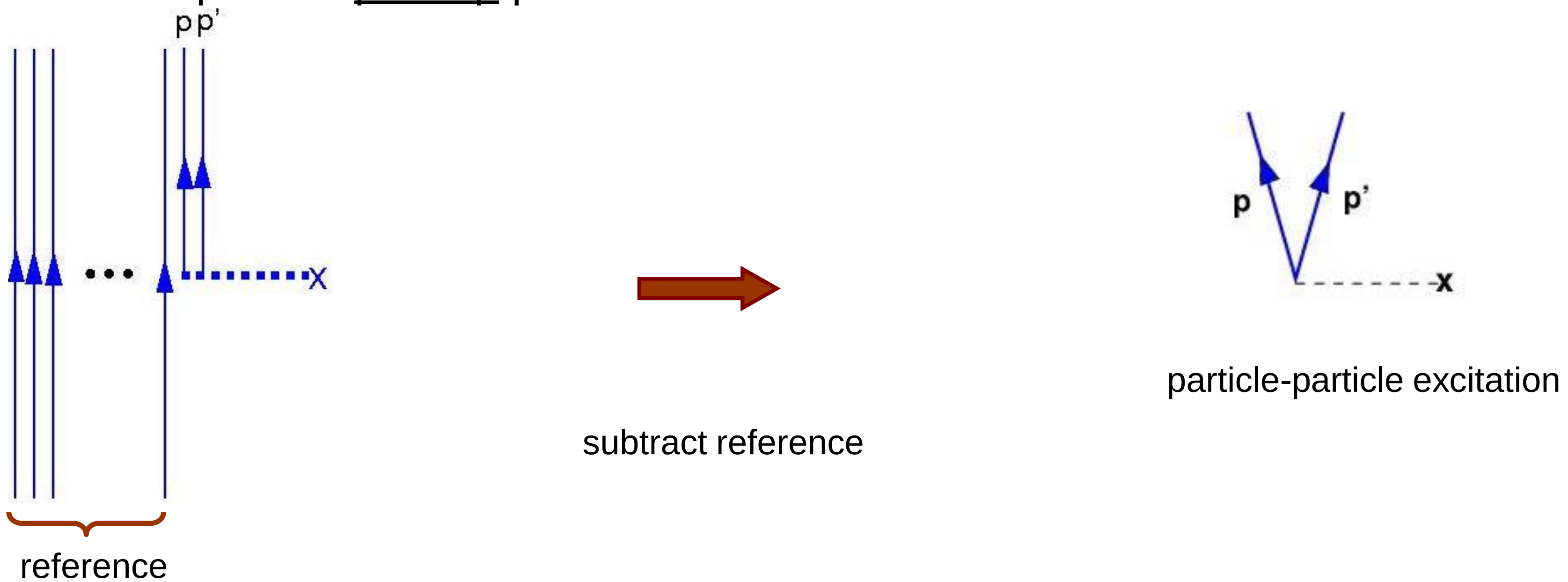
schematically:



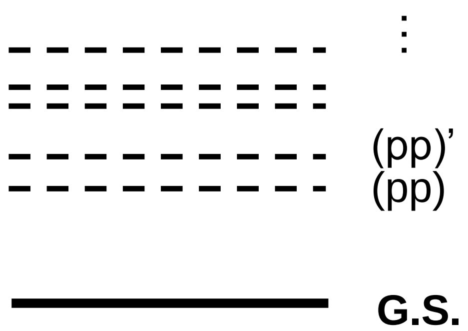


Traditional interpretation: <u>two modes</u> (one <u>addition</u> and one <u>removal</u>)

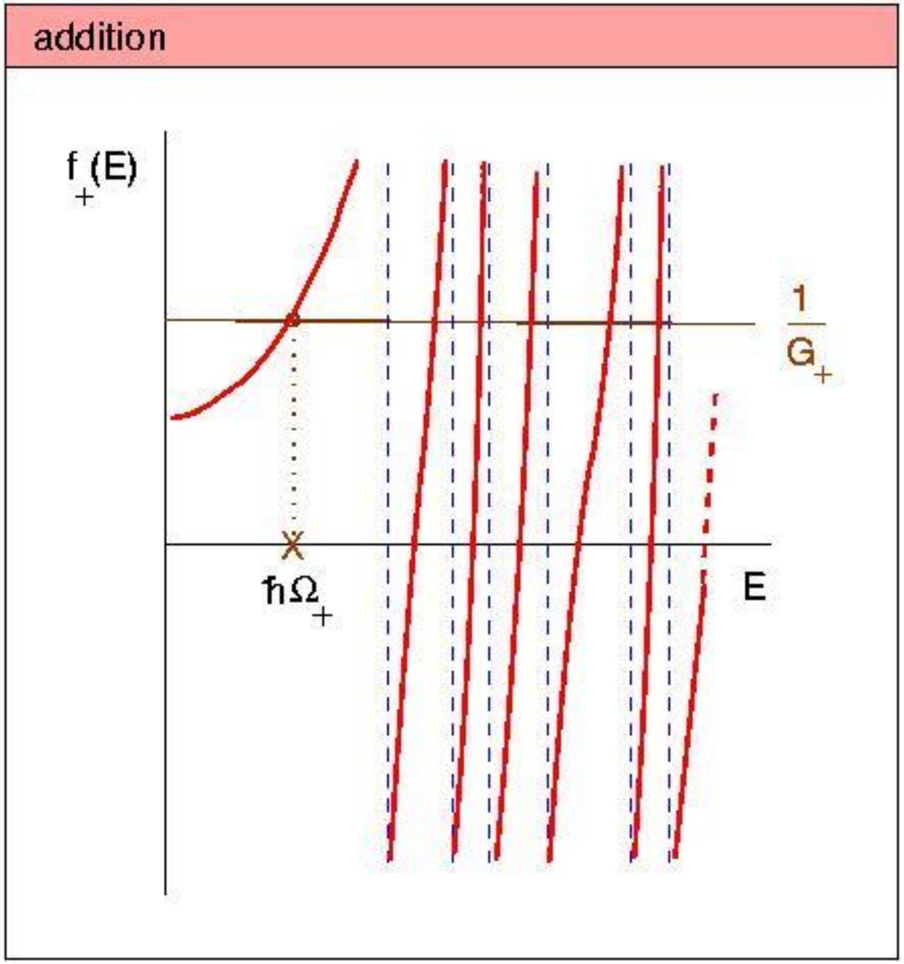
Basic two-particle pick-up process:



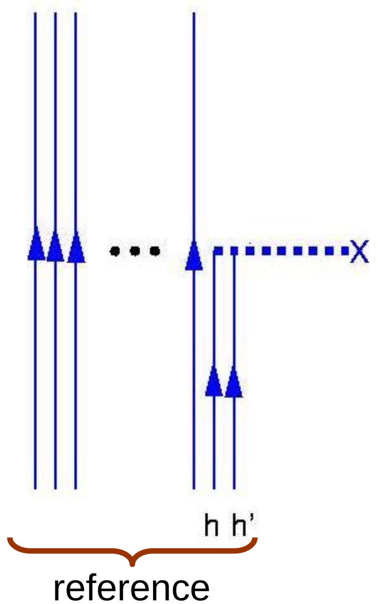
but then, the excitation spectrum would be



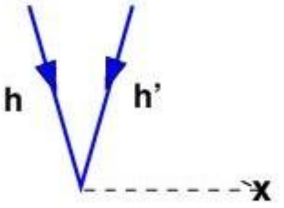
Add residual interaction $H_p = G\,P^\dagger P$ where $P^\dagger = \sum_{k>0} a_k^\dagger a_{\bar{k}}^\dagger$


addition
f+(E)
1/G+
ħΩ+
E

Basic two-particle <u>stripping</u> process:

