Effective Field Theories and Quarkonium Physics

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Heavy Quark Physics in Heavy-Ion Collisions: experiments, phenomenology and theory

ECT*, Villazzano, March 17^{th}

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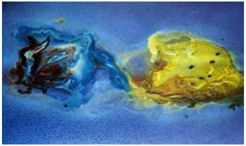
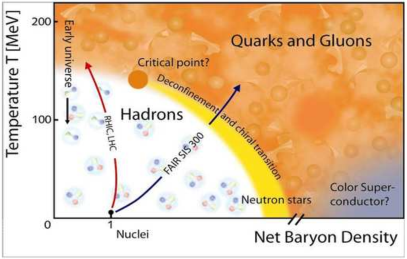


Outline

QCD phase diagram

- QCD phase diagram is explored at present day colliders and experiments
- Transition from a hadronic phase to a deconfined phase of quarks and gluons at $T_c = 154 \pm 9$ MeV [HotQCD Collaboration (2014)]

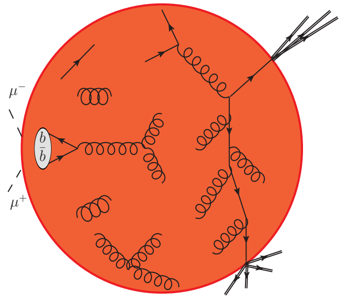


How can we study the properties of quark gluon plasma (QGP)?

- effective hydrodynamics description (bulk properties)
- hard probes, highly energetic particles not in equilibrium with QGP

Heavy Quarkonium and QGP

Heavy $Q\bar{Q}$ in medium



- Medium effects can dissociate the $Q\bar{Q}$ [T. Matsui and H. Satz (1986)]

$$V(r) = -C_F\frac{\alpha_s}{r} \rightarrow -C_F\alpha_s\frac{e^{-m_D(T)r}}{r}$$

- At some T_d the bound state ceases to exist *due to Debye screening*

$\Rightarrow$ Suppressed yield of dilepton decay channel

$$R_{AA}(Q\bar{Q})$$

Potential models naturally follow [F. Karsch, M. T. Mehr and H. Satz (1988)]

- Medium effects entirely encoded in a T-depended potential
- Plug the T-potentials in a Schrödinger equation

  [A. Bazavov, P. Petreczky and A. Velytsky (2009)]
- Still lacking a first principle derivation of the potential from QCD

Heavy particles and energy scales

- We identify a particle of mass M **heavy** if

$$M \gg E \text{ (any other possible scale)}$$

- In particular the particle of mass M is *non-relativistic*

An EFT treatment is possible at a scale $M \gg \mu \gg E$

- Heavy-particle low-energy field H and light fields at the scale E

$$\mathcal{L}_{\rm EFT} = H^\dagger iD_0 H + [\text{operators of dimension } d > 4]\,\frac{1}{M^{d-4}} + \mathcal{L}_{light}$$

- $\mathcal{L}_{\rm EFT}$ as an expansion in $1/M$
- Contribution to physical observables are suppressed by E/M
- $\mathcal{L}_{\rm EFT}$ may be computed by imposing $E = 0$

- If we want to consider $Q\bar{Q}$, the form is $\mathcal{L}_{\rm EFT} \approx S^\dagger(i\partial_0 - h_s)S + \mathcal{L}_{light}$

EFT and thermal scales, E=T

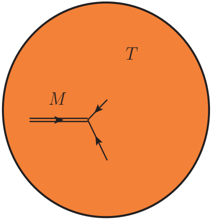
- A special case: heavy particle of mass M in a hot medium with $M \gg T$

An EFT treatment is possible at a scale $M \gg \mu \gg T$

- The EFT Lagrangian reads the same

$$\mathcal{L}_{\mathrm{EFT}} = H^\dagger iD_0 H + [\text{operators of dimension } d > 4] \frac{1}{M^{d-4}} + \mathcal{L}_{light}$$

1. $\mathcal{L}_{\mathrm{EFT}}$ as an expansion in $1/M$
2. Contribution to physical observable are suppressed by T/M
3. $\mathcal{L}_{\mathrm{EFT}}$ may be computed by imposing $T = 0$



