Emergent BCS regime of 2D fermionic Hubbard model: ground-state phase diagram

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Advances in Diagrammatic Monte Carlo Methods for Quantum Field Theory Calculations in Nuclear-, Particle-, and Condensed Matter Physics

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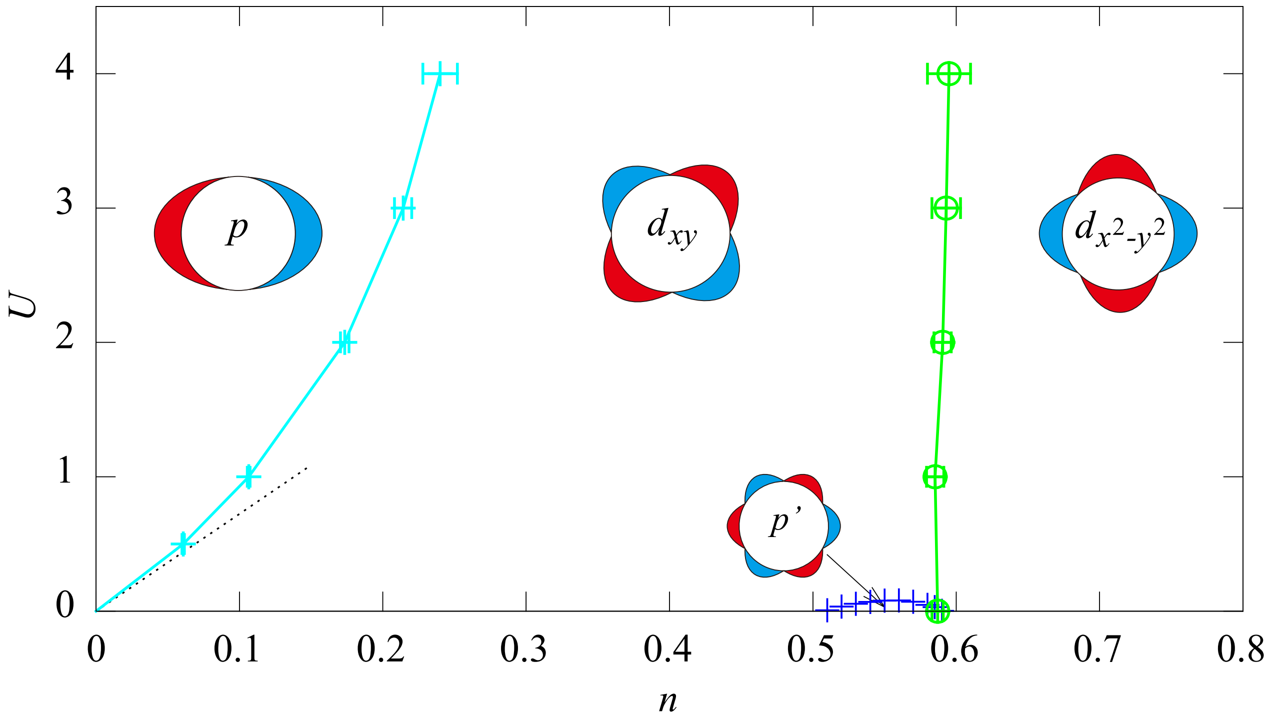
$$H = -t \sum_{\substack{<ij> \\ \sigma=\uparrow,\downarrow}} a^{+}_{\sigma i} a_{\sigma j} + U \sum_{i} n_{\uparrow i} n_{\downarrow i} , \qquad n_{\sigma i} = a^{+}_{\sigma i} a_{\sigma i}$$



Youjin Deng, Evgeny Kozik, N. Prokof'ev and BS, EPL **110**, 57001 (2015)

Cooper instability via linear response

Modify the Hamiltonian: $H \rightarrow H + \left(\eta_{12}^{*} \psi_1 \psi_2 + \mathrm{H.c.} \right)$

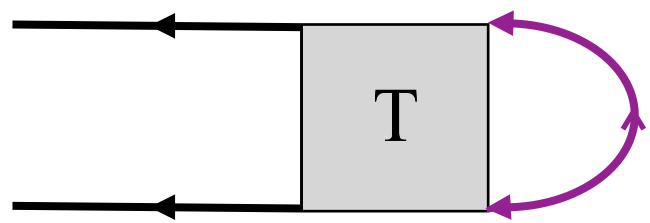
infinitesimally small

Study linear response $\langle \psi_1 \psi_2 \rangle$

Diagrammatically:

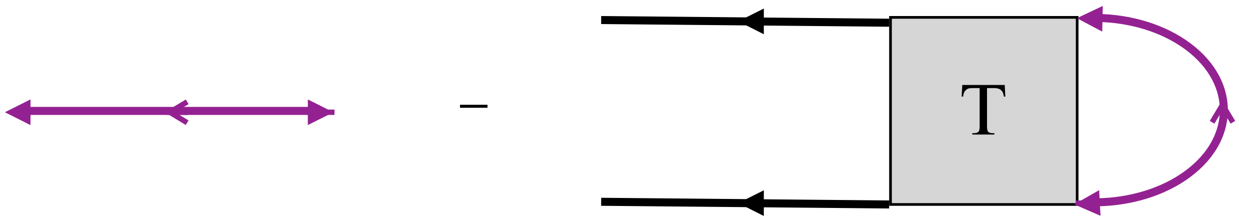
$\langle \psi_1 \psi_2 \rangle$

$= \quad G_{13} G_{24} \eta_{34} \quad +$



irreducible
(in the Cooper channel)
four-pole vertex

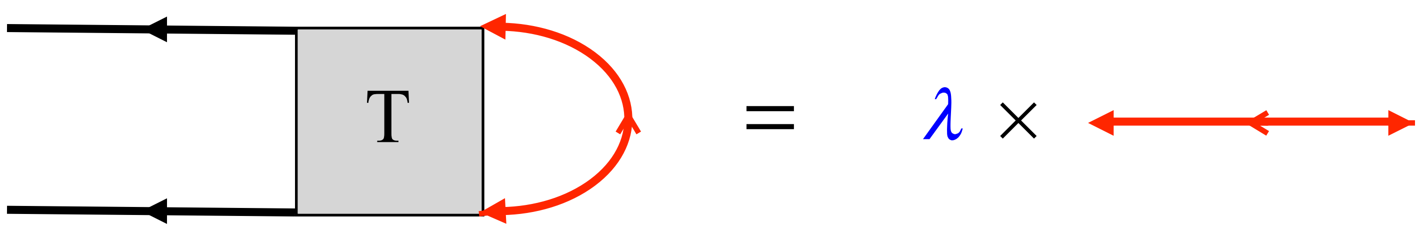
Singular response: eigenvector-eigenvalue problem



$$= G_{13}G_{24}\eta_{34}$$

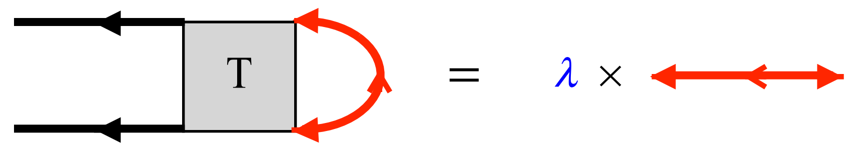
just some
right-hand side…

Response diverges when the following eigenvalue becomes equal **1** .



Corresponding eigenvector defines the pairing channel.

(Emergent) BCS regime



In this regime,

The four-pole vertex T is small and temperature independent.

Green's function has a Fermi-liquid form (close to the Fermi surface):

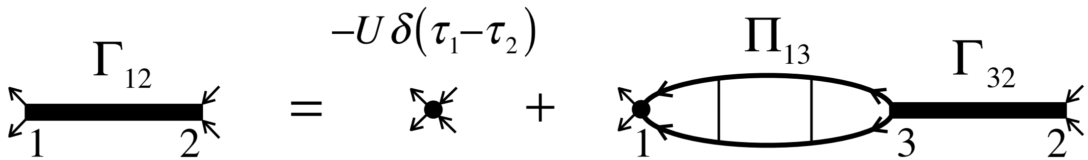
$$G(\mathbf{k},\xi) \approx \frac{z(\hat{k})}{i\xi - \mathbf{v}_F(\hat{k})\bullet[\mathbf{k}-\mathbf{k}_F(\hat{k})]}$$