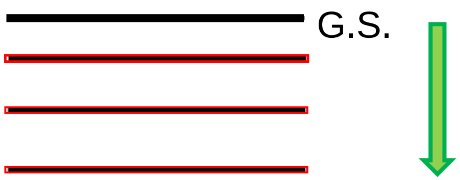


subtract reference

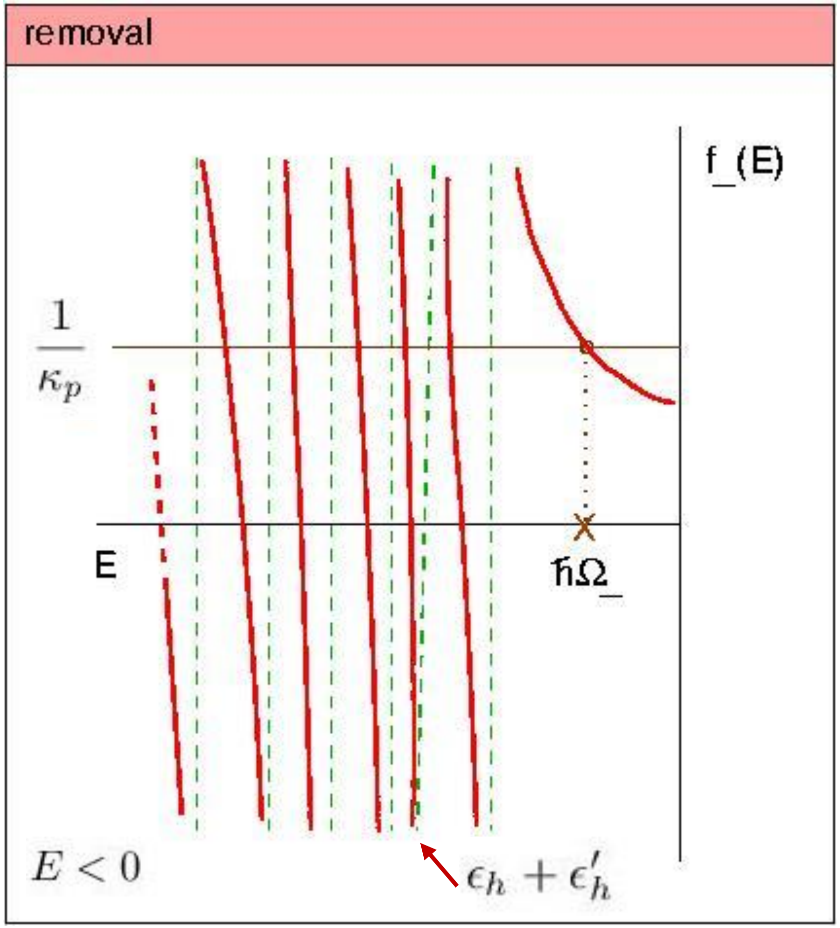


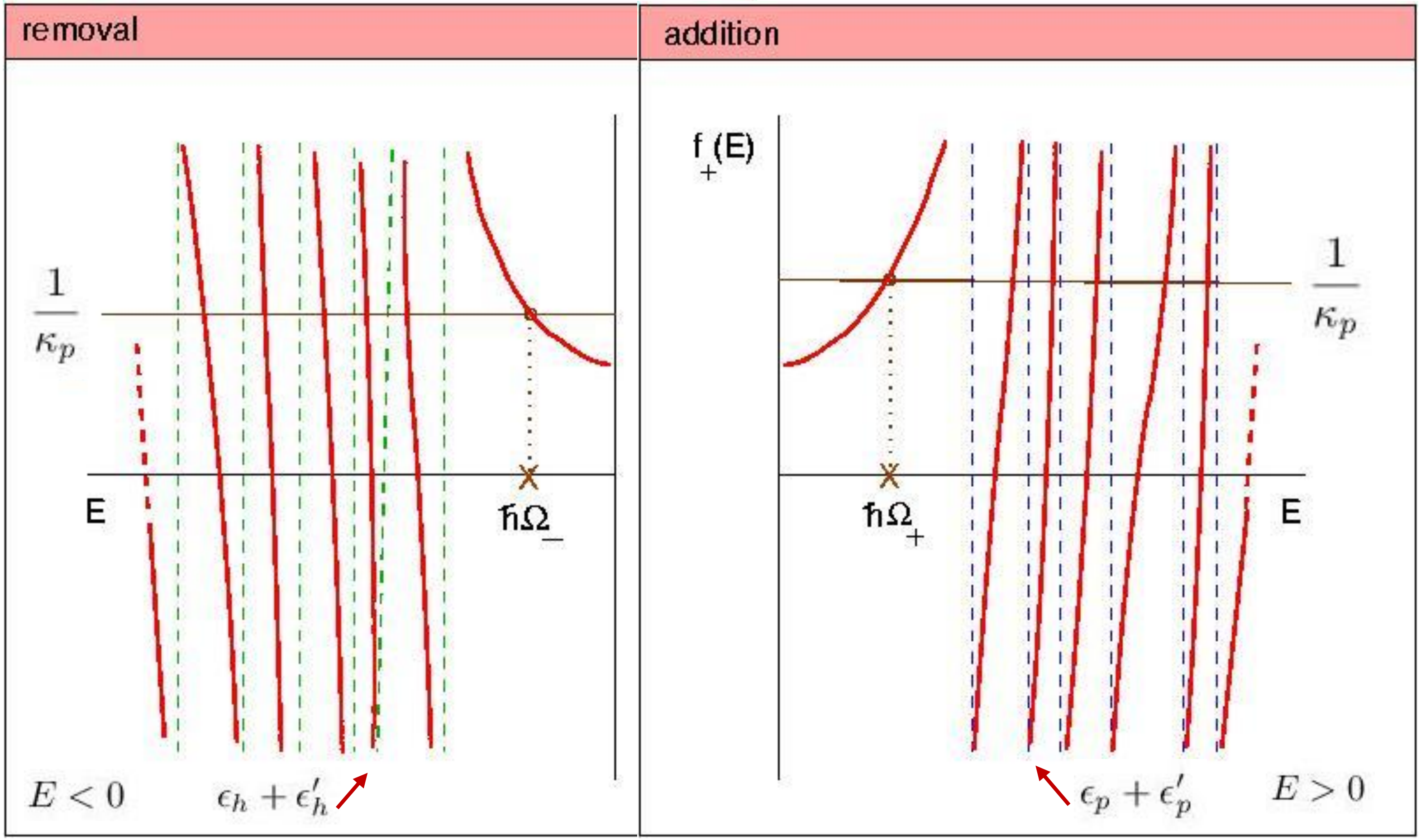
hole-hole excitation

but then, the excitation spectrum would be



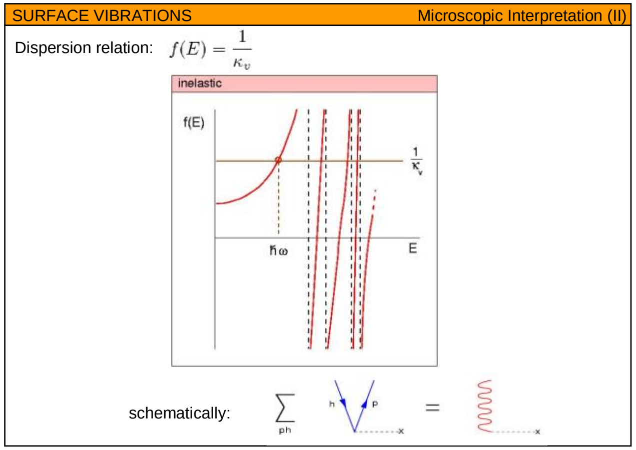
(in disagreement with the experimental evidence …)