$\Rightarrow$ **Wilson coefficients may be computed in vacuum**

Heavy quarkonia in a thermal medium

Application of the EFT formalism

- A heavy quarkonium, $Q\bar{Q}$, in a weakly coupled quark-gluon plasma
- Bound states scales ($v \sim \alpha_s$ for a Coulombic state):
  1. M (mass of the heavy quark)
  2. Mv (momentum transfer, $1/r$)
  3. Mv^2 (kinetic energy, binding energy, potential)

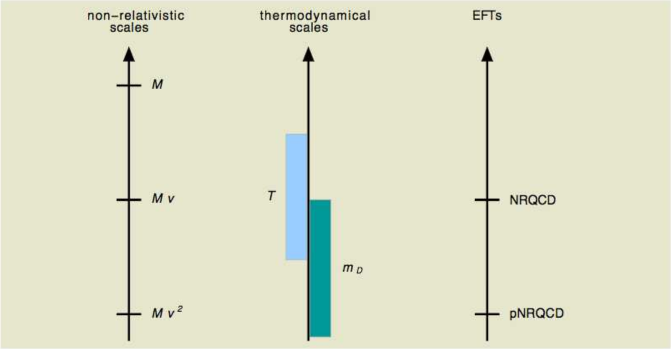
$$M \gg Mv \gg Mv^2$$

- Thermal scales:
  1. πT (temperature)
  2. m_D (Debye mass, screening of the chromoelectric interactions)

$$\pi T \gg m_D \sim gT \text{ (in weak coupling)}$$

- **Main goal**: Extract information at the scale Mv^2
- Study the possible interplay between the two sets of scales: $Q\bar{Q}$ in a hot medium

EFTs for QCD



- Energy modes $\sim M$ integrated out: NRQCD
  [Caswell and Lepage (1986), Bodwin, Braaten and Lepage]
- Energy modes $\sim Mv$ integrated out: pNRQCD
  [Soto and Pineda (1998), Brambilla, Pineda, Soto and Vairo (2000)]

EFTs and thermal scales

What do we want to study?

- We are interested in the $Q\bar{Q}$ potential and spectrum
- EFTs allow for a proper treatment of the thermal scales

$$V_s \to V_s(T, m_D)\,,\; E_{n\ell} \to E_{n\ell}(T, m_D)$$

EFTs describing a static $Q\bar{Q}$ pair for different hierarchies ($\Delta V \approx \frac{N_c \alpha}{2r}$)

[Brambilla, Ghiglieri, Petreczky and Vairo (2008)]

1. $1/r \gg \pi T \gg m_D \gg \Delta V$
2. $\pi T \gg 1/r \gtrsim m_D \gg \Delta V$

[Laine, Philipsen Romatschke and Tassler (2007)]

- $T_{\rm LHC} \approx 2T_c \approx 300$ MeV
- $1/r \simeq 1.5\,\text{GeV} > \pi T_{\rm LHC}$, for $\Upsilon(1S)$

Important result:

- The $Q\bar{Q}$ static potential exhibits a real and an **imaginary part**
- The imaginary part can be related to a thermal width for quarkonium,

$$\Gamma = -2\langle {\rm Im} V_s \rangle$$

Heavy quarkonium in a weakly coupled QGP

We have to choose a scenario for the scales:

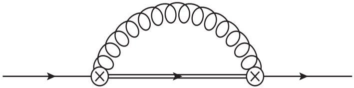
$$M \gg Mv \gg \pi T \gg Mv^2 \gg m_D$$

[Brambilla, Escobedo, Ghiglieri, Soto and Vairo (2010)], Possible candidates: $(\Upsilon(1S), \eta_b)$ [Vairo (2011)]

We start with pNRQCD

$$\mathcal{L}_{\text{pNRQCD}} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \sum_i \bar{q}_i i\not{D} q_i + \int d^3\mathbf{r}\,\mathrm{Tr}\left\{S^\dagger\left(i\partial_0 - h_s\right)S + O^\dagger\left(iD_0 - h_o\right)O\right.$$

$$\left. + V_A\left(O^\dagger \mathbf{r}\cdot g\mathbf{E}S \; + h.c.\right) + \frac{V_B}{2} O^\dagger \left\{\mathbf{r}\cdot g\mathbf{E}, O\right\} + \cdots\right.$$