Temperature dependence of the eigenvalue is due to the Green's function factor:

$$\lambda(T) = g\ln\left(\#\, E_F / T\right) \quad \Rightarrow \quad \boxed{T_c = \#\, E_F\, e^{-1/g}}$$

$$\boxed{g \ll 1}$$

Ladder summation trick

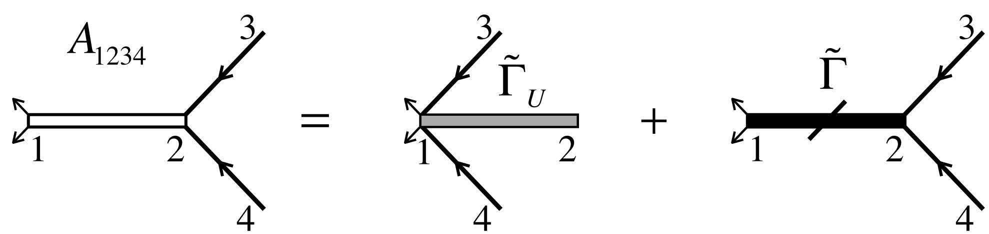


$$\Gamma(\tau,\mathbf{k}) = -U\delta(\tau) + \tilde{\Gamma}(\tau,\mathbf{k})$$

The two terms substantially compensate each other, but only in the integral sense.

Introduce new object: $\int_0^{\beta} \tilde{\Gamma}_U(\tau)\,d\tau = -U$

Now we can combine the two:

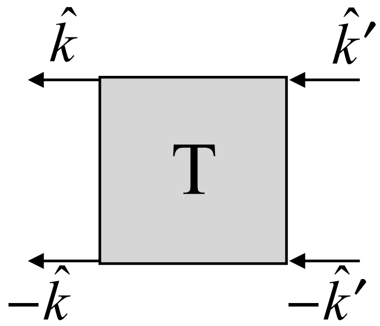


$$A_{1234} = \tilde{\Gamma}_U(\tau_1-\tau_2)G_{\uparrow}(\tau_1-\tau_3)G_{\downarrow}(\tau_1-\tau_4) + \tilde{\Gamma}(\tau_1-\tau_2)G_{\uparrow}(\tau_2-\tau_3)G_{\downarrow}(\tau_2-\tau_4)$$

The eigenvector-eigenvalue problem in 2D

Momenta live on the Fermi surface.

Frequencies approach zero.



$$\hat{k},-\hat{k} \rightarrow \theta$$

$$\hat{k}',-\hat{k}' \rightarrow \theta'$$

Fermi surface is parameterized in terms of polar angle θ .

$$\int_0^{2\pi} \tilde{\mathrm{T}}_{\theta,\theta'}\,\phi_{\theta'}\,\frac{d\theta'}{2\pi} = g\phi_\theta$$

$$\tilde{\mathrm{T}}_{\theta,\theta'} = Q_\theta^{\frac{1}{2}}\mathrm{T}_{\theta,\theta'}Q_{\theta'}^{\frac{1}{2}}$$

$$Q_\theta = \frac{k_F(\theta)\,z^2(\theta)}{2\pi\,\hat{\theta}\cdot\mathbf{v}_F(\theta)}$$