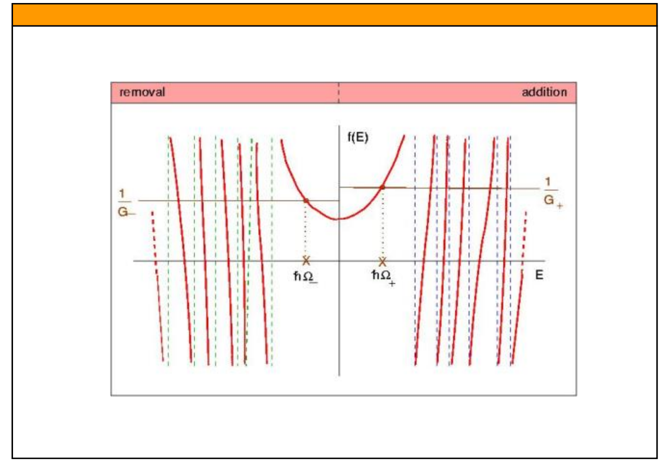

removal
f_(E)
$\frac{1}{\kappa_p}$
E
ħΩ_
$E < 0$
$\epsilon_h + \epsilon'_h$

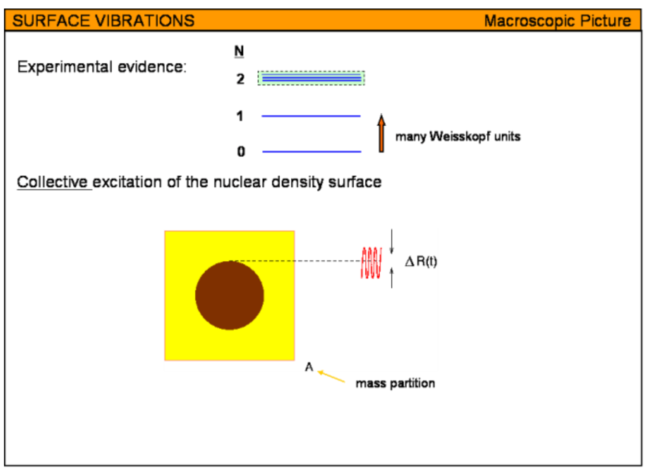


removal
addition
$\frac{1}{\kappa_p}$
E
$\hbar\Omega_-$
$E < 0$
$\epsilon_h + \epsilon_h'$
$f_+(E)$
$\frac{1}{\kappa_p}$
$\hbar\Omega_+$
E
$\epsilon_p + \epsilon_p'$
$E > 0$

So far …

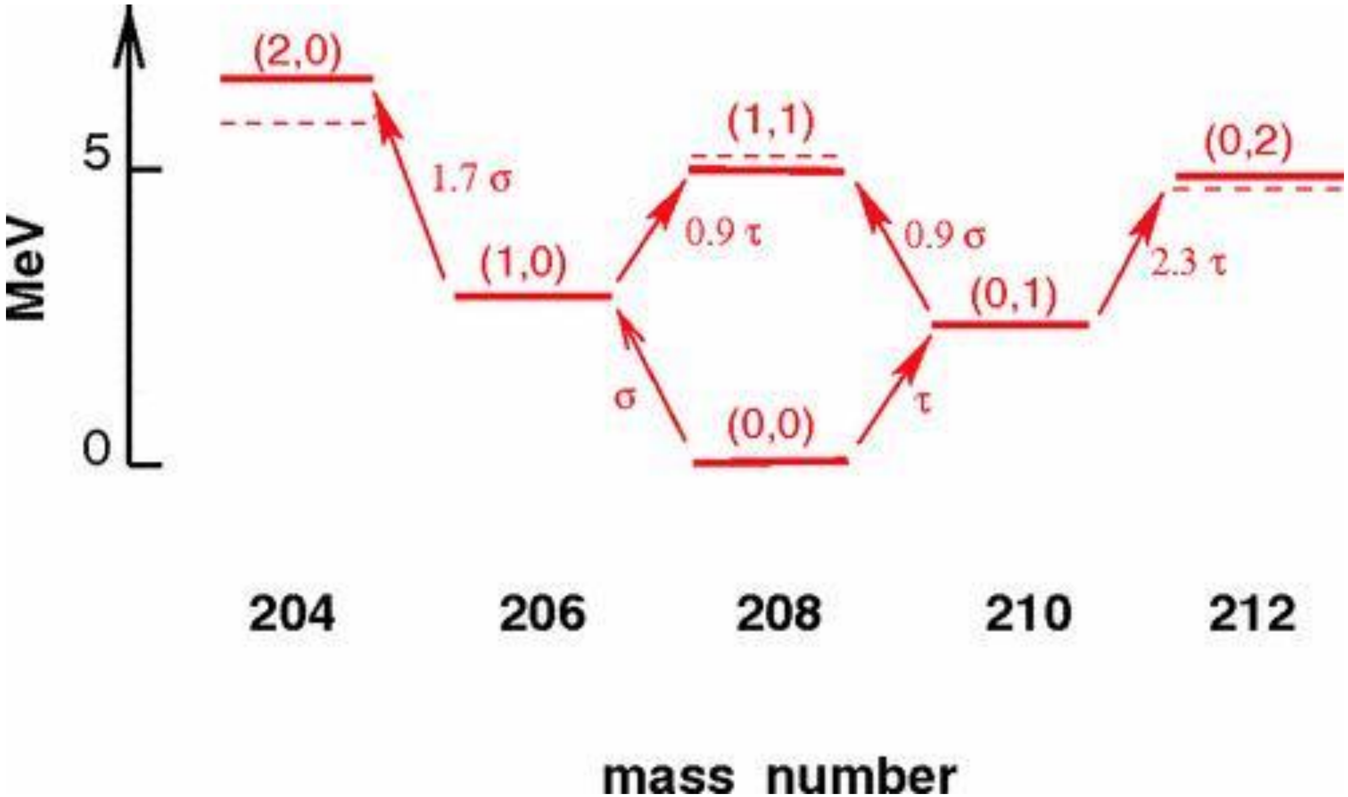

SURFACE VIBRATIONS
Microscopic Interpretation (II)
Dispersion relation: $f(E) = \frac{1}{\kappa_v}$
inelastic
f(E)
$\frac{1}{\kappa_v}$
$\hbar\omega$
E
schematically:
ph
h
p
x
x


removal
addition
f(E)
$\frac{1}{G_-}$
$\frac{1}{G_+}$
$\hbar\Omega_-$
$\hbar\Omega_+$
E

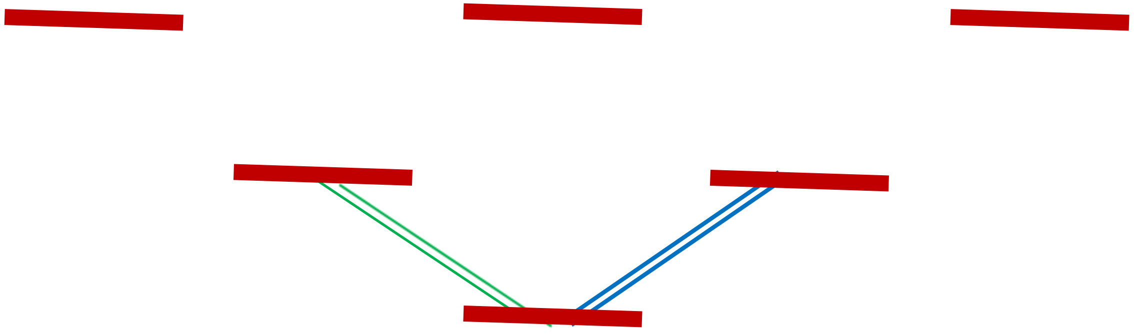

SURFACE VIBRATIONS
Macroscopic Picture
Experimental evidence:
N
2
1
0
many Weisskopf units
Collective excitation of the nuclear density surface
ΔR(t)
A
mass partition

?