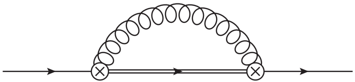


1. Singlet field S, Octet field O
2. $h_{s,o} = \frac{\mathbf{p}^2}{M} + V^{(0)}_{s,o} + \frac{V^{(1)}_{s,o}}{M} + \ldots$
3. $V^{(0)}_s = -C_F \frac{\alpha_s}{r}$, $V^{(0)}_o = \frac{1}{2N_c}\frac{\alpha_s}{r}$

Next step: integrate out the temperature ⇒ pNRQCD$_{\text{HTL}}$

- Yang-Mills Lagrangian becomes HTL Lagrangian
- Coulomb potential receives thermal corrections

Integrating out the temperature at LO



$$-ig^2 C_F \frac{r^i}{D-1}\mu^{4-D}\int \frac{d^D k}{(2\pi)^D}\frac{i}{E-h_o-k_0+i\eta}\left[k_0^2 D_{ii}^{(0)}(k_0,k)+k^2 D_{00}^{(0)}(k_0,k)\right] r^i$$

- Momentum region $k_0 \sim T$ and $k \sim T$. Since $T \gg (E-h_0)$

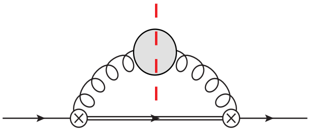
$$\frac{i}{E-h_o-k_0+i\eta} = \frac{i}{-k_0+i\eta} - i\frac{E-k_0}{(-k_0+i\eta)^2} + i\frac{(E-h_0)^2}{(-k_0+i\eta)^3} - i\frac{(E-k_0)^3}{(-k_0+i\eta)^4} + \cdots$$

- At leading order in α_s we obtain

$$\delta V_s(r) = \frac{\pi}{9} N_c C_F \alpha_s^2 T^2 r + \frac{2\pi}{3M} C_F \alpha_s T^2$$

$$+\frac{\alpha_s C_F I_T}{3\pi}\left[-\frac{N_c^3}{8}\frac{\alpha_s^3}{r} - (N_c^2+2N_c C_F)\frac{\alpha_s^2}{Mr^2} + 4(N_c-2C_F)\frac{\pi\alpha_s}{M^2}\delta^3(\mathbf{r}) + N_c\frac{\alpha_s}{M^2}\left\{\nabla_{\mathbf{r}}^2,\frac{1}{r}\right\}\right]$$

Integrating out the temperature at NLO



$$-ig^2 C_F \frac{r^i}{D-1}\mu^{4-D}\int \frac{d^D k}{(2\pi)^D}\frac{i}{E-h_o-k_0+i\eta}\left[k_0^2 D_{ii}(k_0,k)+k^2 D_{00}(k_0,k)\right] r^i$$

- Momentum region $k_0 \sim T$ and $k \sim T$. Since $T \gg (E-h_0)$

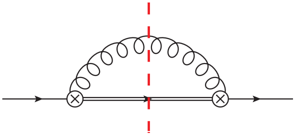
$$\frac{i}{E-h_o-k_0+i\eta}=\frac{i}{-k_0+i\eta}+\cdots\,,\qquad D_{00}^{R,A}(k_0,k)=-\frac{i}{k^4}\left[\Pi_{00}^{R,A}(k_0,k)\right]_{\text{therm.}}$$

- The potential exhibits a real and an imaginary part (Landau damping)

$$\delta V_s^{(2loops)}(r)=-\frac{3}{2}\zeta(3)C_F\frac{\alpha_s}{\pi}r^2 T m_D^2+\frac{2}{3}\zeta(3)N_c C_F\alpha_s^2 r^2 T^3$$

$$+i\left[\frac{C_F}{6}\alpha_s r^2 T m_D^2\left(-\frac{2}{\epsilon}+\gamma_E+\log\pi-\log\frac{T^2}{\mu^2}+\frac{2}{3}-4\log 2-2\frac{\zeta'(2)}{\zeta(2)}\right)+\frac{4\pi}{9}\log 2\, N_C C_F\alpha_s^2 r^2 T^3\right]$$