D_{4h} nomenclature for the square lattice

$$f_s(\theta) = \sum_{m=0}^{\infty} A_m \cos(4m\theta)$$

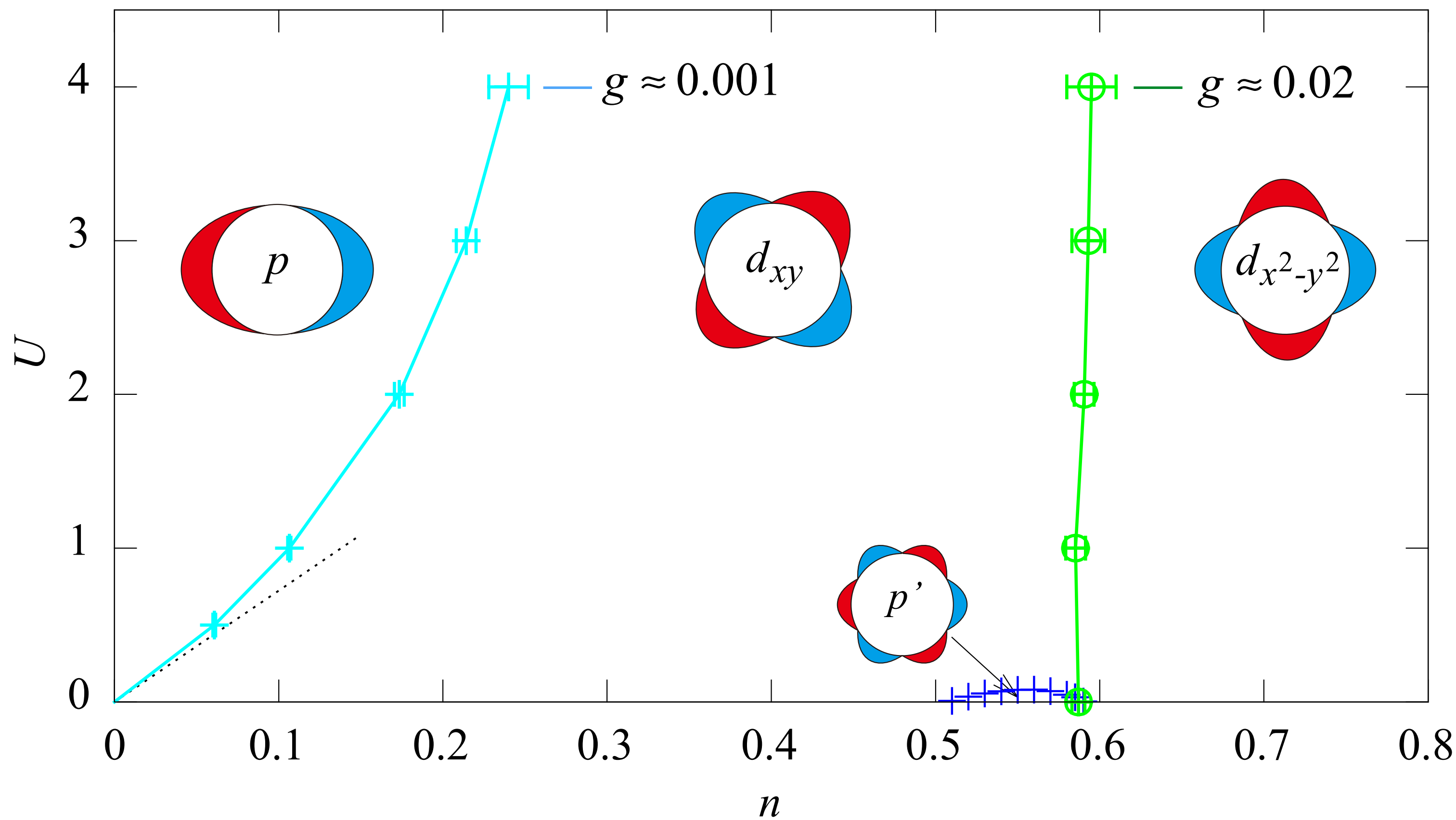
$$f_g(\theta) = \sum_{m=1}^{\infty} B_m \sin(4m\theta)$$

$$f_{\left\{ \begin{array}{c} p_y \\ p_x \end{array} \right\}}(\theta) = \sum_{m=0}^{\infty} C_m \left\{ \begin{array}{c} \cos[(2m+1)\theta] \\ \sin[(2m+1)\theta] \end{array} \right\}$$

$$f_{\left\{ \begin{array}{c} d_{x^2-y^2} \\ d_{xy} \end{array} \right\}}(\theta) = \sum_{m=0}^{\infty} \left\{ \begin{array}{c} D_m \cos[(4m+2)\theta] \\ E_m \sin[(4m+2)\theta] \end{array} \right\}$$

Weakness of BCS coupling

$$T_c = \# E_F \, e^{-1/g}$$



Crisis of bold diagrammatics!

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Nonexistence of the Luttinger-Ward Functional and Misleading Convergence of Skeleton Diagrammatic Series for Hubbard-Like Models

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Shifted-action expansion: controlled dressed diagrammatic schemes

R. Rossi, F. Werner, N. Prokof'ev, and BS, 2015

Partial dressing

original action

$$S[\psi] = \langle \psi \,|\, G_0^{-1} \,|\, \psi \rangle + S_{\text{int}}[\psi]$$

auxiliary action

$$S_\xi^{(N)}[\psi] = \langle \psi \,|\, \tilde{G}_N^{-1} + \xi \Lambda_1 + \ldots + \xi^N \Lambda_N \,|\, \psi \rangle + \xi S_{\text{int}}[\psi]$$

expansion parameter

$$\tilde{G}_N^{-1} + \Lambda_1 + \Lambda_2 + \ldots + \Lambda_N = G_0^{-1} \quad \Rightarrow \quad S_{\xi=1}^{(N)} = S$$

equivalence to the original action

self-energies of corresponding orders, playing the role of counter-terms

Full dressing:
sufficient condition for converging to correct answer

(i) The sequence $\tilde{G}_N$ converges and is uniformly bounded.

(ii) The sequence

$$\xi\Lambda_1\left[\tilde{G}_N\right]+\xi^2\Lambda_2\left[\tilde{G}_N\right]+\ldots+\xi^N\Lambda_N\left[\tilde{G}_N\right]$$

converges and is uniformly bounded within a circle $|\xi|<\xi_0$, where $\xi_0>1$.