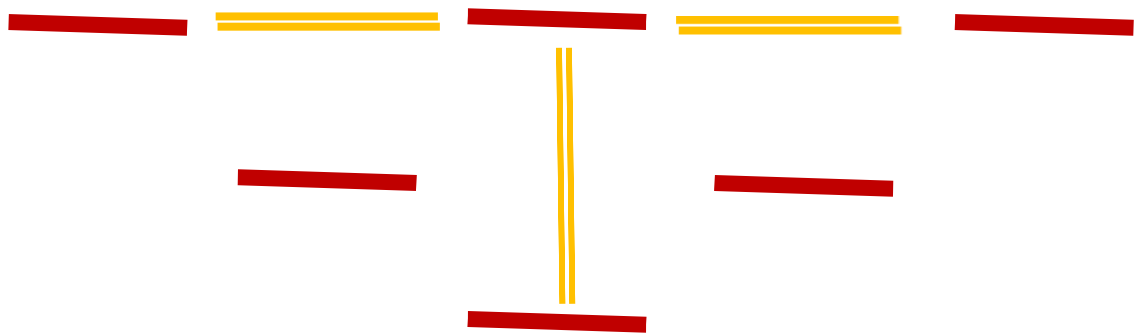
SEARCHING FOR AN ALTERNATIVE



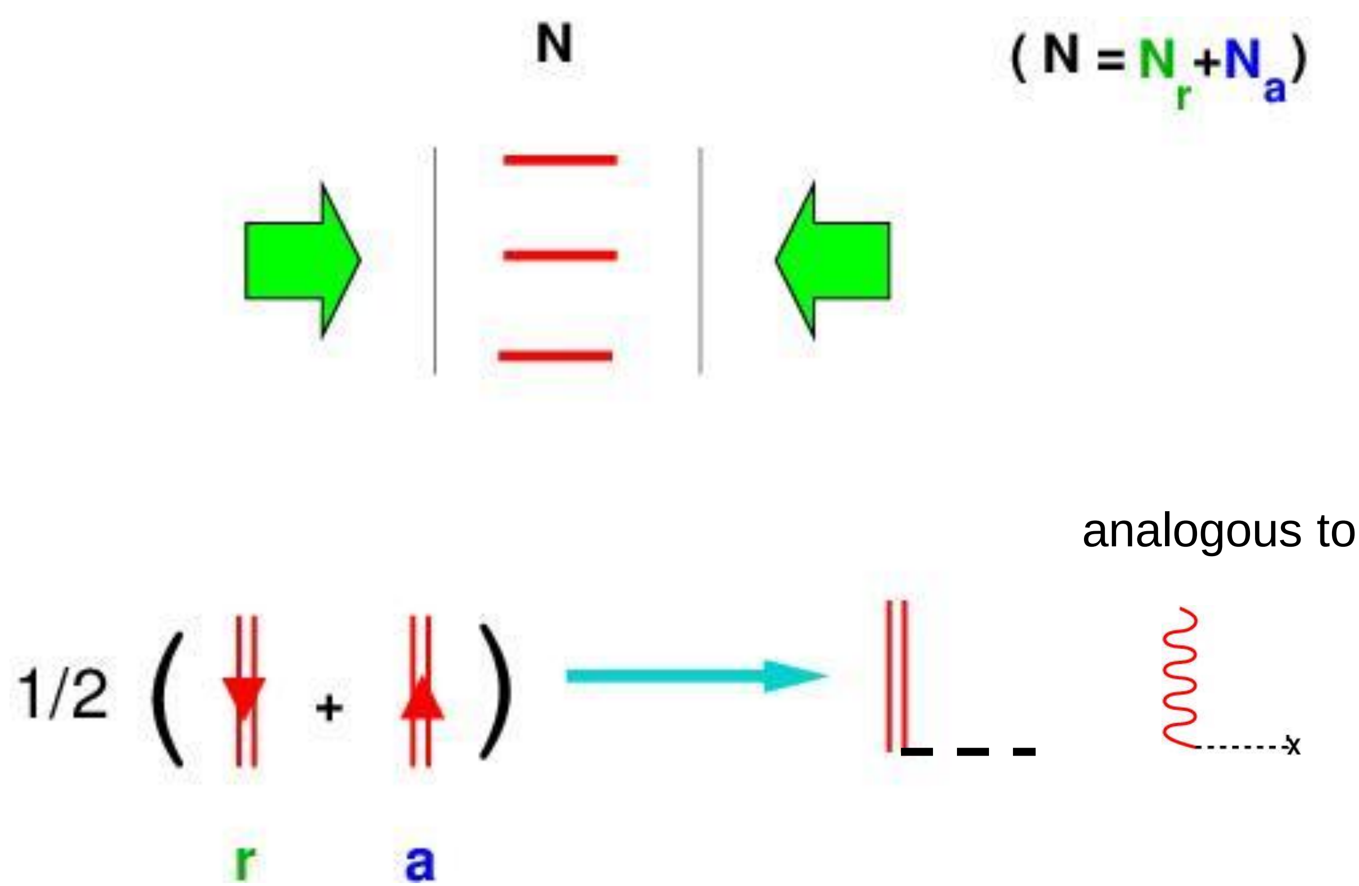
SEARCHING FOR AN ALTERNATIVE

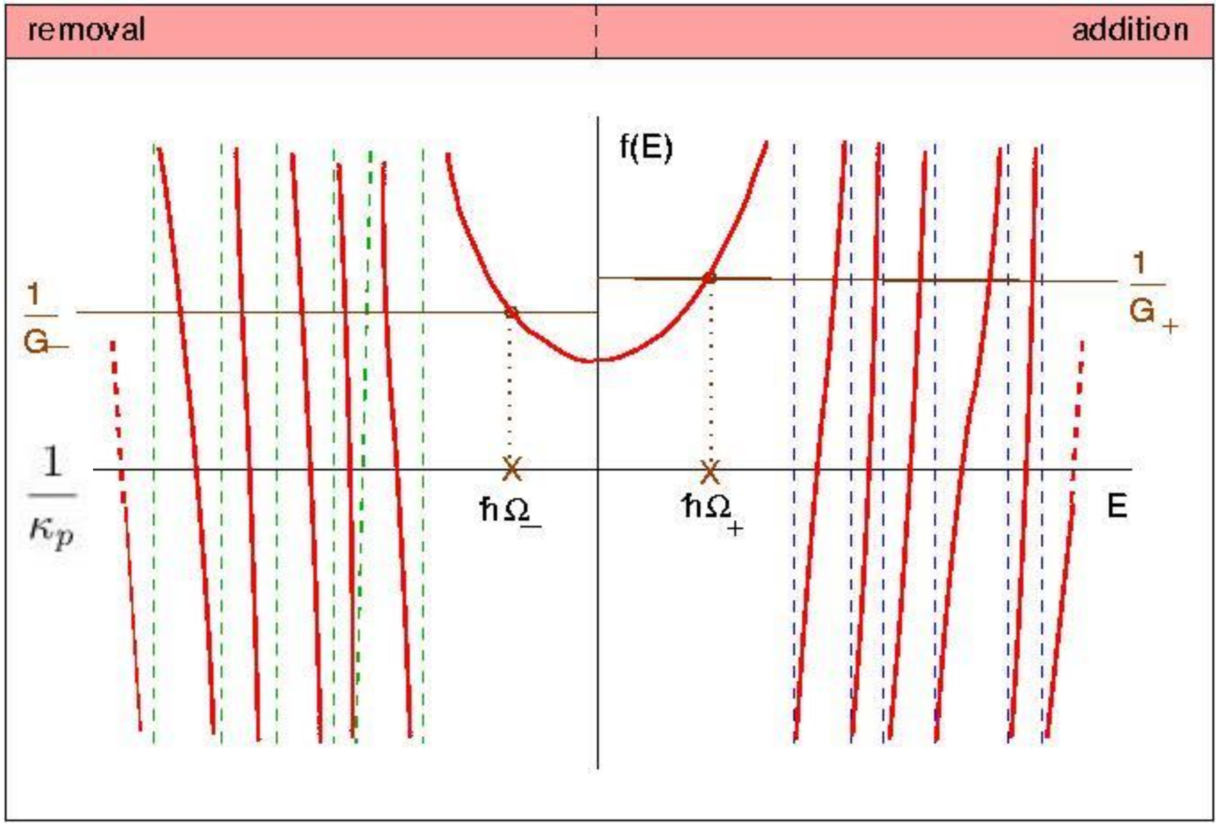


SEARCHING FOR AN ALTERNATIVE



SEARCHING FOR AN ALTERNATIVE





modify

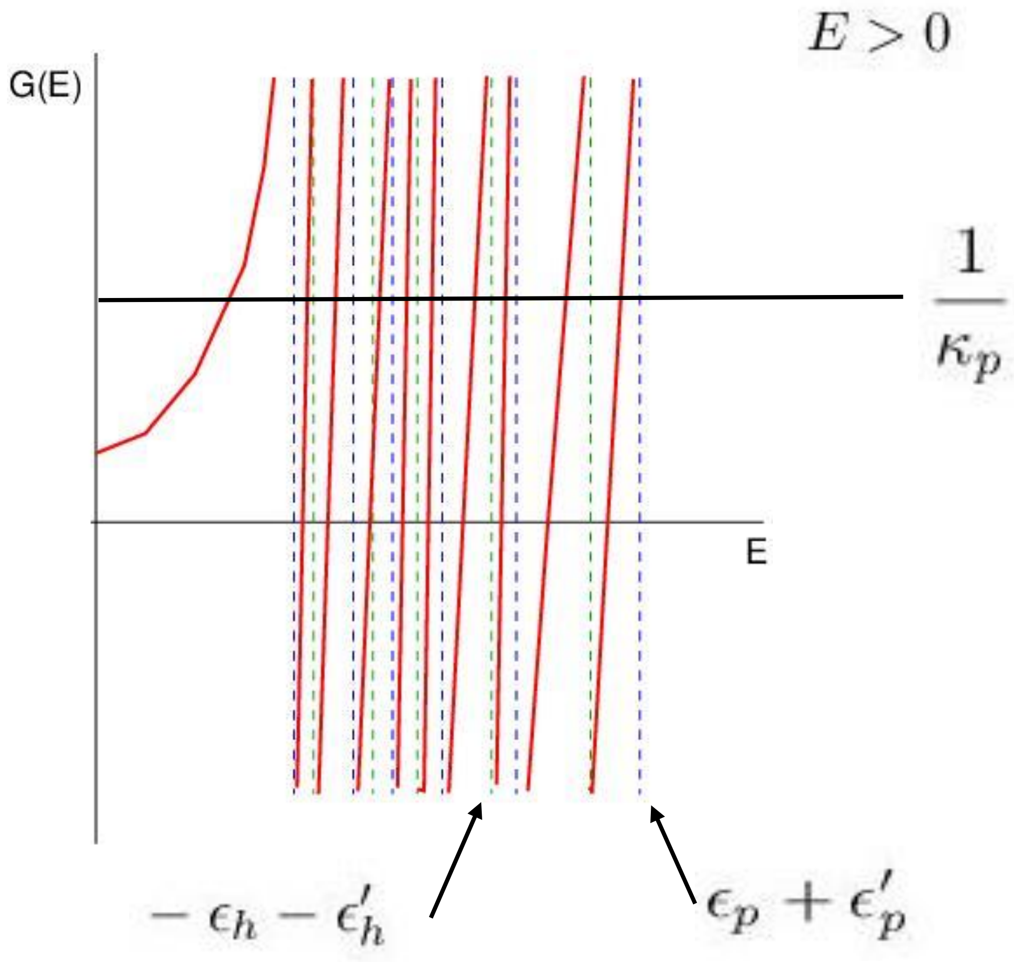
$$H_p \rightarrow \frac{1}{2}\, g\, Q^\dagger\, Q \qquad\qquad P^\dagger \quad \rightarrow \quad Q^\dagger = \frac{1}{\sqrt{2}} \sum_{kk'>0} \left[q_{kk'}\; a^\dagger_{kk'}\; a^\dagger_{kk'} + q^*_{k'k}\; a_{k'k}\; a_{k'k} \right]$$

where

$$q_{kk'} \;=\; \int \Psi^*_k(\vec{r})\; q(r)\; \Psi_{k'}(\vec{r})\; d\vec{r}$$

SEARCHING FOR AN ALTERNATIVE

pairing



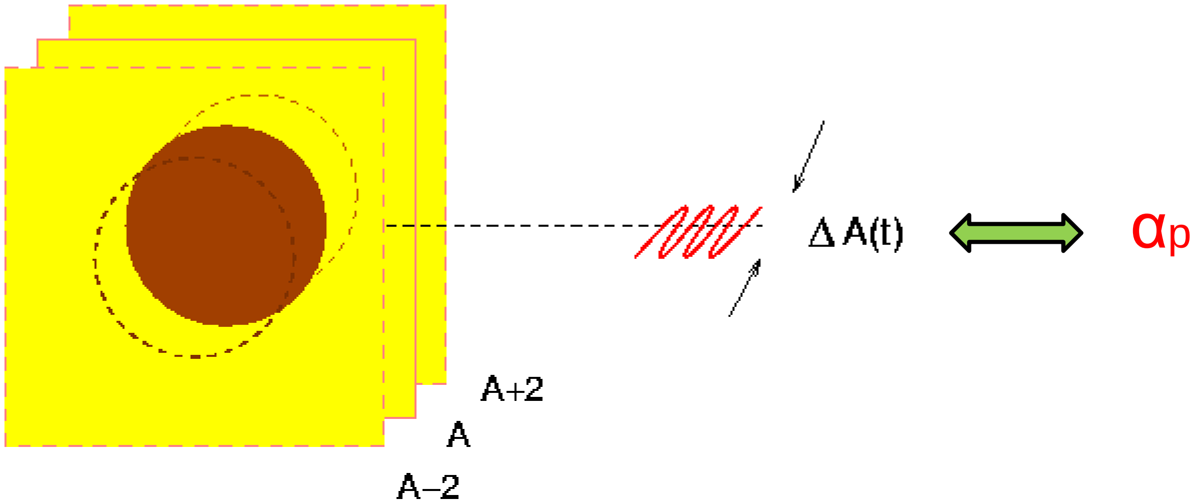
NORDITA LECTURE NOTES, 1979

"As we have seen, the combination $a^\dagger a$ is associated with the one-particle operators such as the density $\rho(\vec{r})$ or the potential V. When viewed in this way there seems to be a natural place for densities and potentials associated with the combinations $a^\dagger a^\dagger$ and $a\,a$. However, you may then argue that since these densities do not conserve the particle number (N) they have no natural role in the description of a system with a fixed number of particles, like a nucleus (e.g. the expectation value for any state or ensemble with fixed number of particles is zero, $\langle N|a^\dagger\ a^\dagger|N\rangle = 0$

In retrospective one might be tempted to say that this attitude held up the development of the present subject for some thirty years."