Contribution from the scale $E \sim mv^2$



- In pNRQCD$_{\mathrm{HTL}}$: momentum region $k_0 \sim E$ and $k \sim E$. Since $k \sim E \ll T$
  1. Octet propagator unexpanded
  2. $n_B(k) = \frac{T}{k} + \frac{1}{2} + \cdots$
  3. HTL popagators, expanded for $E \gg m_D$

- Summing all the contributions, the thermal width is finite $(n = 1, \ell = 0)$

$$\Gamma = \frac{1}{3} N_c^2 C_F \alpha_s^3 T + \frac{4}{3} C_F^2 \alpha_s^3 T (C_F + N_c) \qquad \text{(singlet-to-octet decay)}$$

$$+ \frac{2E_1 \alpha_s^3}{3} \left[4C_F^3 + 5N_c C_F^2 + 2N_c^2 C_F + \frac{N_c^3}{4} \right]$$

$$- a_0^2 \left[C_F \alpha_s T m_D^2 \left(2\gamma_E - \log \frac{T^2}{E_1^2} - 3 - \log 4 - 2\frac{\zeta'(2)}{\zeta(2)} \right) + \frac{8\pi}{3} \log 2 N_C C_F \alpha_s^2 T^3 \right]$$

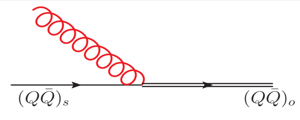
$$+ \frac{8}{3} C_F \alpha_s T m_D^2 a_0^2 \, I_{1,0}$$

[Brambilla, Escobedo, Ghiglieri, Soto and Vairo (2010)]

Take home message: weak coupling regime

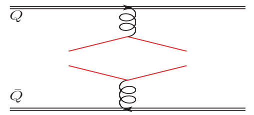
- EFTs allow a systematic derivation of $Q\bar{Q}$ potential and spectrum
- EFTs show the presence of a real and **imaginary** part also in the case quarkonium is below the dissociation regime
  1. Increasing masses of $Q\bar{Q}$ with $T^2 \Rightarrow$ more energetic $\mu^+\mu^-$ or $\gamma\gamma$
  2. Γ responsible for quarkonium states suppression in the medium

Gluo-dissociation



Singlet-octet thermal break up (EFT)

Inelastic parton scattering



Landau damping (EFT)

- Connections with phenomenological definitions of the dissociation mechanism and rigorous EFT derivation from QCD

[Brambilla, Escobedo, Ghiglieri, and Vairo (2013)]

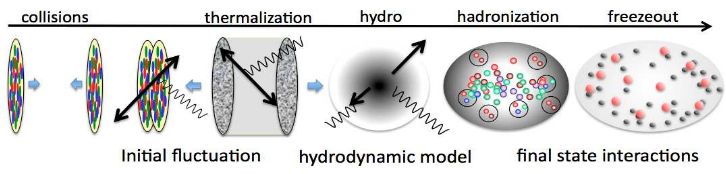
Relaxing some assumptions

Hierarchies of scales

- We often assumed a clear separation of the scales
  1. $1/r \gg \pi T \gg m_D \gg \Delta V$
  2. $M \gg Mv \gg \pi T \gg Mv^2 \gg m_D$
- For example one can relax $1/r \gg \pi T$ to $1/r \sim \pi T$

Isotropic quark-gluon plasma

- Isotropic distribution for partons in the medium
- The system may be more complex and there is at least one preferred direction (beam axis)



Relaxation of the hierarchies

- Relaxing some hierarchy may be important since:
  1. the QGP temperature **evolves** from the maximum T_{LHC} to $T_{\text{freeze-out}}$

     $\Rightarrow \pi T$ and m_D change with time

  2. The maximum temperature will increase in the next run:

     $\Rightarrow \pi\, 3T_c \approx 1/r$ for the $\Upsilon(1S)$

Work in progress...