A. Bohr

We visualize it as follows,



that is, an oscillation across the mass partitions.

Additional ansatz:

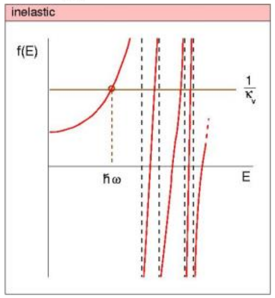
$$q(r) \propto \frac{\partial \rho}{\partial r}$$

i.e. pairing as a surface effect …

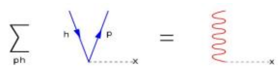
Picture is Complete!!! …

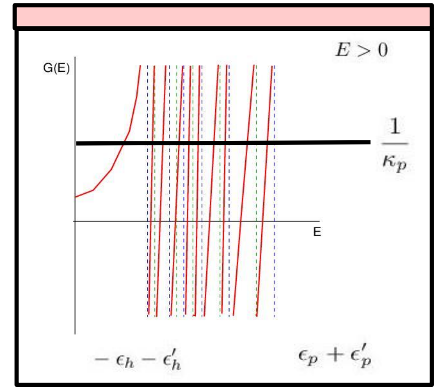
SURFACE VIBRATIONS — Microscopic Interpretation (II)

Dispersion relation: $f(E) = \frac{1}{\kappa_v}$



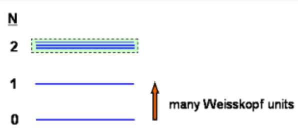
schematically:



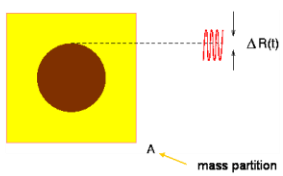


SURFACE VIBRATIONS — Macroscopic Picture

Experimental evidence:



Collective excitation of the nuclear density surface



PAIRING VIBRATIONS — Macroscopic picture

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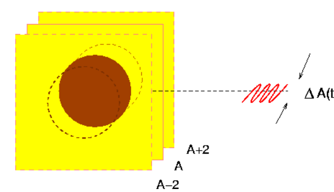


that is, an oscillation across the mass partitions.

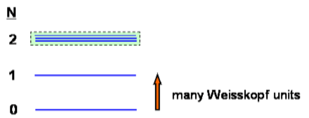
Additional ansatz:

i.e. pairing as a surface effect …

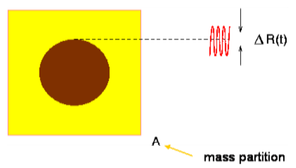
So far …

SURFACE VIBRATIONS — Macroscopic Picture

Experimental evidence:

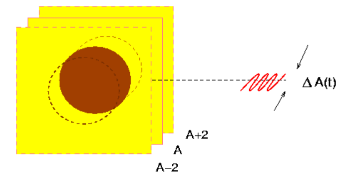


Collective excitation of the nuclear density surface



PAIRING VIBRATIONS — Macroscopic picture

We visualize it as follows,



that is, an oscillation across the mass partitions.

Additional ansatz:

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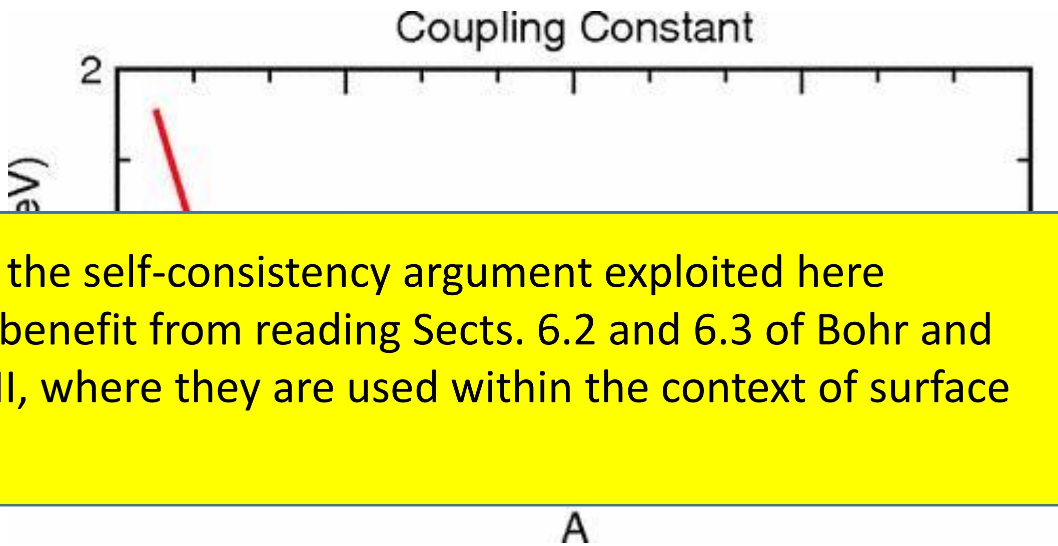
Self-consistent coupling!

X

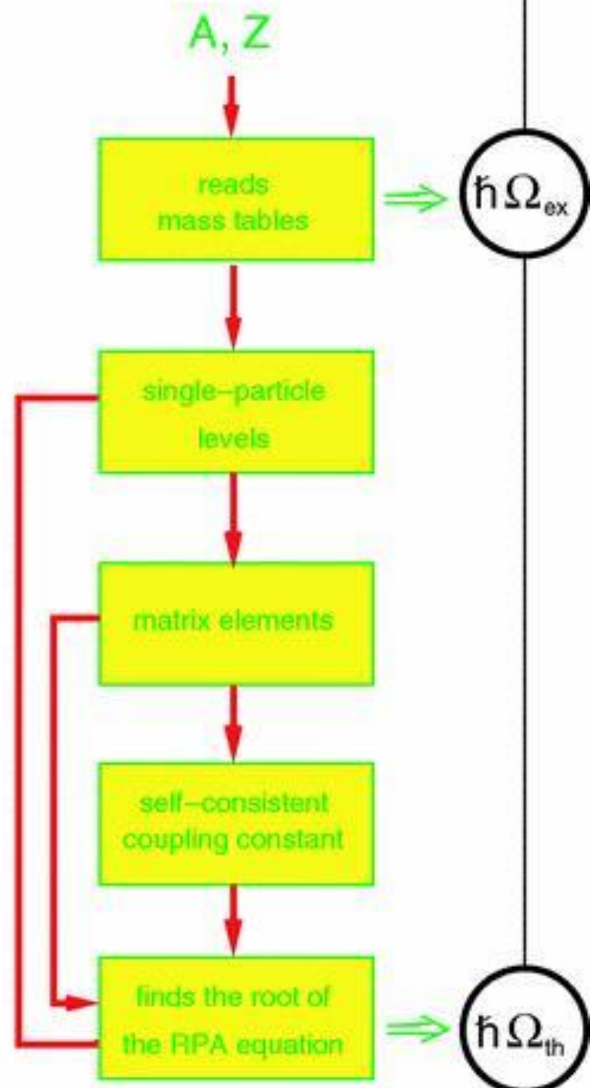
SELF-CONSISTENT COUPLING

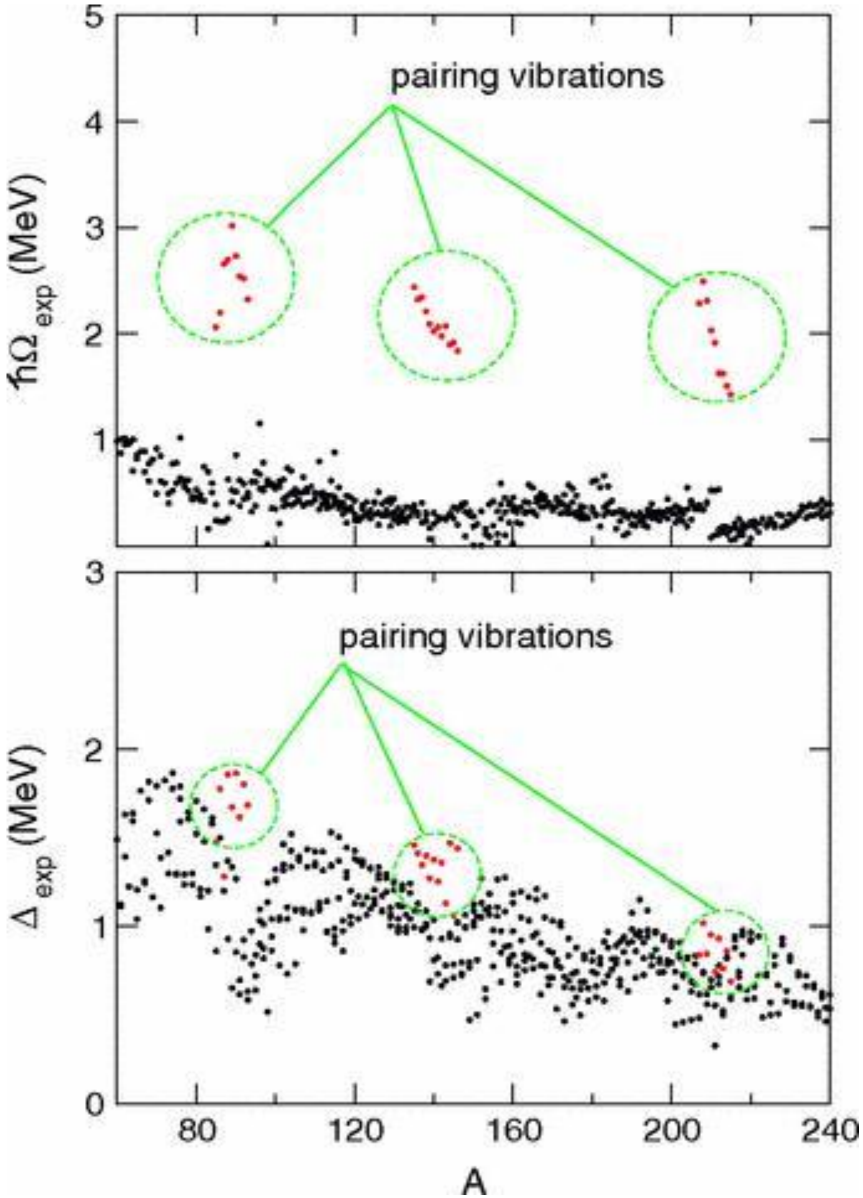


Those who find the self-consistency argument exploited here unfamiliar may benefit from reading Sects. 6.2 and 6.3 of Bohr and Mottelson Vol. II, where they are used within the context of surface vibrations.

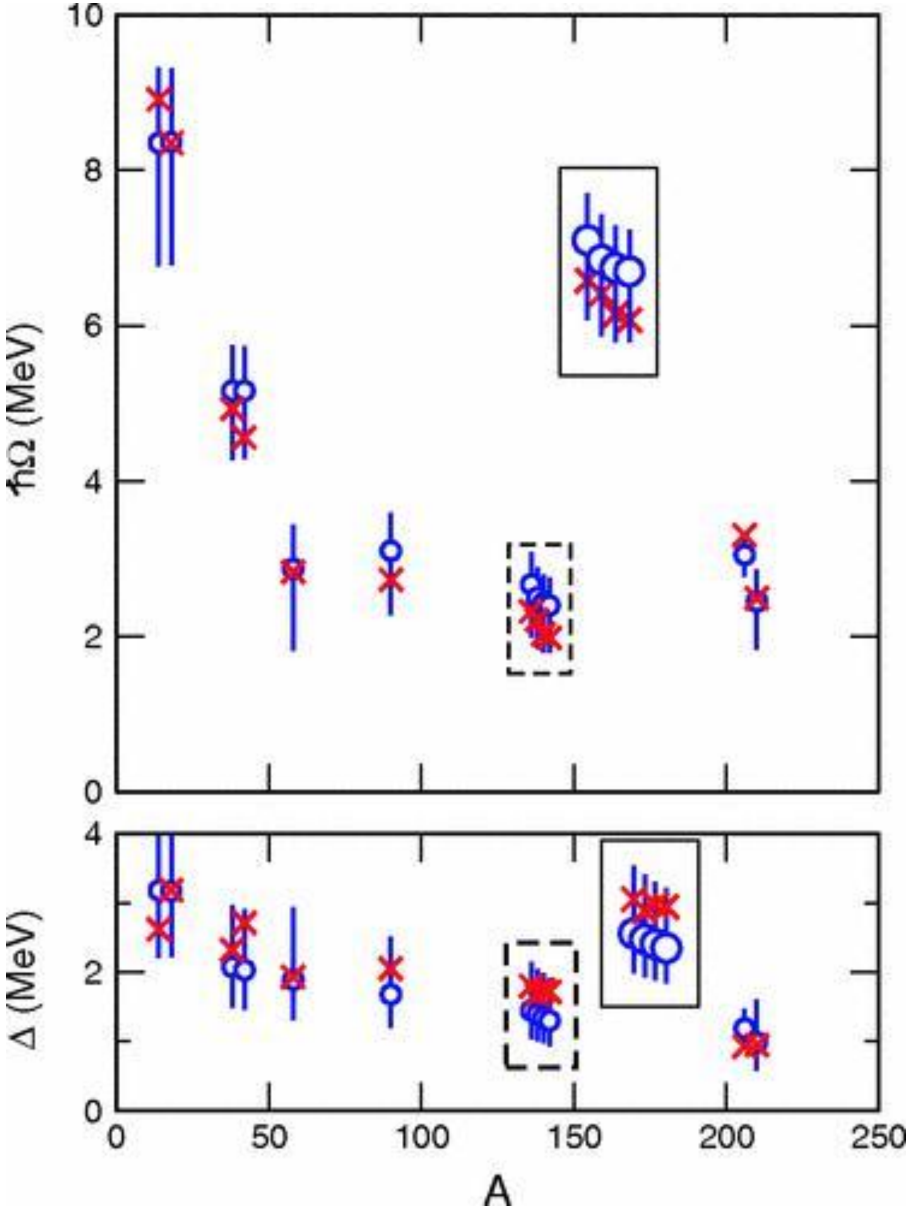
$$\kappa = 4\pi \left(\frac{R}{3A}\right) \int_0^\infty \left(\frac{\partial \rho}{\partial r}\right) \left(\frac{\partial V}{\partial r}\right) r^2 dr$$

```
#
# Datos:
#
a=208
z=82
ich=0
#
# Program I
#
echo ' '
echo ' '
echo ' - Now qqpp is running...'
qqpp << fin1
$a  $z  $ich
fin1
#
# - Program II
#
echo ' '
echo ' '
echo ' - Now splevpp is running...'
splevpp << fin2
$a  $z  $ich
fin2
#
# - Program III
#
echo ' '
echo ' '
echo ' - Now homepp is running...'
homepp << fin3
$a
fin3
#
# - Program IV
#
echo ' '
echo ' '
echo ' - Now constpp is running...'
constpp << fin4
$a
fin4
#
# - Program V
#
echo ' '
echo ' '
echo ' - Now rpapp is running...'
rpapp << fin5
$a  $z  $ich
fin5
```