- Solve the Schrödinger equation numerically with the potentials form EFTs
- One has to handle the imaginary part of the potential

  [M. Laine (2007)]

  [Strickland and Yager-Elorriaga (2010)]

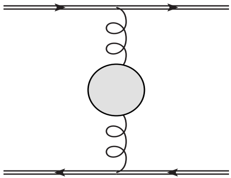
  [Petreczky, Miao and Mocsy (2010)]

  [Margotta,McCarty, McGahan, Strickland and Yager-Elorriaga (2011)]

Real part of the static potential: $1/r \sim \pi T \gg m_D \gg \Delta V$

Strategy of the calculation [M. A. Escobedo and J. Soto (2010)]

- We start with NRCQD at $T = 0$
- the $1/r$ and πT are integrated out at once $\rightarrow$ match with pNRQCD$_{\rm HTL}$



$$V_s(r) = C_F g^2 \mu^{4-D} \int \frac{d^{D-1}p}{(2\pi)^{D-1}} \left(e^{i\mathbf{p}\cdot\mathbf{r}} - 1\right) \Delta_{11}(p)$$

$$\Delta_{11} = \frac{1}{2}\left(\Delta_R + \Delta_A + \Delta_S\right), \; \Delta_R = \Delta_R^0 + \Delta_R^0 \Pi_R \Delta_R$$

Retarded gluon self energy in $p_0 \ll p \sim \pi T$

$$\mathrm{Re}\left[\Pi_{00}^R(p)\right] = \frac{g^2 T_F N_f}{\pi^2} \int_0^\infty dk_0 \, k_0 \, n_F(k_0) \left[2 + \left(\frac{p}{2k_0} - 2\frac{k_0}{p}\right) \ln\left|\frac{p-2k_0}{p+2k_0}\right|\right] \rightarrow \text{light-quarks}$$

$$+ \frac{g^2 N_c}{\pi^2} \int_0^\infty dk_0 \, k_0 \, n_B(k_0) \left[1 - \frac{p^2}{2k_0^2} + \left(-\frac{k_0}{p} + \frac{p}{2k_0} - \frac{p^3}{8k_0^3}\right) \ln\left|\frac{p-2k_0}{p+2k_0}\right|\right] \rightarrow \text{gluons}$$

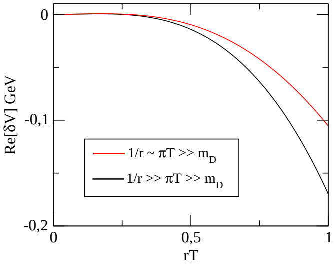
Result and check

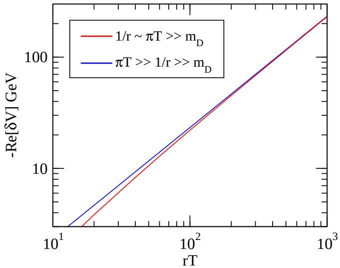
Light quarks contribution: $m_{D,q}^2 = T_F N_f g^2 T^2/3$

$$\mathrm{Re}\,[\delta V^q] = -\frac{C_F}{4}\,\alpha_s\, r\, m_{D,q}^2 - C_F\,\frac{3}{2\pi}\,\alpha_s\, r^2\, T\, m_{D,q}^2\,\zeta(3) + C_F\,\frac{\alpha_s\, m_{D,q}^2}{4\,\pi^2 r T^2}\,F_q(rT)$$

Gluon contribution $m_{D,g}^2 = N_c g^2 T^2/3$

$$\mathrm{Re}\,[\delta V^g] = -\frac{C_F}{4}\alpha_s\, r\, m_{D,g}^2 - C_F\,\frac{\alpha_s\, r^2\, T\, m_{D,g}^2}{\pi}\,\zeta(3) + C_F\,\frac{\alpha_s\, m_{D,g}^2}{8\,\pi^2 r T^2}\,F_g(rT)$$





Anisotropy in QGP

QGP is a rather complicated system...

- Longitudinal (beam axis) expansion is bigger than the radial expansion

1) Different temperatures
2) Anisotropic parton momenta

Local momentum anisotropy : ξ

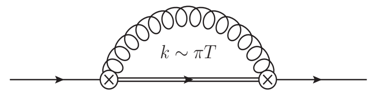
- The anisotropy already included in the $Q\bar{Q}$ potential for $\pi T \gg 1/r \sim m_D$
  [Y. Burnier, M.Laine and M. Vepsalainen (2009), A. Dimitru, Y. Gou, and M. Strickland (2009)]
- We can address within EFTs the case $M \gg Mv \gg \pi T \gg Mv^2 \gg m_D$