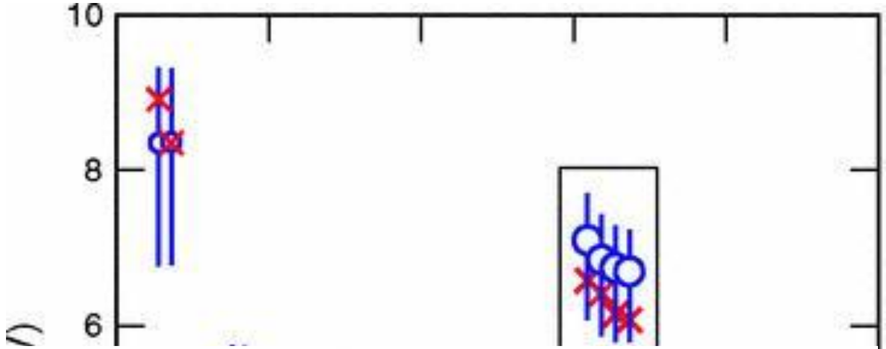




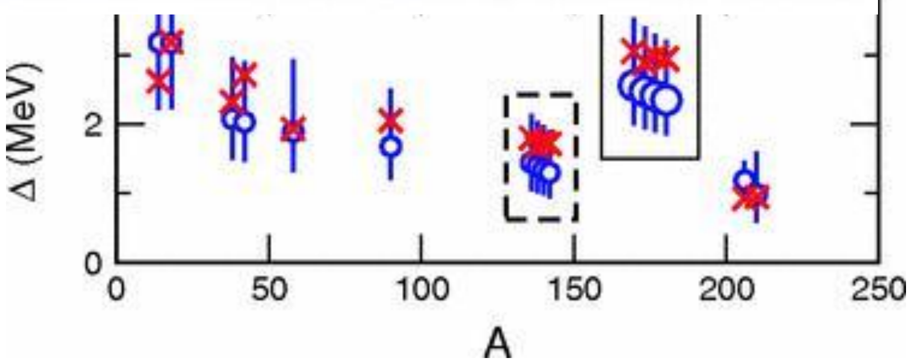
pairing vibrations
ħΩ_exp (MeV)
Δ_exp (MeV)
A
0
1
2
3
4
5
80
120
160
200
240



ħΩ (MeV)
Δ (MeV)
A
10
8
6
4
2
0
4
2
0
0
50
100
150
200
250



N_{max}	Wave function components	$\hbar\Omega_{th}$ (MeV)
7	62	2.9
8	83	2.7
9	109	2.5
10	150	2.3
11	198	2.0



First historical evidence for pairing interactions: Nuclear Masses

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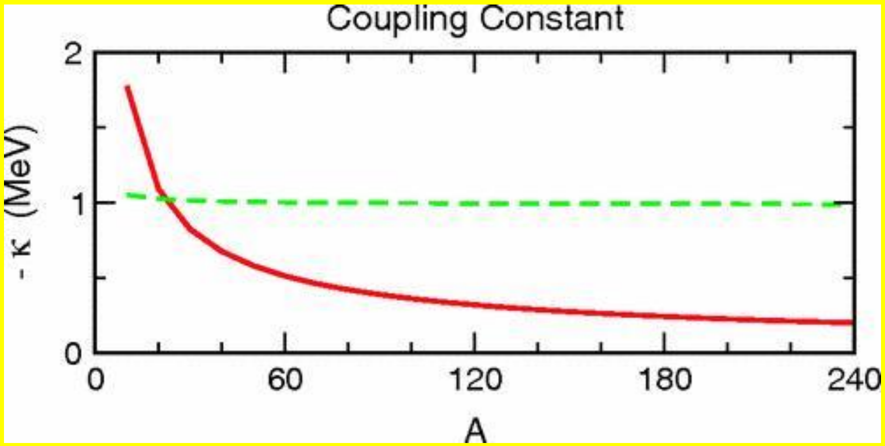
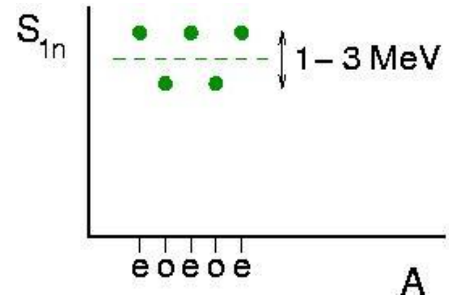


$$\kappa(A) \approx -\frac{7.8}{A^{2/3}}\ MeV$$



Se… (Von Weiszacker, 1935)

$$B(A,Z) = a_v\ A - a_s\ A^{2/3} - a_c\ \frac{Z^2}{A^{1/3}} - a_a\ \frac{(A-2Z)^2}{A} + \delta_p(A)$$

where,

$$\delta_p(A) = \begin{cases} +a_p\ A^{-1/2} & \text{even-even} \quad (a_p\ A^{-3/4}) \\ 0 & \text{even-odd / odd-even} \\ -a_p\ A^{-1/2} & \text{odd-odd} \quad (a_p\ A^{-3/4}) \end{cases}$$

we will try to understand the value (and A-dependence) of this constant …

FINAL…

First historical evidence for pairing interactions: Nuclear Masses

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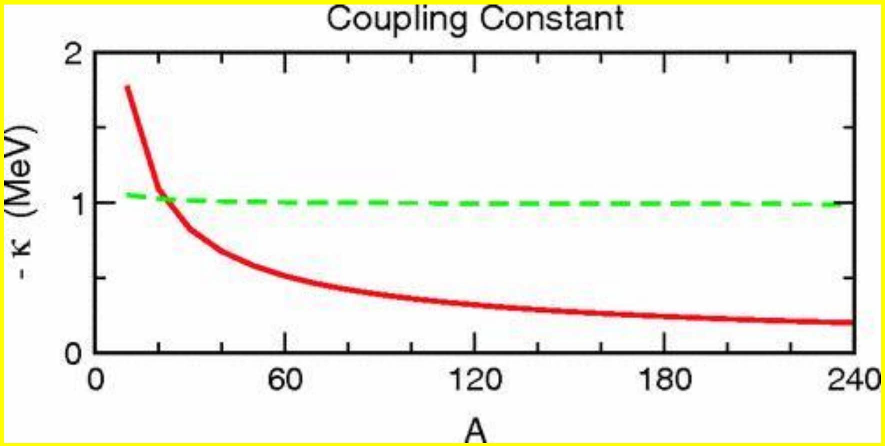

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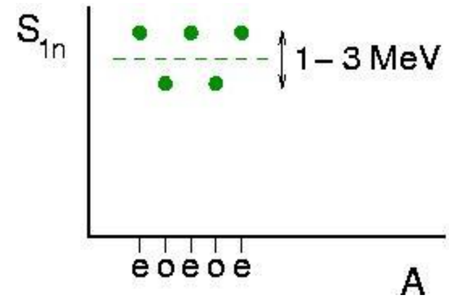

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Notice 1/2 < 2/3 < 3/4 !!

we do have an understanding of the value (and A-dependence) of this constant …