Modelling the anisotropy: $\vec{n} = (0,0,1)$

$$f(\mathbf{k},\mathbf{n}) = f_{iso}\left(\sqrt{\mathbf{k}^2 + \xi(\mathbf{k}\cdot\mathbf{n})^2}\right) = \left(e^{\frac{\sqrt{\mathbf{k}^2+\xi(\mathbf{k}\cdot\mathbf{n})^2}}{T}} - 1\right)^{-1}$$

We start with pNRQCD

$$\mathcal{L}_{\rm pNRQCD} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \sum_i \bar{q}_i i\not{D} q_i + \int d^3\mathbf{r}\,{\rm Tr}\left\{S^\dagger\left(i\partial_0 - h_s\right)S + O^\dagger\left(iD_0 - h_o\right)O\right.$$
$$\left. + V_A\left(O^\dagger \mathbf{r}\cdot g\mathbf{E}S\; + h.c.\right) + \frac{V_B}{2}O^\dagger\left\{\mathbf{r}\cdot g\mathbf{E}, O\right\} + \cdots\right.$$



- Match pNRQCD onto pNRQCD$_{\rm HTL}$
- T encoded in a redefined potential

$$\delta\Sigma(E) = -ig^2 C_F \frac{r^i}{D-1}\mu^{4-D}\int\frac{d^Dk}{(2\pi)^D}\frac{i}{E-h_o-k_0+i\eta}\,k_0^2 D^{(0)}_{ii}(k_0,k)r^i$$

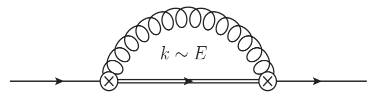
- Momentum region $k_0 \sim \pi T$ and $k \sim \pi T$. Since $\pi T \gg (E - h_0)$

$$\frac{i}{E-h_o-k_0+i\eta} = \frac{i}{-k_0+i\eta} - i\frac{E-k_0}{(-k_0+i\eta)^2} + \cdots$$

- At leading order in α_s we obtain

$$\delta V_s(T,\xi) = \frac{\pi\alpha_s C_F T^2}{3}\left(\frac{2}{m} + \frac{N_c\alpha_s r}{4} + \frac{N_c\alpha_s(\mathbf{r}\cdot\mathbf{n})^2}{4r}\right)\frac{\arctan\xi}{\xi}$$
$$+\frac{\pi N_c C_F \alpha_s^2 T^2}{12\xi r}\left(1 - \frac{\arctan\sqrt{\xi}}{\sqrt{\xi}}\right)\left(r^2 - 3(\mathbf{r}\cdot\mathbf{n})^2\right)$$

Strategy of the calculation: $M \gg Mv \gg \pi T \gg Mv^2 \gg m_D$



- Effect of the scale E within pNRQCD$_{\mathrm{HTL}}$
- Octet unexpanded,
- $f(\mathbf{k}) \simeq \frac{T}{k\sqrt{1+\xi\cos^2\theta}} + \dots$

- Thermal width from the scale Mv^2: $\Gamma = -2\,\langle n, l|\mathrm{Im}\,\delta\Sigma(E)|n, l\rangle$

$$\Gamma(T,\xi) = \left(\frac{1}{3}N_c^2 C_F \alpha_s^3 T + \frac{4}{3}\frac{C_F^2\alpha_s^3}{n^2}T(C_F+N_c)\right)\frac{\sinh^{-1}(\sqrt{\xi})}{\sqrt{\xi}}$$

$$+\left(\frac{1}{4}N_c^2 C_F \alpha_s^3 T + \frac{C_F^2\alpha_s^3}{n^2}T\left(C_F - \frac{N_c}{2}\right)\right)\frac{\left(1+\frac{2}{3}\xi\right)\sinh^{-1}(\sqrt{\xi})-\sqrt{\xi(1+\xi)}}{\sqrt{\xi^3}}\langle 2\,\ell\,0\,0|\ell\,0\rangle\langle 2\,\ell\,0\,m|\ell\,m\rangle$$

- Check with $\xi \to 0$, we recover the known result
  [N. Brambilla, M. A. Escobedo, J. Ghiglieri, J. Soto and A. Vairo (2010)]

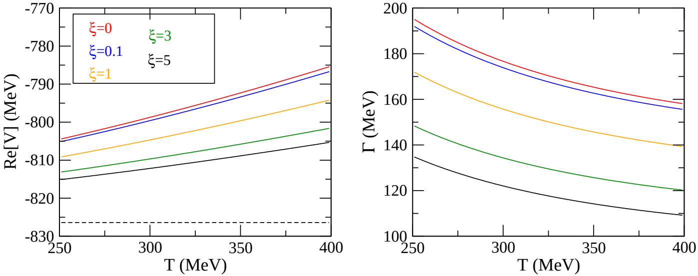
Check with known limits

Real part of the potential (for $\Upsilon(1S)$)

$$V_s(T,\xi) \to -c_F\frac{\alpha_s}{r} + \frac{\pi}{9}N_c C_F \alpha_s^2 T^2 r + \frac{2\pi}{3M}C_F\alpha_s T^2 + \mathcal{O}(\xi)$$

Thermal width

$$\Gamma(T,\xi) \to \frac{1}{3}N_c^2 C_F \alpha_s^3 T + \frac{4}{3}C_F^2\alpha_s^3 T(C_F+N_c) + \mathcal{O}(\xi)$$



Conclusions

- EFTs are a handy tool to study multi-scale problems
- Application to quarkonium physics at finite temperature

- EFTs allow the derivation of $Q\bar{Q}$ spectrum at $T \neq 0$ in a systematic way:
- Real part may differ from the Debye screened potential

  $(\pi T \gg 1/r)$: $V_s \sim -\alpha_s \frac{e^{-rm_d}}{r}$, $(1/r \gg \pi T)$: $V_s \sim -\frac{\alpha_s}{r} + \text{polinomial}(T)$

- Imaginary part of the potential → Thermal width for quarkonium:
  1. Dissociation by gluo-dissiciation (EFT: singlet-octet thermal break up)
  2. Dissociation by inelastic parton scattering (EFT: Landau damping)

Work in progress...

- Relaxation of some hierarchy of scales
- Inclusion of the anisotropy in the case $M \gg Mv \gg \pi T \gg Mv^2 \gg m_D$

Outlook

Improvements and generalization

- Non-perturbative determination of the potential by using EFTs
  1. Talk by A. Rothkopf
     [References therein]
  2. Talk by P. Petreczky
     [References therein]
- Extension to the case of a finite relative velocity between the $Q\bar{Q}$ bound state and the medium

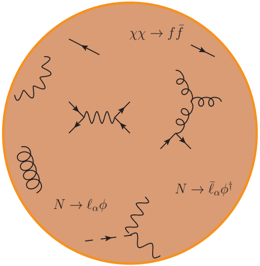
  [Escobedo, Soto and Mannarelli (2011)]

Outlook

- The EFT approach is quite general
- Treatment of different systems at finite T, if $M \gg T$

Examples:

- Heavy Majorana neutrinos decaying in a SM thermal bath, leptogenesis
- Dark matter particle production, being non-relativistic at decoupling



- $\Gamma_N = \Gamma_0 + \sum_n g_{\rm SM} \left(\frac{T}{M}\right)^n$
- $\epsilon = \epsilon_0 + \sum_n g_{\rm SM} \left(\frac{T}{M}\right)^n$

[Biondini, Brambilla, Escobedo, Vairo (2013)]

Back up

Example of the hierarchy: [Vairo (2011)]

$$m_b \approx 5\,\text{Gev} > m_b\alpha_s \approx 1.5\,\text{GeV} > \pi T \sim 1\,\text{GeV} > m_b\alpha_s^2 \approx 0.5\,\text{GeV} \geq m_D$$

Anisotropy

- More general form

$$f(\mathbf{k},\mathbf{n}) = \frac{A(\xi)}{e^{\frac{\sqrt{\mathbf{k}^2+\xi(\mathbf{k}\cdot\mathbf{n})^2}}{TB(\xi)}} - 1}$$

- In particular limits

$$\xi = \frac{10}{T\tau}\frac{\eta}{s}$$

where η is the shear viscosity, and $1/\tau$ the expansion rate

[Romatschke and Strickland (2003)]

[Dumitru, Gou and Strickland (2009)]

[Y. Burnier, M.Laine and M. Vepsalainen (2009)]